HW1: Propositional and Predicate Logic

CS40 Summer'24

Due: Tuesday, July 3, 2024 at 11:59PM on Gradescope

In this assignment,

You will practice reading and applying definitions to get comfortable working with mathematical language and propositional logic. You will also practice defining and using sets and functions in multiple ways, translating English statements to predicate logic and negating quantified statements.

Typed answers are preferred for this HW. Diagrams may be hand-drawn and scanned and included in the typed document. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions[1].

You are allowed to submit handwritten answers provided they are neat and legible. Points will be docked for unclear and illegible submissions.

Integrity reminders

- You may not collaborate on this homework with anyone. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza. You *cannot* use any online resources about the course content other than the text book and class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions we will use this quarter.
- Do not share written solutions or partial solutions for homework with other students in the class. Doing so would dilute their learning experience and detract from their success in the class.

You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "HW1".

[1]To use this template, you will need to copy both the source file (extension `.tex`) you'll be editing and the file containing all the "shortcut" commands we've defined for this class

Assigned Questions

1. Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.

   (a) How tall is Storke Tower?

   (b) Storke Tower is as tall as 33 people stacked on top of each other.

2. Express each English statement using the logical operations $\vee$, $\wedge$, $\neg$ and the propositional variables t, n, and m defined below. The use of the word "or" means inclusive or.

   - t: My first flight was delayed.
   - n: I had a snack at the airport.
   - m: I made my final connection.

   (a) I never snack at the airport.

   (b) Even though my first flight was delayed, I still made my final connection.

   (c) Whenever I have a snack at the airport, I miss my final connection.

   (d) I miss my final connection only if I have a snack in the airport, or my flight was delayed.

3. Practice finding the truth values for conditional statements in English. Which of the following conditional statements are true and why?

   (a) If 51 is an even number, then the sky is green.

   (b) If 51 is an odd number, then the sky is blue.

4. In this question, practice expressing English colloquialisms using logical operations.

   Consider the following situations:

   - i: It is raining.
   - j: It is pouring.
   - k: It is shining.
   - l: We are having a parade.

   Write a logical expression to capture each of the following colloquialisms[2]:

   (a) It either rains or it pours.

   (b) It is raining on our parade.

   (c) We are having this parade, come rain or shine.

5. Give an English sentence in the form "If...then...." that is equivalent to each sentence.

   (a) Rafael can ride the elephant only if he is not afraid of heights.

   (b) Rafael can ride the elephant if he is not afraid of heights.

   (c) Being able to swim is a necessary skill needed for Tyra to learn to surf.

[2] Inclusive or is assumed unless explicitly stated otherwise.

(d) Being able to swim is a sufficient skill needed for Tyra to learn to surf.

6. Use the laws of propositional logic listed in Section 1.5, Table 1.5.1 of the book to prove the following. Explicitly specify which laws are being used.

   (a) $\neg p \to q \equiv \neg q \to p$

   (b) $\neg p \to (q \wedge \neg q) \equiv p$

   (c) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \equiv p \wedge \neg r$

7. Use any technique of your choice to show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology.

8. There are three kinds of people on an island: knaves who always lie, knights who always tell the truth, and spies who can either lie or tell the truth. You encounter three people, Andromeda, Brunhilda, and Clytemnestra. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the identity of the other two. Andromeda says "Clytemnestra is the knave," Brunhilda says "Andromeda is the knight," and Clytemnestra says "I am the spy." Determine whether or not a unique solution exists, and if so, state who the knave, knight, and spy are. If there is no unique solution, list all possible solutions or state that there are no solutions.

9. Use the Disjunctive Normal form to construct a proposition Q whose truth table is as below:

p	q	r	Q
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T

10. The sets A, B, and C are defined as follows:

    $A = \{\text{tall, grande, venti}\}$ $B = \{\text{foam, no-foam}\}$ $C = \{\text{non-fat, whole}\}$

    Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

    (a) Write an element from the set $B \times A \times C$.

    (b) Write the set $B \times C$ using roster notation.

11. Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

    (a) $\forall x \exists y (P(x, y) \wedge Q(x, y))$

    (b) $\exists x \forall y (P(x, y) \to Q(x, y))$

12. In the following question, the domain is a set of students who show up for a test. Define the following predicates:

$$P(x) : x \text{ showed up with a pencil}$$

$$C(x) : x \text{ showed up with a calculator}$$

Translate each statement given below into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question and response that can be used as reference for the detail expected in your answer:

Sample question: Every student showed up with a calculator.

Sample response:

$\forall x C(x)$

Negation: $\neg \forall x C(x)$

Applying De Morgan's law: $\exists x \neg C(x)$

English: Some student showed up without a calculator.

(a) At least one of the students showed up with a pencil.

(b) Every student showed up with a pencil or a calculator (or both).

(c) Every student who showed up with a calculator also had a pencil.

(d) There is a student who showed up with both a pencil and a calculator.