University of California, Santa Barbara
Department of Computer Science

CS 40 Final Exam

Winter 2024

Full Name: ____

Perm Number: ____

UCSB Email: ____________________________________ @ucsb.edu

Instructions:

- This exam is closed book, closed notes. No electronics are allowed in the exam. However, hard copy of lecture handouts/slides for any three weeks is allowed.
- Do not detach any pages from the booklet and return the cheat sheet with your exam at the end.
- Print your full name clearly on all pages. Failure to include your name or if your name is unreadable on any page will result in the loss of 0.5 points.
- You have 150 minutes to complete this exam.
- Write all your answers in the provided boxes in pen or dark pencil. Answers written outside boxes will NOT be considered. Ensure your writing is clear and legible; unreadable work cannot be evaluated or graded.
- $\forall$ questions $\in$ this exam: You receive full credit $\rightarrow$ You show all your work.

By signing your name below, you are asserting that all work on this exam is yours alone, and that you will not provide any information to anyone else taking the exam. In addition, you are agreeing that you will not discuss any part of this exam with anyone who is not currently taking the exam in this room. This includes posting any information about this exam on Piazza or any other social media. Discussing any aspect of this exam with anyone outside of this room constitutes a violation of the academic integrity agreement for CS 40 .

Signature: ____

DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO.

Good Luck!

Content here will NOT be graded

1. [15 pts] Translate the following sentences into equivalent logical statements using the following propositional variables:

$$p = \text{I will pass this class}$$
$$q = \text{I study for the exam}$$
$$r = \text{I will stay up all night}$$

(a) [5 pts] I will pass this class only if I both study for the exam and don't stay up all night.

(b) [5 pts] I will stay up all night if and only if I both study for the exam and don't pass the class.

(c) [5 pts] Not staying up all night and studying for the exam is sufficient for passing the class.

Content here will NOT be graded

2. [15 pts] Suppose $P(x)$ is a predicate over a domain D. Translate the following statements into symbolic form using quantifiers.

   (a) [5 pts] There are **exactly** two elements in D where the predicate P evaluates to true.

   (b) [5 pts] There are **at least** two elements in D where the predicate P evaluates to true.

   (c) [5 pts] There are **at most** two elements in D where the predicate P evaluates to true.

Content here will NOT be graded

3. [10 pts] Use laws of propositional logic to prove that:

$$p \leftrightarrow q \equiv (\neg p \wedge \neg q) \vee (q \wedge p)$$

Content here will NOT be graded

4. [15 pts] Write each of the following set descriptions in set builder notation with logical and mathematical symbols. Do not use descriptions with words.

   (a) [5 pts] The set of all natural numbers that are perfect squares.

   (b) [5 pts] The set of all integers that are even and divisible by 7.

   (c) [5 pts] The set of all two-tuples where both elements are integers and the second element is 3 times bigger than the first element.

Content here will NOT be graded

5. [20 pts] In parts (a)–(d), mappings are given between the sets S, T, and R to define different functions.

Using the following set definitions, determine if each function is *well-defined* or not. If a function is well-defined, also indicate if it is *surjective* (onto), *injective* (one-to-one), or neither. You do not need to prove your answer, but do briefly justify your reasoning in the space provided.

$$S = \{1, 2, 3, 4, 5\}, \quad T = \{3, 4, 5, 6, 7\}, \quad R = \{3, 4, 5\}$$

(a) [5 pts] $f : S \to T$, $f = \{(2, 3), (5, 5), (2, 4), (1, 3), (1, 7)\}$

(b) [5 pts] $h : S \to R$, $h = \{(4, 3), (2, 3), (3, 3), (1, 4), (5, 5)\}$

(c) [5 pts] $g : R \to S$, $g = \{(3, 5), (4, 3), (5, 3)\}$

(d) [5 pts] $j : T \to S$, $j = \{(7, 2), (3, 3), (4, 5), (6, 4), (5, 1)\}$

Content here will NOT be graded

6. [10 pts] Prove distributive property for sets A, B, C: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   Do not use the identity. Hint: How do you show two sets are equal?

Content here will NOT be graded

7. [10 pts] Calculate $(9^{120} + 8^{2024}) \bmod 7$ using congruence theorems.

8. [10 pts] Let B be the set of all strings of the form $b^n a b^n$, where $n \in \mathbb{Z}^{\geq 0}$, a is a specific string that is not the empty string, and b^2 represents the string bb, b^3 represents the string bbb, and so forth. Provide a recursive definition for B without using the b^n notation.

Content here will NOT be graded

9. [25 pts] Let S be a subset of $\mathbb{Z}$ defined recursively as follows:

$$\begin{array}{ll} \text{Basis Step:} & 5 \in S \\ \text{Recursive Step:} & x \in S \to (x - 15) \in S \land x^2 \in S \end{array}$$

Prove that every element in S is divisible by 5.

Content here will NOT be graded

10. [5 pts] Prove or disprove the following claim:

$$\forall a, d \in \mathbb{N} : (a \mid d^2) \;\rightarrow\; (a \mid d)$$

Content here will NOT be graded

11. [25 pts] Prove by induction that for all natural numbers n where $n > 3$, $n! > 2n$.

Content here will NOT be graded

12. [25 pts] Recall that $\mathbb{R}^{\geq 0}$ represents all non-negative real numbers. Prove by contradiction that

$$\forall x, y \in \mathbb{R}^{\geq 0} : \sqrt{xy} \leq \frac{x+y}{2}$$

Content here will NOT be graded

13. [15 pts] In CS40 office hours, there is 1 TA, 1 ULA, and 3 students present. There are 5 seats available in the room. Use this setup to answer the following questions, you may leave your answer in a non-reduced form (as an equation).

    (a) [5 pts] How many different ways can all the people be seated?

    (b) [10 pts] If we imagine each seat is numbered 1 - 5, how many ways are there for the people to sit if the TA has to be sitting at a seat with a smaller number than the ULA?

– The End –

Instructor Use Only
Do not fill, or you will lose points.
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