Solutions To HW1

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Solution To Question 1

(a) : *How tall is Stroke Tower* is not a proposition as a question is neither true or false.

(b) : *Storke Tower is as tall as 33 people stacked on top of each other* is a proposition.

The negation is : **Storke Tower is not as tall as 33 people stacked on top of each other.**

Solution To Question 2

(a) : $\neg n$

(b) : $t \wedge m$

(c) : $n \to \neg m$

(d) : $\neg m \to (n \vee t)$

Solution To Question 3

(a) If 51 is an even number, then the sky is green.

- P: "51 is an even number." This is False because 51 is an odd number.

- Q: "The sky is green." This is also False because, typically, the sky is blue.

Since P is false and Q is also false, the statement $P \to Q$ is True.

(b) If 51 is an odd number, then the sky is blue.

- P: "51 is an odd number." This is true.

- Q: "The sky is blue." This is true.

Since both P and Q are true, the statement $P \to Q$ is True.

Solution To Question 4

(a) It either rains or it pours.

$$i \vee j$$

(b) It is raining on our parade.

$$i \wedge l$$

(c) We are having this parade, come rain or shine.

$$l$$

Solution To Question 5

(a) Rafael can ride the elephant only if he is not afraid of heights.

Equivalent statement:
> If Rafael rides the elephant, then he is not afraid of heights.

(b) Rafael can ride the elephant if he is not afraid of heights.

Equivalent statement:
> If Rafael is not afraid of heights, then he rides the elephant.

(c) Being able to swim is a necessary skill needed for Tyra to learn to surf.

Equivalent statement:
> If Tyra learns to surf, then she is able to swim.

(d) Being able to swim is a sufficient skill needed for Tyra to learn to surf.

Equivalent statement:
> If Tyra is able to swim, then she learns to surf.

Solution To Question 6

(a) $\neg p \to q \equiv \neg q \to p$

Proof:

$$
\begin{aligned}
\neg p \to q \quad &\text{(Given)} \\
\neg\neg p \vee q \quad &\text{(Conditional Identities)} \\
p \vee q \quad &\text{(Double Negative Law)} \\
q \vee p \quad &\text{(Commutative Laws)} \\
\neg\neg q \vee p \quad &\text{(Double Negative Law)} \\
\neg q \to p \quad &\text{(Conditional Identities)}
\end{aligned}
$$

(b) $\neg p \to (q \wedge \neg q) \equiv p$

Proof:

$$
\begin{aligned}
\neg p \to (q \wedge \neg q) \quad &\text{(Given)} \\
\neg p \to F \quad &\text{(Complement Laws)} \\
\neg\neg p \vee F \quad &\text{(Conditional Identities)} \\
p \vee F \quad &\text{(Double Negative Laws)} \\
p \quad &\text{(Identity Laws)}
\end{aligned}
$$

(c) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \equiv p \wedge \neg r$

Proof:

$$
\begin{aligned}
(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \quad &\text{(Given)} \\
(\neg r \wedge p \wedge q) \vee (\neg r \wedge p \wedge \neg q) \quad &\text{(Commutative Laws)} \\
\neg r \wedge ((p \wedge q) \vee (p \wedge \neg q)) \quad &\text{(Distributive Laws)} \\
\neg r \wedge (p \wedge (q \vee \neg q)) \quad &\text{(Distributive Laws)} \\
\neg r \wedge (p \wedge T) \quad &\text{(Complement Laws)} \\
\neg r \wedge p \quad &\text{(Domination Laws)} \\
p \wedge \neg r \quad &\text{(Commutative Laws)}
\end{aligned}
$$

Solution To Question 7

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the truth table, we prove that this is a tautology.

Solution To Question 8

Given the statements:

- Andromeda: "Clytemnestra is the knave."
- Brunhilda: "Andromeda is the knight."
- Clytemnestra: "I am the spy."

Case 1: Andromeda is the knight

If Andromeda is the knight, she tells the truth. Therefore, her statement "Clytemnestra is the knave" is true. Thus:

- Clytemnestra is the knave.
- Brunhilda must be the spy.

We verify the statements:

- Brunhilda says, "Andromeda is the knight." As the spy, she can tell the truth.
- Clytemnestra says, "I am the spy." As the knave, she lies.

Case 2: Brunhilda is the knight

If Brunhilda is the knight, she tells the truth. Therefore, her statement "Andromeda is the knight" must be true. This is a contradiction.

Case 3: Clytemnestra is the knight

If Clytemnestra is the knight, she tells the truth. Therefore, her statement "I am the spy" must be true. This is a contradiction.

Conclusion

- Andromeda is the knight.
- Brunhilda is the spy.
- Clytemnestra is the knave.

Solution To Question 9

$$Q = (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$$

Solution To Question 10

(a) An element from the set $B \times A \times C$

$$(\text{foam}, \text{tall}, \text{non-fat})$$

(b) The set $B \times C$ in roster notation

$$B \times C = \{(\text{foam}, \text{non-fat}), (\text{foam}, \text{whole}), (\text{no-foam}, \text{non-fat}), (\text{no-foam}, \text{whole})\}$$

Solution To Question 11

(a) Negation of $\forall x \exists y (P(x,y) \wedge Q(x,y))$

$$\neg(\forall x \exists y (P(x,y) \wedge Q(x,y)))$$
$$\exists x \neg (\exists y (P(x,y) \wedge Q(x,y)))$$
$$\exists x \forall y \neg (P(x,y) \wedge Q(x,y))$$
$$\exists x \forall y (\neg P(x,y) \vee \neg Q(x,y))$$

(b) Negation of $\exists x \forall y (P(x,y) \rightarrow Q(x,y))$

$$\neg(\exists x \forall y (P(x,y) \rightarrow Q(x,y)))$$
$$\forall x \neg (\forall y (P(x,y) \rightarrow Q(x,y)))$$
$$\forall x \exists y \neg (P(x,y) \rightarrow Q(x,y))$$
$$\forall x \exists y (P(x,y) \wedge \neg Q(x,y))$$

Solution To Question 12

- $P(x)$: x showed up with a pencil. - $C(x)$: x showed up with a calculator.

(a) At least one of the students showed up with a pencil.

$$\exists x\, P(x)$$

Negation:

$$\neg(\exists x\, P(x))$$
$$\forall x\, \neg P(x)$$

English translation: *Every student showed up without a pencil.*

(b) Every student showed up with a pencil or a calculator (or both).

$$\forall x \, (P(x) \lor C(x))$$

Negation:

$$\neg(\forall x \, (P(x) \lor C(x)))$$
$$\exists x \, \neg(P(x) \lor C(x))$$
$$\exists x \, (\neg P(x) \land \neg C(x))$$

English translation: *There is at least a student who showed up without a pencil and without a calculator.*

(c) Every student who showed up with a calculator also had a pencil.

$$\forall x \, (C(x) \to P(x))$$

Negation:

$$\neg(\forall x \, (C(x) \to P(x)))$$
$$\exists x \, \neg(C(x) \to P(x))$$
$$C(x) \to P(x) \equiv \neg C(x) \lor P(x)$$
$$\exists x \, (\neg(\neg C(x) \lor P(x)))$$
$$\exists x \, (C(x) \land \neg P(x))$$

English translation: *There is at least a student who showed up with a calculator and without a pencil.*

(d) There is a student who showed up with both a pencil and a calculator.

$$\exists x \, (P(x) \land C(x))$$

Negation:

$$\neg(\exists x \, (P(x) \land C(x)))$$
$$\forall x \, \neg(P(x) \land C(x))$$
$$\forall x \, (\neg P(x) \lor \neg C(x))$$

English translation: *Every student showed up without a pencil or without a calculator (or both).*