# 1 Concepts

- Set
- Element
- Subset
- Superset

# 2 Special sets

- Universal set: U
- Empty set: $\emptyset$
- Natural numbers: $\mathbb{N}$
- Integer: $\mathbb{Z}$
- Rational numbers: $\mathbb{Q}$
- Real numbers: $\mathbb{R}$
- Complex numbers: $\mathbb{C}$

# 3 Set operations

## 3.1 Union and interception

### 3.1.1 Union

$$A \cup B = \{x \mid x \in A \lor x \in B\}$$

### 3.1.2 Interception

$$A \cap B = \{x \mid x \in A \land x \in B\}$$

Properties:

- $A \cap B \subseteq A, A \cap B \subseteq B$
- $A \subseteq A \cup B, B \subseteq A \cup B$

Theorem 1.3: For any sets A and B, we have:

$$A \cap B \subseteq A \subseteq A \cup B$$
$$A \cap B \subseteq B \subseteq A \cup B$$

Theorem 1.4: The followings are equivalent:

$$A \subseteq B, A \cap B = A, A \cup B = B$$

## 3.2 Complement

### 3.2.1 Absolute complement

$$A^c = \{x \mid x \in \mathrm{U}, x \notin A\}$$

### 3.2.2 Relative Complement

The relative complement of A and B, donoted by $A \setminus B$, is:

$$A \setminus B = \{x \mid x \in A, x \notin B\}$$

### 3.2.3 Symmetric difference

Symmetric difference of sets A and B, denoted by $A \oplus B$, is:

$$A \oplus B = \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$$

That is:

$$A \oplus B = (A \cup B) \setminus (A \cap B)$$
$$A \oplus B = (A \setminus B) \cup (B \setminus A)$$

## 3.3 Fundamental products

Consider $n$ distinct sets $A_1, A_2, \cdots, A_n$, a fundanmental product is a set of form

$$A_1^* \cap A_2^* \cap \cdots \cap A_n^* (A_i^* = A_i \vee A_i^* = A_i^c)$$

# 4 Algebra of sets, duality

Theorem 1.5: Sets satisfy the laws in table:

# 5 Venn diagrams

Table 1: Laws of the algebra of sets

| | | |
|---|---|---|
| Idempotent laws | $A \cup A = A$ | $A \cap A = A$ |
| Associative laws | $(A \cup B) \cup C = A \cup (B \cup C)$ | $(A \cap B) \cap C = A \cap (B \cap C)$ |
| Commutative laws | $A \cup B = B \cup A$ | $A \cap B = B \cap A$ |
| Distributive laws | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ | $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| Identity laws | $A \cup \emptyset = A$<br>$A \cup \mathrm{U} = \mathrm{U}$ | $A \cap \mathrm{U} = A$<br>$A \cap \emptyset = \emptyset$ |
| Involution laws | $(A^c)^c = A$ | |
| Complement laws | $A \cup A^c = \mathrm{U}$<br>$\mathrm{U}^c = \emptyset$ | $A \cap A^c = \emptyset$<br>$\emptyset^c = \mathrm{U}$ |
| DeMorgan's laws | $(A \cup B)^c = A^c \cap B^c$ | $(A \cap B)^c = A^c \cap B^c$ |