1. Prove: Let A, B, C, D be sets. Suppose R is a relation from A to B, S is a relation from B to C and T is a relation from C to D. Then $(R \circ S) \circ T = R \circ (S \circ T)$

2. Suppse C is a collection of relations S on a set A, and let T be the intersection of the relations S in C, that is $T = \cap(S|S \in C)$. Prove: a. If every S is symmetric, then T is symmetric. b. If every S is transitive, then T is transitive.

3. let R be a relation on a set A, and let P be a property of relations, such as symmetry and transitivity. Then P will be called R-closable if P satisfies: i. There is a P-relation S containing R. ii. The intersection of P-relations is a P-relation. a. Show that symmetry and transitivity are R-closable for any relation R. b. Suppose P is R-closable. Then $P(R)$, the P-closure of R, is the intersection of all P-relations S containing R, that is:
$$P(R) = \cap(S|\text{S is a P-relation and } R \subseteq S)$$

4. Consider the $\mathbf{Z}$ of integers and an integer $m > 1$. We say that x is congruent to y modulo m, written
$$x \equiv y(modm)$$
if $x - y$ is divisible by m. Show that this defines an equivalence relation on $\mathbf{Z}$.

5. Let A be a set of nonzero integers and let $\sim$ be the relation on AA defined by
$$(a, b) \sim (c, d)\text{whenever}ad = bc$$
Prove that $\sim$ is an equivalence relation.

6. Prove: Let R be an equivalence relation in a set A. Then the quotient set A/R is a partition of A. Specifically, i. $\forall a \in A \to a \in [a]$ ii. $[a] = [b] \iff (a, b) \in R$ iii. $[a] \neq [b] \to [a] \cap [b] = \emptyset$

7. Prove: Let L be any collection of sets, the relation of set inclusion $\subseteq$ a partial order on L.

8. Suppose R and S are relations on a set A, and R is antisymmetric. Prove that $R \cap S$ is antisymmetric.

9. Prove that if R is an equivalence relation on set A, the R^{-1} is also an equivalence relation on A.