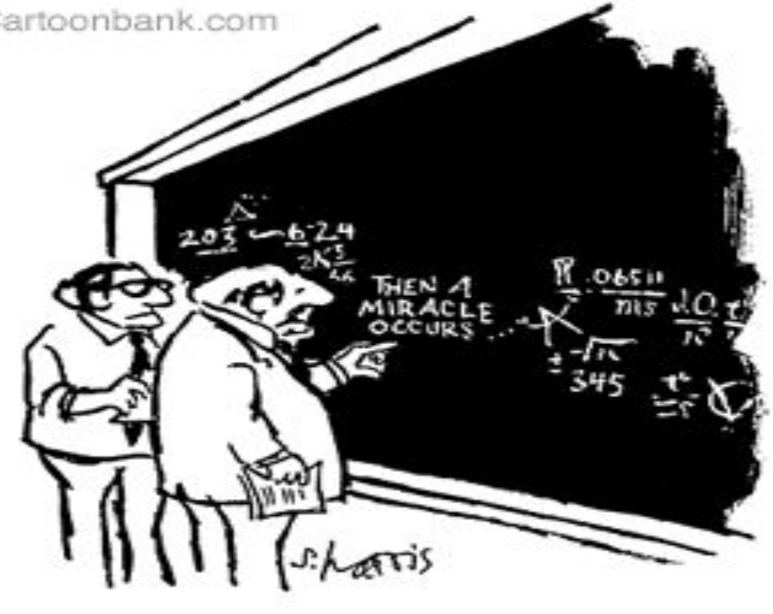
CS 40
FOUNDATIONS OF CS

Summer 2024
Session A

Adapted for CS40 by Diba Mirza

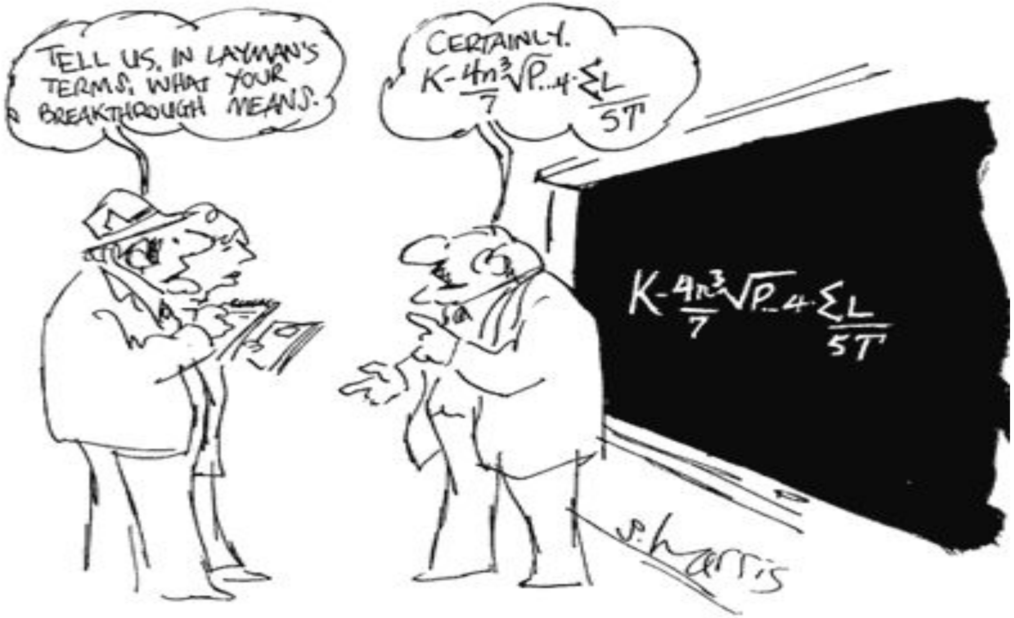
CS40 Learning goals

Technical Skepticism



"I think you should be more explicit here in step two."

Multiple Representations



About the team

Instructor: Vaishali Surianarayanan (she/her)

TA: Ajaykrishnan ES

ULA: Hao Yi

- Communication with staff via **Piazza**
- Lectures and sections will be in person
- Office hours posted on Gauchospace

** Ask questions about class examples, assignment questions, or other CS topics. **

Assignment Schedule

Week	M	T	W	R	F	S	S
1							
2			HW1(collab) Released 04/06		Zybook-Ch 1&2 Due 04/08		
3		HW1(collab) Due 04/12	HW2(individual) Released 04/13	Quiz 1 04/14			
4		HW2(individual) Due 04/19	HW3(collab) Released 04/20		Zybook-Ch 3, 4 Due 04/22		
5		HW3(collab) Due 04/26	HW4(individual) Released 04/27	Quiz 2 04/28			
6		HW4(individual) Due 05/03	HW5(collab) Released 05/04		Zybook-Ch 5, 6 Due 05/06		
7		HW5(collab) Due 05/10	HW6(individual) Released 05/11	Quiz 3 05/12			
8		HW6(individual) Due 05/17	HW7(collab) Released 05/18		Zybook-Ch 7, 8 Due 05/20		
9		HW7(collab) Due 05/24	HW8(individual) Released 05/25	Quiz 4 05/26			
10		HW6(individual) Due 05/31			Zybook-Ch 9, 10 Due 06/03		
11		**Final exam:** 06/07 4p - 7p, NH 1006					

All homeworks are due at 11:59p on Tuesdays
All quizzes will be online via Gradscope for 30 - 45 mins, open Thursdays 5p - 10p
All zybook activites are due at 11:59p on Fridays
See the syllabus for the academic integrity policy for each type of assignment

Zybook:	15%
Collaborative HWs:	15%
Individual HWs:	20%
Quizzes:	15%
Final:	35%

Attributions

Professor Mia Minnes: http://cseweb.ucsd.edu/~minnes

- Many years of experience teaching *Discrete Mathematics*
- Created **awesome** material with Professor Joe Politz that tie discrete math concepts to CS problems and applications
- Has kindly agreed to share her materials with us for this class

You will see attributions to Professors Minnes and Politz throughout our course material, particularly on slides & weekly handouts.

Tuesday's learning goals

- Practice with some **definitions** and **notation**
- Explore mathematical **definitions** related to a specific **application** (Netflix)
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.

n-tuples, preferences, and Netflix

NETFLIX

→ preferences

→ categorize movies

→ watch history

Multiple Representations

What data should we encode about each Netflix account holder to help us make effective recommendations?

n-tuples, preferences, and Netflix

n-tuple	(x_1, x_2, x_3)	The 3-tuple of x_1, x_2, and x_3
	$(3, 4)$	The 2-tuple or ordered pair of 3 and 4

Person	Fyre	Frozen II	Picard
P_1	✗	•	✓
P_2	✓	✓	✗
P_3	✓	✓	✓
P_4	•	✗	✓

✗ Did not like -1
• No preference 0
✓ Liked 1

n-tuples, preferences, and Netflix

n-tuple	(x_1, x_2, x_3)	The 3-tuple of x_1, x_2, and x_3
	$(3, 4)$	The 2-tuple or ordered pair of 3 and 4

Person	Fyre	Frozen II	Picard	Ratings written as a 3-tuple
P_1	✗	•	✓	$(-1, 0, 1)$
P_2	✓	✓	✗	$(1, 1, -1)$
P_3	✓	✓	✓	$(1, 1, 1)$
P_4	•	✗	✓	$\boxed{(0, -1, 1)}$

✗ Did not like: represent with -1
• No preference: represent with 0
✓ Liked: represent with 1

Similarity check

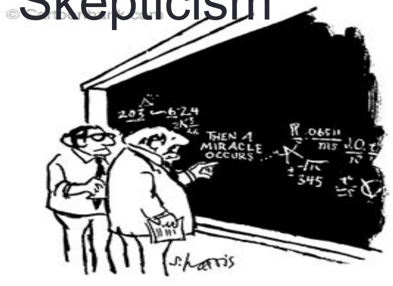
How similar are people's preferences?

Which of P_1, P_2, P_3 has movie preferences most similar to P_4?

A: P1
B: P2
C: P3
D: There is a tie

	Person	Fyre	Frozen II	Picard
60%	P_1	✗ −1	• 0	✓ 1
← 1	P_2	✓	✓	✗
	P_3	✓	✓	✓
	P_4	• 0	✗ −1	✓ 1

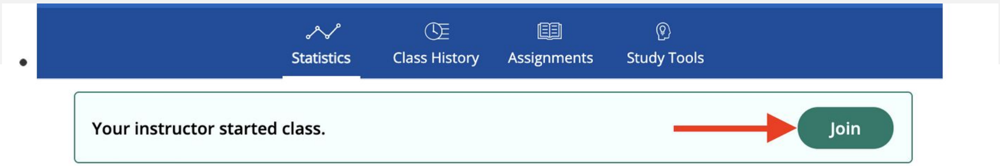
Technical Skepticism



"I think you should be more explicit here in step two."

Check into our class on iclicker cloud:

1. Login: https://app.reef-education.com/#/login
2. Join the class: CS40: Foundations of CS



One approach: functions

function definition	$f(x) = x + 4$	Define f of x to be $x + 4$
function application	$f(7)$	f of 7 **or** f applied to 7 **or** the image of 7 under f
	$f(z)$	f of z **or** f applied to z **or** the image of z under f
	$f(g(z))$	f of g of z **or** f applied to the result of g applied to z

Page 1 of worksheet:

This page has some useful notation that will be used throughout the course. Find the definitions for each of these terms by looking in the Appendix of the course textbook (zybook).

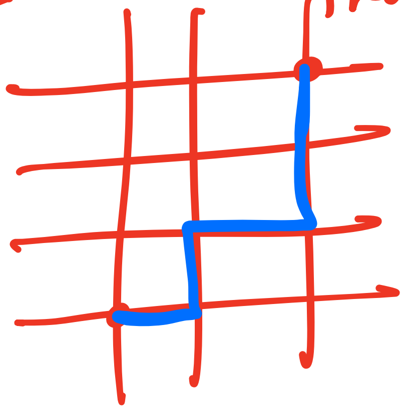
Define the following functions whose inputs are ordered pairs of 3-tuples each of whose components comes from the set $\{-1, 0, 1\}$

$$d_1(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \underbrace{\sum_{i=1}^{3} (|\ \underline{x_i - y_i}\ |)}$$

Person	Fyre	Frozen II	Picard	Ratings written as a 3-tuple
P_1	✗	•	✓	$(-1, 0, 1)$
P_2	✓	✓	✗	$(1, 1, -1)$
P_3	✓	✓	✓	$(1, 1, 1)$
P_4	•	✗	✓	

$\sum |x_i - y_i|$ Manhattan distance

P_4 is more similar to P_1 than P_2



5 > 2

$d_1(\underline{P_4}, \underline{P_1})$ = (-1,0,1) (0,-1,1)

d(,) = 2

takes as i/p two tuples

$d(P_4, P_2)$ = (1,1,-1) (0,-1,1) = 5

roster method	$\{43, 7, 9\}$	The set whose elements are 43, 7, and 9

Define the following functions whose inputs are ordered pairs of 3-tuples each of whose components comes from the set $\{-1, 0, 1\}$

Person	Fyre	Frozen II	Picard	Ratings written as a 3-tuple
P_1	✗	•	✓	$(-1, 0, 1)$
P_2	✓	✓	✗	$(1, 1, -1)$
P_3	✓	✓	✓	$(1, 1, 1)$
P_4	•	✗	✓	

$$\underbrace{d_2(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \sqrt{\sum_{i=1}^{3}(x_i - y_i)^2}}_{\text{Euclidean dis}}$$

$$d_2(P_4, P_1) = (-1, 0, 1)\ (0, -1, 1)$$

$$= \sqrt{1 + 1 + 0}$$

$$= \sqrt{2}$$

Define the following functions whose inputs are ordered pairs of 3-tuples each of whose components comes from the set $\{-1, 0, 1\}$

$d_1(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \sum_{i=1}^{3} (\mid x_i - y_i \mid)$	$d_2(\ (x_1, x_2, x_3), (y_1, y_2, y_3)\ ) = \sqrt{\sum_{i=1}^{3} (x_i - y_i)^2}$

least $\therefore$ most similar

$d_1(P_4, P_1)$	$d_1(P_4, P_2)$	$d_1(P_4, P_3)$
$d_2(P_4, P_1)$	$d_2(P_4, P_2)$	$d_2(P_4, P_3)$

Logic

- Precisely express true facts and invariant statements.
- Identify valid arguments (patterns of reasoning) that could be used in proofs.

Definitions

zybook 1.1, Rosen pp. 2-4,

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Which of the following are propositions?

- **Proposition**: declarative sentence that is T or F (not both)

- I have a pet turtle. T
- My pet turtle is purple. T
- Do you have a pet turtle? F
- Don't paint on my pet turtle. F
- I have a pet turtle and a pet elephant. T

$x > 3$ F $\quad$ $5 > 3$ T

Logical operators aka propositional connectives

Input		Output
p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

"Both p and q are true"

Input		Output
p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

"At least one of p and q is true"

Fill in the output for disjunction (reading top to bottom):

A. T-T-T-F
B. T-F-T-F
C. F-F-F-T
D. T-F-F-T
E. None of the above

Conjunction

Disjunction

Logical operators aka propositional connectives

Input		Output
p	q	p ^ q
T	T	T
T	F	F
F	T	F
F	F	F

"Both p and q are true"

Conjunction

Input		Output
p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

"At least one of p and q is true"

Disjunction

Input	Output
p	$\neg p$
T	F
F	T

p is false

Negation

Logical operators aka propositional connectives

Input		Output
p	q	$p \wedge q$ ✓
T	T	T
T	F	F
F	T	F
F	F	F

"Both p and q are true"

Conjunction

Input		Output
p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

"Exactly one of p and q is true"

XOR

Exclusive or

Input		Output
p	q	$p \vee q$ ✓
T	T	T
T	F	T
F	T	T
F	F	F

"At least one of p and q is true"

Disjunction

Truth tables

zybook 1.1 -1.2, Rosen p. 10

We can use truth tables to compute value of compound proposition.

Input						Output
p	q	r	$p \wedge q$	$p \oplus q$	$(p \oplus q) \wedge r$	$(p \wedge q) \oplus ((p \oplus q) \wedge r)$
T	T	T	T	F	F	$(T \wedge T) \oplus ((T \oplus T) \wedge T)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

$= T \oplus (F \wedge T)$

$= T \oplus F = T$

2^3 rows

2^i rows for i variables

$\oplus$

Logical Equivalence

zybook 1.4, Rosen p. 25

We can use truth tables to compute value of compound proposition.

logically equivalent

Input						Output	
p	q	r	$p \wedge q$	$p \oplus q$	$(p \oplus q) \wedge r$	$(p \wedge q) \oplus ((p \oplus q) \wedge r)$	$(p \wedge q) \vee ((p \oplus q) \wedge r)$
T	T	T	T	F	F	T	T
T	T	F	F	F	F	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Compound propositions that have the same truth values for all settings of truth values to their propositional variables are **logically equivalent**, denoted

Tuesday’s learning goals contd…

Preclass: Read Chapter 1 (all sections)

- Prove propositional equivalences using truth tables
- Prove propositional equivalences using other known equivalences, e.g.
  - DeMorgan’s laws
  - Double negation laws
  - Distributive laws, etc.
- Reverse engineer compound propositions from their truth tables (Disjunctive Normal Form)
- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Solve logic puzzles using propositional logic

Tautology and contradiction

zybook 1.4, Rosen p. 25

Tautology: compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Which of the following is a tautology?

A. $p \wedge p$ $= p$

B. $p \oplus p$ $T \oplus T$ or $F \oplus F$ $= F$ Contradiction

C. $p \vee p$ $= p$

D. $p \vee \neg p$ $= T$ Tautology

E. $p \wedge \neg p$ $= F$ Contradiction.

F T F T

Which (if any) is a contradiction?

(Some) logical equivalences *zybook 1.5, Table 1.5.1, Rosen p. 26-28*

$$p \vee q \equiv q \vee p \qquad p \wedge q \equiv q \wedge p$$ commutative

$$(p \vee q) \vee r \equiv p \vee (q \vee r) \qquad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$ associative

$$p \wedge F \equiv F \qquad p \vee T \equiv T$$ domination

$$p \wedge T \equiv p \qquad p \vee F \equiv p$$ identity

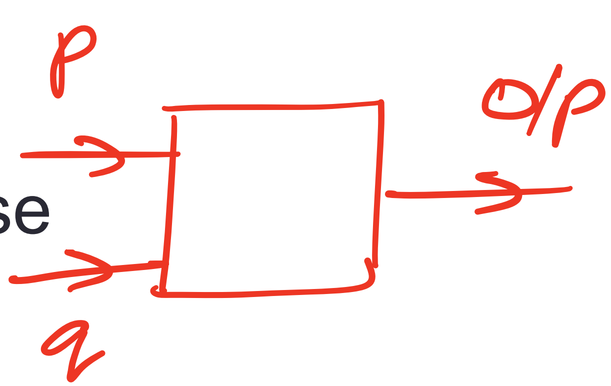
$$\neg(p \wedge q) \equiv \neg p \vee \neg q \qquad \neg(p \vee q) \equiv \neg p \wedge \neg q$$ DeMorgan's

$\sim(p \wedge q \wedge r) \equiv \sim p \vee \sim q \vee \sim r$

.... 11 equivalences listed in zybook!

Can replace *p* and *q* with any (compound) proposition

Going backwards



Given a compound proposition, we can use

- Truth tables
- Logical equivalences

to compute its truth value for specific input values.

What about the opposite problem? Given truth table settings, want a compound proposition with that output.

- Apply to design a circuit

Truth table $\longrightarrow$ compound proposition $\longrightarrow$ circuit

Reverse-engineering

look at row that evaluate to T

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

DNF
Disjunctive normal form

ORs of ANDs

$mystery_2 \equiv \sim p \wedge \sim q$

$mystery_1 \equiv (p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

An Algorithmic approach

ANDs of ORs

conjunctive normal form

$\boxed{\text{CNF}}$

Which situations guarantee output T?

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

mystery 1 $\equiv \sim(\sim p \wedge q) \equiv p \vee \sim q$

mystery 2 $\equiv \sim(p \wedge q) \wedge \sim(p \wedge \sim q) \wedge \sim(\sim p \wedge q)$

$(\sim p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee \sim q)$

An Algorithmic approach

Which situations guarantee output T?

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

ONLY THIS ROW for *$mystery_2$*

An Algorithmic approach

Which situations guarantee output T?

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

ONLY THIS ROW for *$mystery_2$*

"p is False and q is False"

$$\boxed{\neg p \wedge \neg q}$$

An Algorithmic approach

Which situations guarantee output T?

Input		Outp
p	q	$mystery_1$
T	T	T
T	F	T
F	T	F
F	F	T

Which compound proposition gives output *mystery*$_1$?

A. $\neg p \wedge q$

B. $(p \wedge q) \wedge (p \wedge \neg q) \wedge (\neg p \wedge \neg q)$

C. $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

D. More than one of the above

E. None of the above

DNF and CNF

Rosen p. 35 #42-53

Disjunctive normal form: OR of ANDs (of variables or their negations).

For *mystery*$_1$ a DNF is $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$
For *mystery*$_2$ a DNF is $\neg p \wedge \neg q$

Conjunctive normal form: AND of ORs (of variables or their negations).

For *mystery*$_1$ a CNF is $p \vee \neg q$
For *mystery*$_2$ a CNF is $(\neg p \vee \neg q) \wedge (\neg p \vee q) \wedge (p \vee \neg q)$

Conditional

zybook 1.3, Rosen p. 6-10

The hypothesis of p□q is p
The premise of p□q is p
The conclusion of p□q is q
The consequent of p□q is q

$\wedge, \vee, \oplus, \sim, p \rightarrow q$

If p, then q

p implies q

Input		Output
p	q	p □ q
T	T	T
T	F	F
F	T	T
F	F	T

The only way to make a conditional statement false is to …

Conditionals: vocabulary

zybook 1.3, Rosen p. 6-10

- The converse of p□q is ___ $q \to p$ ___
- The inverse of p□q is ___ $\sim p \to \sim q$ ___
- The contrapositive of p□q is ___ $\sim q \to \sim p$ ___

Which of the following is true?

A. $p \to q \equiv q \to p$

B. $p \to q \equiv \neg p \to \neg q$

C. $p \to q \equiv \neg q \to \neg p$

D. More than one of the above

E. None of the above

Conditional and biconditional

zybook 1.3,

Rosen p. 6-10

$p \leftrightarrow q$ p iff q

$p \to q$ is F

$\underset{T}{p} \to \underset{F}{q}$

$\equiv \underset{T}{\neg p} \vee \underset{F}{q}$ is F

Which of the following is NOT true?

A. $p \to q \equiv \neg p \vee q$ ✗

B. $p \leftrightarrow q \equiv p \wedge q$

C. $\neg(p \leftrightarrow q) \equiv p \oplus q$

D. $p \to q \equiv q \to p$

E. $p \leftrightarrow q \equiv q \leftrightarrow p$

Input		Output
p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

Conditional

Input		Output
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

"p iff q"
"p if and only if q"

Biconditional

Translation

zybook 1.1 -1.3, Rosen p. 14 #22a

Express the sentence
w
"A sufficient condition for the warranty to be good is that you bought the computer less than a year ago" using the propositions
b

w: "the warranty is good"
b: "you bought the computer less than a year ago"

$b \rightarrow w$

If b then w

Translation

zybook 1.1 -1.3, See more on Rosen p. 14

Express the sentence
"I will complete my to-do list only if I put a reminder in my calendar" using the propositions

r   c

r: "I will complete my to-do list"
c: "I put a reminder in my calendar"

$r \longrightarrow c$

If r then c

Logic Puzzles

Knaves: Always Lie

An island has two types of inhabitants: Knights and Knaves. You meet two people on the island: **A** and **B**.

A says: " I am a knave or B is a knight"
B says nothing

What are A and B?

Ans : A - Knight
B - Knight

Knights: Always tell the truth

Logic Puzzles

Define propositions:
p : **A is a knight**
q : **B is a knight**

Knaves: Always Lie

Knights: Always tell the truth

A says: " I am a knave or B is a knight"
B says nothing
What are A and B?

The puzzle can be completely described by which of the following statements?

A. $p \to \neg q$
B. $p \to (\neg p \vee q)$
C. $p \leftrightarrow (\neg p \vee q)$
D. More than one of the above
E. None of the above

Logic Puzzles

Define propositions:
p : **A is a knight**
q : **B is a knight**

Knaves: Always Lie

Knights: Always tell the truth

A says: " I am a knave or B is a knight"
B says nothing
What are A and B?

What truth values for p and q make the proposition $p \leftrightarrow (\neg p \vee q)$ true?

A. TT
B. TF
C. FT
D. FF

① p: A is a knight $\quad q$: B is a knight

→ define propositions

② Write given statement

A says: “ I am a knave or B is a knight”

$$\sim p \lor q$$

③ Write ~~statement~~ along with the info

If p then statement is T. If $\sim p$ then statement is F

$$(p \longrightarrow \sim p \lor q) \land (\sim p \longrightarrow \sim(\sim p \lor q))$$

↓ using $s \longrightarrow r$, $\sim r \longrightarrow \sim s$ are equivalent

$$\Rightarrow (p \longrightarrow \sim p \lor q) \land (\sim p \lor q \longrightarrow p)$$

$$p \longleftrightarrow \sim p \lor q$$

④ Construct Truth table

p	q	$\sim p$	$\sim p \lor q$	$p \longleftrightarrow \sim p \lor q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

only T when A & B are knights