MF IN FINTECH

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HW1

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Q.1 Use truth tables to decide whether or not the following two propositions are equivalent.

a.

The truth table of $p \veebar q$ is:

p	q	$p \veebar q$
True	True	False
True	False	True
False	True	True
False	False	False

(1)

The truth table of $\neg p \vee \neg q$ is:

p	q	$\neg p \vee \neg q$
True	True	False
True	False	True
False	True	True
False	False	True

(2)

Not equivalent.

b.

The truth table of $\neg q \wedge \neg (p \rightarrow q)$ is:

p	q	$\neg q \wedge \neg (p \Rightarrow q)$
True	True	False
True	False	True
False	True	False
False	False	False

(3)

Which is depend on weather q is true or not, so they are not equivalent.

c.

The truth table of $p \vee q \rightarrow r$ is:

p	q	r	$p \,\|\, q \Rightarrow r$
True	True	True	True
True	True	False	False
True	False	True	True
True	False	False	False
False	True	True	True
False	True	False	False
False	False	True	True
False	False	False	True

(4)

The truth table of $(p \rightarrow r) \wedge (q \rightarrow r)$ is:

p	q	r	$(p \Rightarrow r) \wedge (q \Rightarrow r)$
True	True	True	True
True	True	False	False
True	False	True	True
True	False	False	False
False	True	True	True
False	True	False	False
False	False	True	True
False	False	False	True

(5)

Equivalent

d.

The truth table of $(p \to \neg q) \leftrightarrow (r \to p \vee \neg q)$ is :

p	q	r	$(p \Rightarrow \neg q) \Leftrightarrow (r \Rightarrow p \vee \neg q)$
True	True	True	False
True	True	False	False
True	False	True	True
True	False	False	True
False	True	True	False
False	True	False	True
False	False	True	True
False	False	False	True

(6)

The truth table of $q \vee \neg p \wedge \neg r$ is :

p	q	r	$q \vee (\neg p \wedge \neg r)$
True	True	True	True
True	True	False	True
True	False	True	False
True	False	False	False
False	True	True	True
False	True	False	True
False	False	True	False
False	False	False	True

(7)

Not equivalent.

Q.2 Use logical equivalences to prove the following statements

a.

$$\begin{aligned} (\neg (p \to q) \to p) &\equiv (p \wedge \neg q) \to p \\ &\equiv \neg (p \wedge \neg q) \vee p \\ &\equiv \neg p \vee q \vee p \\ &\equiv T \vee q \\ &\equiv T \end{aligned} \tag{8}$$

b.

$$\begin{aligned} (p \wedge \neg q) \to r &\equiv \neg (p \wedge \neg q) \vee r \\ &\equiv \neg p \vee q \vee p \end{aligned} \tag{9}$$

$$p \Rightarrow (q \vee r) \equiv \neg p \vee (q \vee r) \equiv \neg p \vee q \vee p \tag{10}$$

c.

$$\begin{aligned} \neg p \to (q \to r) &\equiv p \vee (q \to r) \\ &\equiv p \vee \neg q \vee r \end{aligned} \tag{11}$$

$$q \to (p \vee r) \equiv \neg q \vee p \vee r \equiv p \vee \neg q \vee r \tag{12}$$

d.

$$\begin{aligned} \neg (p \oplus q) &\equiv \neg ((p \wedge \neg q) \vee (q \wedge \neg p)) \\ &\equiv \neg (p \wedge \neg q) \wedge \neg (q \wedge \neg p) \\ &\equiv (\neg p \vee q) \wedge (\neg q \vee p) \\ &\equiv (p \Rightarrow q) \wedge (q \Rightarrow p) \\ &\equiv p \Leftrightarrow q \end{aligned} \tag{13}$$

e.

$$\begin{aligned} (p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q)) &\equiv (p \Rightarrow q) \Rightarrow (q \vee \neg p \vee \neg r) \\ &\equiv \neg (p \Rightarrow q) \vee (q \vee \neg p \vee \neg r) \\ &\equiv (p \wedge \neg q) \vee q \vee \neg p \vee \neg r \\ &\equiv (p \vee q \vee \neg p \vee \neg r) \wedge (\neg q \vee q \vee \neg p \vee \neg r) \\ &\equiv T \wedge T \\ &\equiv T \end{aligned} \tag{14}$$

Q.3 Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

$$\neg ((p \Rightarrow q) \wedge (q \Rightarrow r)) \equiv \neg (p \Rightarrow q) \vee \neg (q \Rightarrow r) \equiv (p \wedge \neg q) \vee (q \wedge \neg r) \tag{15}$$

Hence

$$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r) \equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r) \tag{16}$$

If which is not tautology, we can find some p, q, r to make it false. Firstly, p should be true and r should be false otherwise $(\neg p \vee r)$ is true. However:

$$(p \wedge \neg q) \vee (q \wedge \neg r) \equiv -q \vee q \equiv T \tag{17}$$

So it’s tautology.

Q.4 Determine whether or not the following two are logically equivalent, and explain your answer.

a.

Equivalent:

$$\begin{aligned} (p \Rightarrow q) \vee (p \Rightarrow r) &\equiv (\neg p \vee q) \vee (\neg p \vee r) \\ &\equiv \neg p \vee (q \vee r) \\ &\equiv p \Rightarrow (q \vee r) \end{aligned} \tag{18}$$

b.

$$(p \Rightarrow q) \Rightarrow r \equiv p \wedge \neg q \vee r \tag{19}$$

$$p \Rightarrow (q \Rightarrow r) \equiv \neg p \vee \neg q \vee r \tag{20}$$

They are not equivalent

Q.5 Prove that if p ∧ q, p → ¬(q ∧ r), s → r, then ¬s.

$$(p \wedge q) \wedge (\neg p \vee \neg (q \wedge r) \wedge (\neg s \vee r) \equiv (p \wedge q) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (\neg s \vee r) \equiv T \tag{21}$$

This statement is true, so $p = q = T$:

$$(\neg p \vee \neg q \vee \neg r) \equiv T \Rightarrow r = F \tag{22}$$

Then:

$$(\neg s \vee r) \equiv T \Rightarrow s = F \Rightarrow \neg S \equiv T \tag{23}$$

Q.6 Let C(x) be the statement "x has a cat", let D(x) be the statement "x has a dog" and let F(x) be the statement "x has a ferret." Express each of these sentences in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a.

$$\exists x\, C(x) \wedge D(x) \wedge F(x) \tag{24}$$

b.

$$\forall x\, C(x) \vee D(x) \vee F(x) \tag{25}$$

c.

$$\exists x\, C(x) \wedge F(x) \wedge \neg D(x) \tag{26}$$

d.

$$\forall x \neg C(x) \vee \neg D(x) \vee \neg F(x) \tag{27}$$

e.

Rewrite $C(x)$ as “x has a cat as pet”, do similar operations for others.

$$(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x)) \tag{28}$$

Q.7 Let L(x, y) be the statement "x loves y", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statement.

a.

$$\forall x\, L(x, \text{ Jerry}) \tag{29}$$

b.

$$\forall x\, \exists y\, L(x, y) \tag{30}$$

c.

$$\exists x\, \forall y\, L(y, x) \tag{31}$$

d.

$$\forall x\, \exists x \neg L(x, y) \tag{32}$$

e.

$$\exists x \neg L(\text{Lydia}, x) \tag{33}$$

f.

$$\exists x\, \forall y \neg L(y, x) \tag{34}$$

g.

$$\exists !\, x\, \forall y\, L(y, x) \tag{35}$$

h.

$$\exists_2 L(\text{Lynn}, x) \tag{36}$$

i.

$$\forall x\, L(x, x) \tag{37}$$

j.

$$\exists x\, L(x, y) \Rightarrow x = y \tag{38}$$

Q.8 Express the negations of each of these statements so that all negation symbols immediately precede predicates

a.

$$\exists x\, \forall y\, \exists z \neg T(x, y, z) \tag{39}$$

b.

$$\exists x\, \forall y \neg P(x, y) \wedge \exists x\, \forall y \neg Q(x, y) \tag{40}$$

c.

$$\exists x\, \forall y(\neg P(x, y) \vee \forall z \neg R(x, y, z)) \tag{41}$$

d.

$$\exists x\, \forall y\, (P(x, y) \wedge \neg Q(x, y)) \tag{42}$$

Q.9

a.

$$\neg (p \Leftrightarrow (q \vee \neg p)) \equiv \neg (p \Rightarrow (q \vee \neg p)) \vee \neg ((q \vee \neg p) \Rightarrow p) \tag{43}$$

$$\neg (p \Rightarrow (q \vee \neg p)) \equiv p \wedge \neg (q \vee \neg p) \equiv p \wedge \neg q \tag{44}$$

$$\neg ((q \vee \neg p) \Rightarrow p) \equiv (q \vee \neg p) \wedge \neg p \equiv \neg p \tag{45}$$

Hence:

$$\neg (p \Leftrightarrow (q \vee \neg p)) \equiv (p \wedge \neg q) \vee \neg p \equiv (\neg p \vee \neg q) \tag{46}$$

b.

P has only $2^{2^2} = 16$ possible truth tables.

Then we can use computer to prove $A \square B$ can create all 16 truth tables.

```
In[29]:= BooleanTable /@
         └真值表
           Flatten[Table[{p, q, ToExpression[x <> o <> y]}, {x, {"p", "q", "¬p", "¬q"}},
           └压平   └表格      └转换为表达式
             {o, {"∧", "∨", "⇔"}}, {y, {"p", "q", "¬p", "¬q"}}], 2] // Union //
                                                                     └并集
          Length
          └长度

Out[29]= 16
```

Q.10 For each of these arguments, explain which rules of inference are used for each step

a.

Let p be the proposition “Someone has taken a course in discrete mathematics” q the proposition “Some one can take a course in algorithms”. By modus ponens:

$$\begin{array}{c} p \\ p \Rightarrow q \\ \therefore q \end{array} \tag{47}$$

b.

Let S be the set:

$$S = \{m \mid m \text{ is produced bt John Sayles}\} \tag{48}$$

Let $w(x)$ be the statement:

$$x \text{ is wonderful} \tag{49}$$

Let C be the set:

$$\{m \mid m \text{ is about coal miners}\} \tag{50}$$

$$\because \forall\, x \in S\, w(x) \wedge \exists\, x \in C,\ x \in S \tag{51}$$

$$\therefore \exists\, x \in C,\ w(x) \tag{52}$$

Q.11 Prove or disprove that there is a rational number x and an irrational number y such that x^y is irrational.

Let

$$\begin{cases} x = 2 \\ y = \sqrt{\frac{1}{2}} \end{cases} \tag{53}$$

y is irrational because $y = \frac{1}{\sqrt{2}}$ and $\sqrt{2}$ is irrational.

Then if x^y is rational, let:

$$\begin{cases} x = 2^{\sqrt{\frac{1}{2}}} \\ y = \sqrt{\frac{1}{2}} \end{cases} \tag{54}$$

$$x^y = 2^{1/2} = \sqrt{2} \tag{55}$$

Which is irrational.

Q.12 Prove that $\sqrt[3]{2}$ is irrational.

If $\sqrt[3]{2}$ is rational, there are integers m and n such that:

$$\sqrt[3]{2} = \frac{m}{n} \tag{56}$$

i.e.

$$n^3 + n^3 = m^3 \tag{57}$$

However by Fermat's Last Theorem, there aren't exist such m and n.

Q.13 Give a direct proof that: Let a and b be integers. If $a^2 + b^2$ is even, then $a + b$ is even.

$$a \equiv a^2 (\text{mod } 2) \tag{58}$$

$$b \equiv b^2 (\text{mod } 2) \tag{59}$$

$$\therefore a^2 + b^2 \equiv 0 \; (\text{mod } 2) \Rightarrow a + b \equiv 0 \; (\text{mod } 2) \tag{60}$$

Q.14 Prove that between every two rational numbers there is an irrational number

For any two rational numbers $a = \frac{n}{m}$ and $b = \frac{q}{p}$, w.l.o.g $a > b$:

$$a - b = \frac{np - qm}{mp} \geq \frac{1}{mp} > \frac{1}{\sqrt{2}\; mp} \tag{61}$$

therefore, we find an irrational number $b + \frac{1}{\sqrt{2}\; np}$ which is between a and b.