MF IN FINTECH

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HW5

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Q.1 Which of these are posets?

c.

Q.2 Answer these questions for the partial order represented by this Hasse diagram.

a.

l,m

b.

a,b,c,

c.

No

d.

No

e.

k,l,m

f.

k

g.

No exist

h.

No exist

Q.3 Let G be a simple graph with n vertices.

a.

For every w vertices, there can be an edges:

$$|E|_{\max} = \binom{n}{2} = \frac{n(n-1)}{2} \tag{1}$$

b.

Suppose the edges of G is $\{v_0, v_1\}$, $\{v_1, v_2\}$, ... $\{v_{n-1}, v_n\}$, there is $n-1$ edges.

Then we prove a connected have no less than $n-1$ edges.

For n = 1 there is nothing to prove.

Now assume the inductive hypothesis, and let G be a connected graph with n + 1 vertices and fewer than n edges, where $n \geq 1$. Since the sum of the degrees of the vertices of G is

$$2|E| \leq 2n \leq 2(n+1) \tag{2}$$

Therefore some vertex has degree less than 2. Since G is connected, this vertex is not isolated, so it must have degree 1. Remove this vertex and its edge. Clearly the result is still connected, and it has n vertices and fewer than n − 1 edges, contradicting the inductive hypothesis. Therefore the statement holds for G, and the proof is complete.

c.

If G is not connected, it has at least two connected subgraph, assume G_1 has k vertices and G_2 has $n-k$ vertices, because for $v_0 \in V_1$

$$\deg(v_0) \geq \frac{n-1}{2} \tag{3}$$

hence

$$k \geq 1 + \frac{n-1}{2} = \frac{n+1}{2} \tag{4}$$

for same reason

$$n-k \geq \frac{n+1}{2} \tag{5}$$

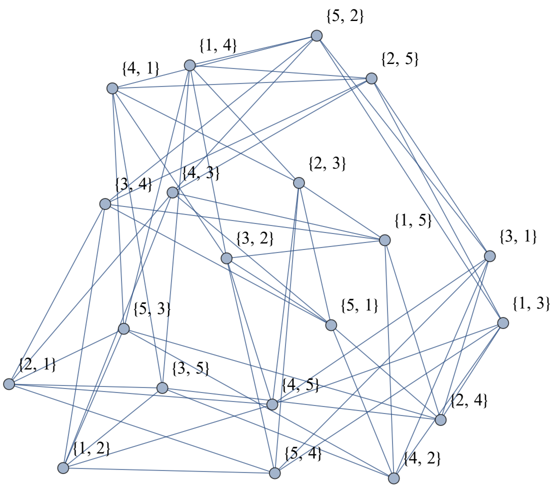
contradicting

$$n-k \leq n - \frac{n+1}{2} = \frac{n-1}{2} \tag{6}$$

Hence G must be connected.

Q.4 Let n ≥ 5 be an integer. Consider the graph Gn whose vertices are the sets {a, b}, where a, b ∈ {1, . . . , n} and a 6= b, and whose adjacency rule is disjointness, that is, {a, b} is adjacent to {a

a.



(7)

b.

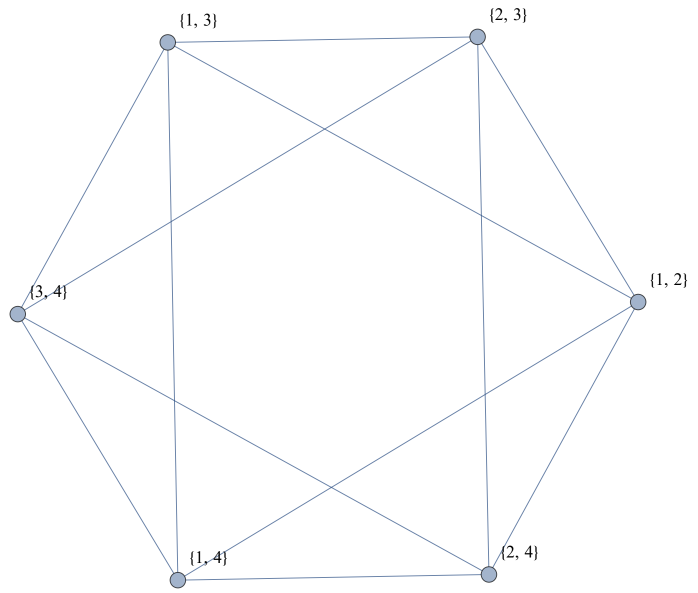
For each vertex, the number of vertex it connected is

$$2 \times \binom{n-2}{2} = (n-2)(n-3) \tag{8}$$

Q.5 Let G = (V, E)s be a graph on n vertices. Construct a new

a.

Suppose the edges in G which connect v_i and v_j is $\{i,\ j\}$ in G'



b.

$$\left|E'\right| = \sum_{v\in V} \deg(v) \tag{10}$$

Q.6 Let G be a connected graph, with the vertex set V. The distance between two vertices u and v, denoted by dist(u, v), is defined as the minimal length of a path from u to v. Show that dist(u, v) is a metric, i.e., the following properties hold for any u, v, w ∈ V :

Suppose the path from u to v is

$$u,\ w_1,\ w_2,\ \ldots v \tag{11}$$

If dist $(u,\ v) = 0$ and $u \neq v$, there is no exist a path, hence

$$\text{dist}(u,\ v) \Rightarrow u = v \tag{12}$$

If $u = v$, the path is

$$u,\ u \tag{13}$$

hence

$$u = v \Rightarrow \text{dist}(u,\ v) \tag{14}$$

The path from u to v also can be written as the path from v to u:

$$v,\ \ldots,\ w_2,\ w_1,\ u \tag{15}$$

that is, for every path p from u to v, there exist p' from v to u

$$\text{Length}(p) = \text{Length}(p') \tag{16}$$

hence their minimal length is equal:

$$\text{dist}(u,\ v) = \text{dist}(v,\ u) \tag{17}$$

The path from u to w and the path from w to v can construct a path from u to v. However dist(u,v) has the minimal length in all the path from u to v:

$$\text{dist}(u,\ v) \leq \text{dist}(u,\ w) + \text{dist}(w,\ v) \tag{18}$$

Q.7 Show that if G is bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$

Suppose the two subset of V is V_1 and V_2, and assume $\left|V_1\right| = k$, we know that

$$e \leq \text{edges number of } K_{k,v-k} = k(v-k) \tag{19}$$

Note

$$k(v-k) \leq \frac{v^2}{4} \tag{20}$$

Hence

$$e \leq \frac{v^2}{4} \tag{21}$$

Q.8
(a) What is the sum of the entries in a row of the adjacency matrix for an undirected graph? For a directed graph?
(b) What is the sum of the entries in a column of the adjacency matrix for an undirected graph? For a directed graph?

a

For undirected graph, the sum of the entries in ith row is $\deg(v_i)$ while for directed graph is $\deg^{-}(v_i)$

b.

For undirected graph, the sum of the entries in ith column is $\deg(v_i)$ while for directed graph is $\deg^{+}(v_i)$

Q.9 Show that isomorphism of simple graphs is an equivalence relation.

isomorphism is reflexive:

Let

$$f(v) = v \tag{22}$$

Then G and G are isomorphism.

isomorphism is symmetric:

If G and G' are isomorphism, there exist reversible function

$$f : V \to V' \tag{23}$$

Let

$$f^{-1} : V' \to V \tag{24}$$

Then G' and G are isomorphism.

isomorphism is transitive:

If

$$G \text{ isomorphism } G',\ G' \text{ isomorphism } G'' \tag{25}$$

There exist

$$f : G \to G',\ g : G' \to G'' \tag{26}$$

Then $f \circ g$ let G isomorphism G''

Q.10 Show that every connected graph with n vertices has at least n − 1 edges

For n = 1 there is nothing to prove.

Now assume the inductive hypothesis, and let G be a connected graph with n + 1 vertices and fewer than n edges, where $n \geq 1$. Since the sum of the degrees of the vertices of G is

$$2\,|\,E\,|\ \leq 2\,n \leq 2\,(n+1) \tag{27}$$

Therefore some vertex has degree less than 2. Since G is connected, this vertex is not isolated, so it must have degree 1. Remove this vertex and its edge. Clearly the result is still connected, and it has n vertices and fewer than n − 1 edges, contradicting the inductive hypothesis. Therefore the statement holds for G, and the proof is complete.

Q.11 Explain how to find a path with the least number of edges between two vertices in an undirected graph by considering it as a shortest path problem in a weighted graph.

Suppose $G = (V, E)$, where $V = v_0, v_1, \ldots v_n$ and the for every vertex pair

$$w(v_i, v_j) = \begin{cases} \text{weight} & \{v_i, v_j\} \text{ is an edge} \\ \infty & \text{otherwise} \end{cases} \tag{28}$$

Then find the shortest path from v_0 to v_n:

```
For i In range(n):
    L(vi)=∞
L(v0)=0
S=∅
While vn∉S:
    u=Min[V-S,key=lambda v:L(v)]
    S=S.append(u)
    For j In V-S:
        If L(u)+w(u,v)<L(v):
            L(v)=L(u)+w(u,v)
Return L(vn)
```

Q.12 Which of the these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a palnar graph?

a,c