This page has some useful notation that will be used throughout the course. Find the definitions for each of these terms by looking in the Appendix of the course textbook (zybook).

Term	Notation Example(s)	We say in English
n-tuple	(x_1, x_2, x_3)	The 3-tuple of x_1, x_2, and x_3
	$(3, 4)$	The 2-tuple or **ordered pair** of 3 and 4
sequence	$x_1, \ldots, x_n$	A sequence x_1 to x_n
	$x_1, \ldots, x_n$ where $n = 0$	An empty sequence
	$x_1, \ldots, x_n$ where $n = 1$	A sequence containing just x_1
	$x_1, \ldots, x_n$ where $n = 2$	A sequence containing just x_1 and x_2 in order
	x_1, x_2	A sequence containing just x_1 and x_2 in order
set		Unordered collection of objects. The set of ...
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. **Note**: we use the convention that 0 is a natural number.
roster method	$\{43, 7, 9\}$	The set whose elements are 43, 7, and 9
	$\{9, \mathbb{N}\}$	The set whose elements are 9 and $\mathbb{N}$
set builder notation	$\{x \in \mathbb{Z} \mid x > 0\}$	The set of all x from the integers such that x is greater than 0
	$\{3x \mid x \in \mathbb{Z}\}$	The set of all integer multiples of 3 **Note**: we use the convention that writing two numbers next to each other means multiplication.
function definition	$f(x) = x + 4$	Define f of x to be $x + 4$
function application	$f(7)$	f of 7 **or** f applied to 7 **or** the image of 7 under f
	$f(z)$	f of z **or** f applied to z **or** the image of z under f
	$f(g(z))$	f of g of z **or** f applied to the result of g applied to z
absolute value	$\lvert -3 \rvert$	The absolute value of -3
square root	$\sqrt{9}$	The non-negative square root of 9
summation notation	$\sum_{i=1}^{n} i$	The sum of the integers from 1 to n, inclusive
	$\sum_{i=1}^{n} i^2 - 1$	The sum of $i^2 - 1$ (i squared minus 1) for each i from 1 to n, inclusive
quotient, integer division	n **div** m	The (integer) quotient upon dividing n by m; informally: divide and then drop the fractional part
modulo, remainder	n **mod** m	The remainder upon dividing n by m

Themes for CS 40

- Technical skepticism
- Multiple representations

Recurring examples in CS 40

- Clustering and recommendation systems (machine learning, Netflix)
- Genomics and bioinformatics (DNA and RNA)
- Codes and information (secret message sharing and error correction)
- "Under the hood" of computers (number representation, data structures)

Week 1 Part A highlights

- Use and apply definitions and notation
- Explore mathematical definitions related to a specific application (Netflix)
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Prove propositional equivalences using truth tables
- Prove propositional equivalences using other known equivalences, e.g.
  - DeMorgan's laws
  - Double negation laws
  - Distributive laws, etc.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.

What data should we encode about each Netflix account holder to help us make effective recommendations?

In machine learning, clustering can be used to group similar data for prediction and recommendation. For example, each Netflix user's viewing history can be represented as a n-tuple indicating their preferences about movies in the database, where n is the number of movies in the database. People with similar tastes in movies can then be clustered to provide recommendations of movies for one another. Mathematically, clustering is based on a notion of distance between pairs of n-tuples.

In the table below, each row represents a user's ratings of movies: ✓ (check) indicates the person liked the movie, ✗ (x) that they didn't, and • (dot) that they didn't rate it one way or another (neutral rating or didn't watch).

Person	Fyre	Frozen II	Picard	Ratings written as a 3-tuple
P_1	✗	•	✓	$(-1, 0, 1)$
P_2	✓	✓	✗	$(1, 1, -1)$
P_3	✓	✓	✓	$(1, 1, 1)$
P_4	•	✗	✓	

Which of P_1, P_2, P_3 has movie preferences most similar to P_4?

One approach to answer this question: use **functions** to define distance between user preferences.

Define the following functions whose inputs are ordered pairs of 3-tuples each of whose components comes from the set $\{-1, 0, 1\}$	
$d_1(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \sum_{i=1}^{3} (\mid x_i - y_i \mid)$	$d_2(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \sqrt{\sum_{i=1}^{3} (x_i - y_i)^2}$

$d_1(P_4, P_1)$	$d_1(P_4, P_2)$	$d_1(P_4, P_3)$
$d_2(P_4, P_1)$	$d_2(P_4, P_2)$	$d_2(P_4, P_3)$

Extra example: A new movie is released, and P_1 and P_2 watch it before P_3, and give it ratings; P_1 gives ✓ and P_2 gives ✗. Should this movie be recommended to P_3? Why or why not?

Extra example: Define the new functions that would be used to compare the 4-tuples of ratings encoding movie preferences now that there are four movies in the database.

Proposition	Declarative sentence that is true or false (not both).
Propositional variable	Variable that represents a proposition.
Compound proposition	New propositions formed from existing propositions (potentially) using logical operators.
Truth table	Table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Note: A propositional variable is one example of a compound proposition.

Logical operators aka propositional connectives

Conjunction	AND	$\wedge$	`\land`	2 inputs	Evaluates to T when **both** inputs are T
Exclusive or	XOR	$\oplus$	`\oplus`	2 inputs	Evaluates to T when **exactly one** of inputs is T
Disjunction	OR	$\vee$	`\lor`	2 inputs	Evaluates to T when **at least one** of inputs is T
Negation	NOT	$\neg$	`\lnot`	1 input	Evaluates to T when its input is F

Input		Output		
		Conjunction	**Exclusive or**	**Disjunction**
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Input	Output
	Negation
p	$\neg p$
T	F
F	T

Input			Output		
p	q	r		$(p \wedge q) \oplus (\ (p \oplus q) \wedge r\ )$	$(p \wedge q) \vee (\ (p \oplus q) \wedge r\ )$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Logical equivalence	Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.
Tautology	A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.
Contradiction	A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.
Contingency	A compound proposition that is neither a tautology nor a contradiction.

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					

(Some) logical equivalences (zybook, Chapter 1.5, Table 1.5.1):

Laws of propositional logic.

$p \vee q \equiv q \vee p$		$p \wedge q \equiv q \wedge p$		**Commutativity** Ordering of terms
$(p \vee q) \vee r \equiv p \vee (q \vee r)$		$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$		**Associativity** Grouping of terms
$p \wedge F \equiv F$	$p \vee T \equiv T$	$p \wedge T \equiv p$	$p \vee F \equiv p$	**Absorption** aka short circuit evaluation
$\neg(p \wedge q) \equiv \neg p \vee \neg q$		$\neg(p \vee q) \equiv \neg p \wedge \neg q$		**DeMorgan's Laws**

Can replace p and q with any compound proposition

Given an compound proposition, we can use

- Truth tables
- Logical equivalences

to compute its truth value for specific input values.

Now, given a truth table, how do we find a compound proposition that has the specified output values?

Application: design a circuit given a desired input-output relationship.

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

Input			Output
p	q	r	?
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

A compound proposition that gives output $mystery_1$ is: ______________________________

A compound proposition that gives output $mystery_2$ is: ______________________________

Definition A compound proposition is in **disjunctive normal form** (DNF) means that it is an OR of ANDs of variables and their negations.

Definition A compound proposition is in **conjunctive normal form** (CNF) means that it is an AND of ORs of variables and their negations.

Extra example: A compound proposition that gives output ? is:

The only way to make the conditional statement $p \to q$ false is to ____________________________________

The **hypothesis** of $p \to q$ is ____________ The **premise** of $p \to q$ is ____________

The **conclusion** of $p \to q$ is ____________ The **consequent** of $p \to q$ is ____________

Input		Output				
		Conjunction	Exclusive or	Disjunction	Conditional	Biconditional
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	F	T	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

Examples

$p \to q \equiv \neg p \vee q$ because ____

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because ____

$\neg(p \leftrightarrow q) \equiv p \oplus q$ because ____

$p \to q$ is not logically equivalent to $q \to p$ because ____

$p \leftrightarrow q \equiv q \leftrightarrow p$ because ____

The **converse** of $p \to q$ is __________________

The **inverse** of $p \to q$ is __________________ Which of these is logically equivalent to $p \to q$?

The **contrapositive** of $p \to q$ is __________________

Translation: Express each of the following sentences as compound propositions, using the given propositions.

"A sufficient condition for the warranty to be good is that you bought the computer less than a year ago"

w is "the warranty is good"
b is "you bought the computer less than a year ago"

"Whenever the message was sent from an unknown system, it is scanned for viruses."

s is "The message is scanned for viruses"
u is "The message was sent from an unknown system"

"I will complete my to-do list only if I put a reminder in my calendar"

r is "I will complete my to-do list"
c is "I put a reminder in my calendar"

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