



bsplyne is a Python library for working with N-dimensional B-splines. It implements the Cox-de Boor algorithm for basis evaluation, order elevation, knot insertion, and provides a connectivity class for multi-patch structures. Additionally, it includes visualization tools with export capabilities to Paraview.

Note: This library is not yet available on PyPI. To install, please clone the repository and install it manually.

Installation

Since **bsplyne** is not yet on PyPI, you can install it locally as follows:

```
git clone https://github.com/Dorian210/bsplyne
cd bsplyne
pip install -e .
```

Dependencies

Make sure you have the following dependencies installed:

- `numpy`
- `numba`
- `scipy`
- `matplotlib`
- `meshio`
- `tqdm`
- `pathos`

Main Modules

- **BSplineBasis**
Implements B-spline basis function evaluation using the Cox-de Boor recursion formula.
- **BSpline**
Provides methods for creating and manipulating N-dimensional B-splines, including order elevation and knot

insertion.

- **MultiPatchBSplineConnectivity**
Manages the connectivity between multiple N-dimensional B-spline patches.
- **CouplesBSplineBorder** (less documented)
Handles coupling between B-spline borders.

Examples

Several example scripts demonstrating the usage of **bsplyne** can be found in the [examples/](#) directory. These scripts cover:

- Basis evaluation on a curved line
- Plotting with Matplotlib
- Order elevation
- Knot insertion
- Surface examples
- Exporting to Paraview

Documentation

The full API documentation is available in the [docs/](#) directory of the project or via the [online documentation portal](#).

Contributions

At the moment, I am not actively reviewing contributions. However, if you encounter issues or have suggestions, feel free to open an issue.

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```
@nb.njit(cache=True)
def wide_product_max_nnz(a_indptr: numpy.ndarray, b_indptr: numpy.ndarray, height: int) -> int:
```

- [View Source](#)

Compute the maximum number of nonzeros in the result.

Parameters

- **a_indptr** (np.ndarray): CSR pointer array for matrix A.
- **b_indptr** (np.ndarray): CSR pointer array for matrix B.
- **height** (int): Number of rows (must be the same for both A and B).

Returns

- **max_nnz** (int): Total number of nonzero elements in the resulting matrix.

```
@nb.njit(cache=True)
def wide_product_row(
    a_data: numpy.ndarray,
    a_indices: numpy.ndarray,
    b_data: numpy.ndarray,
    b_indices: numpy.ndarray,
    b_width: int,
    out_data: numpy.ndarray,
    out_indices: numpy.ndarray
) -> int:
```

- [View Source](#)

Compute the wide product for one row.

For each nonzero in the row of A and each nonzero in the row of B, it computes: $\text{out_index} = \text{a_indices}[i] * \text{b_width} + \text{b_indices}[j]$ $\text{out_value} = \text{a_data}[i] * \text{b_data}[j]$

Parameters

- **a_data** (np.ndarray): Nonzero values for the row in A.
- **a_indices** (np.ndarray): Column indices for the row in A.
- **b_data** (np.ndarray): Nonzero values for the row in B.
- **b_indices** (np.ndarray): Column indices for the row in B.
- **b_width** (int): Number of columns in B.
- **out_data** (np.ndarray): Preallocated output array for the row's data.
- **out_indices** (np.ndarray): Preallocated output array for the row's indices.

Returns

- **off** (int): Number of nonzero entries computed for this row.

```
@nb.njit(cache=True)
def wide_product_numba(
    height: int,
    a_data: numpy.ndarray,
    a_indices: numpy.ndarray,
    a_indptr: numpy.ndarray,
    a_width: int,
    b_data: numpy.ndarray,
    b_indices: numpy.ndarray,
    b_indptr: numpy.ndarray,
    b_width: int
):
```

• [View Source](#)

Compute the row-wise wide (Khatri-Rao) product for two CSR matrices.

For each row i , the result $[i, :] = \text{kron}(A[i, :], B[i, :])$, i.e. the Kronecker product of the i -th rows of A and B .

Parameters

- **height** (int): Number of rows in A and B .
- **a_data** (np.ndarray): Data array for matrix A (CSR format).
- **a_indices** (np.ndarray): Indices array for matrix A .
- **a_indptr** (np.ndarray): Index pointer array for matrix A .
- **a_width** (int): Number of columns in A .
- **b_data** (np.ndarray): Data array for matrix B (CSR format).
- **b_indices** (np.ndarray): Indices array for matrix B .
- **b_indptr** (np.ndarray): Index pointer array for matrix B .
- **b_width** (int): Number of columns in B .

Returns

- **out_data** (np.ndarray): Data array for the resulting CSR matrix.
- **out_indices** (np.ndarray): Indices array for the resulting CSR matrix.
- **out_indptr** (np.ndarray): Index pointer array for the resulting CSR matrix.
- **total_nnz** (int): Total number of nonzero entries computed.

```
def my_wide_product(
    A: scipy.sparse._base.spmatrix,
    B: scipy.sparse._base.spmatrix
) -> scipy.sparse._csr.csr_matrix:
```

• [View Source](#)

Compute a "1D" Kronecker product row by row.

For each row i , the result $C[i, :] = \text{kron}(A[i, :], B[i, :])$. Matrices A and B must have the same number of rows.

Parameters

- **A** (scipy.sparse.spmatrix): Input sparse matrix A in CSR format.
- **B** (scipy.sparse.spmatrix): Input sparse matrix B in CSR format.

Returns

- **C** (scipy.sparse.csr_matrix): Resulting sparse matrix in CSR format with shape $(A.\text{shape}[0], A.\text{shape}[1]*B.\text{shape}[1])$.



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p = 2

knot = array([0. , 0. , 0. , 0.25, 0.5 , 0.75, 1. , 1. , 1.])

C = array([1.00002282, 1.00002282, 0.84721156, 0.56754107, 0.19634954, 0.])

base_quarter_circle = [show](#)

def new_quarter_circle(center, normal, radius):

- [View Source](#)

base_circle = [show](#)

def new_circle(center, normal, radius):

- [View Source](#)

base_disk = [show](#)

def new_disk(center, normal, radius):

- [View Source](#)

base_degenerated_disk = [show](#)

def new_degenerated_disk(center, normal, radius):

- [View Source](#)

base_quarter_pipe = [show](#)

def new_quarter_pipe(center_front, orientation, radius, length):

- [View Source](#)

base_pipe = [show](#)

def new_pipe(center_front, orientation, radius, length):

- [View Source](#)

base_quarter_cylinder = [show](#)

def new_quarter_cylinder(center_front, orientation, radius, length):

- [View Source](#)

base_cylinder = [show](#)

def new_cylinder(center_front, orientation, radius, length):

- [View Source](#)

base_degenerated_cylinder = [show](#)

def new_degenerated_cylinder(center_front, orientation, radius, length):

• [View Source](#)

p_closed = 2

knot_closed = [show](#)

a = 0.998323859441081

b = -0.4135192822211451

C_closed = [show](#)

base_closed_circle = [show](#)

def new_closed_circle(center, normal, radius):

• [View Source](#)

base_closed_disk = [show](#)

def new_closed_disk(center, normal, radius):

• [View Source](#)

base_closed_pipe = [show](#)

def new_closed_pipe(center_front, orientation, radius, length):

• [View Source](#)

base_closed_cylinder = [show](#)

def new_closed_cylinder(center_front, orientation, radius, length):

• [View Source](#)

def new_quarter_strut(center_front, orientation, radius, length):

• [View Source](#)

def new_cube(center, orientation, side_length):

• [View Source](#)



- [View Source](#)

```
@nb.njit(cache=True)
def find(parent, x):
```

- [View Source](#)

```
@nb.njit(cache=True)
def union(parent, rank, x, y):
```

- [View Source](#)

```
@nb.njit(cache=True)
def get_unique_nodes_inds(nodes_couples, nb_nodes):
```

- [View Source](#)

```
class MultiPatchBSplineConnectivity:
```

- [View Source](#)

Contains all the methods to link multiple B-spline patches. It uses 3 representations of the data :

- a unique representation, possibly common with other meshes, containing only unique nodes indices,
- a unpacked representation containing duplicated nodes indices,
- a separated representation containing duplicated nodes indices, separated between patches. It is here for user friendliness.

Attributes

- **unique_nodes_inds** (numpy.ndarray of int): The indices of the unique representation needed to create the unpacked one.
- **shape_by_patch** (numpy.ndarray of int): The shape of the separated nodes by patch.
- **nb_nodes** (int): The total number of unpacked nodes.
- **nb_unique_nodes** (int): The total number of unique nodes.
- **nb_patches** (int): The number of patches.
- **npa** (int): The dimension of the parametric space of the B-splines.

```
MultiPatchBSplineConnectivity(unique_nodes_inds, shape_by_patch, nb_unique_nodes)
```

- [View Source](#)

Parameters

- **unique_nodes_inds** (numpy.ndarray of int): The indices of the unique representation needed to create the unpacked one.
- **shape_by_patch** (numpy.ndarray of int): The shape of the separated nodes by patch.
- **nb_unique_nodes** (int): The total number of unique nodes.

unique_nodes_inds: numpy.ndarray

shape_by_patch: numpy.ndarray

nb_nodes: int

nb_unique_nodes: int

nb_patches: int

npa: int

```
@classmethod
def from_nodes_couples(cls, nodes_couples, shape_by_patch):
```

- [View Source](#)

Create the connectivity from a list of couples of unpacked nodes.

Parameters

- **nodes_couples** (numpy.ndarray of int): Couples of indices of unpacked nodes that are considered the same. Its shape should be (# of couples, 2)
- **shape_by_patch** (numpy.ndarray of int): The shape of the separated nodes by patch.

Returns

- **MultiPatchBSplineConnectivity**: Instance of [MultiPatchBSplineConnectivity](#) created.

@classmethod

```
def from_separated_ctrlPts(
    cls,
    separated_ctrlPts,
    eps=1e-10,
    return_nodes_couples: bool = False
):
```

• [View Source](#)

Create the connectivity from a list of control points given as a separated field by comparing every couple of points.

Parameters

- **separated_ctrlPts** (list of numpy.ndarray of float): Control points of every patch to be compared in the separated representation. Every array is of shape : (*NPh* , nb elem for dim 1, ..., nb elem for dim *npa*)
- **eps** (float, optional): Maximum distance between two points to be considered the same, by default 1e-10
- **return_nodes_couples** (bool, optional): If *True* , returns the *nodes_couples* created, by default False

Returns

- **MultiPatchBSplineConnectivity**: Instance of [MultiPatchBSplineConnectivity](#) created.

```
def unpack(self, unique_field):
```

• [View Source](#)

Extract the unpacked representation from a unique representation.

Parameters

- **unique_field** (numpy.ndarray): The unique representation. Its shape should be : (field, shape, ..., *self* . *nb_unique_nodes*)

Returns

- **unpacked_field** (numpy.ndarray): The unpacked representation. Its shape is : (field, shape, ..., *self* . *nb_nodes*)

```
def pack(self, unpacked_field, method='mean'):
```

• [View Source](#)

Extract the unique representation from an unpacked representation.

Parameters

- **unpacked_field** (numpy.ndarray): The unpacked representation. Its shape should be : (field, shape, ..., *self* . *nb_nodes*)
- **method** (str): The method used to group values that could be different

Returns

- **unique_nodes** (numpy.ndarray): The unique representation. Its shape is : (field, shape, ..., *self* . *nb_unique_nodes*)

```
def separate(self, unpacked_field):
```

• [View Source](#)

Extract the separated representation from an unpacked representation.

Parameters

- **unpacked_field** (numpy.ndarray): The unpacked representation. Its shape is : (field, shape, ..., `self.nb_nodes`)

Returns

- **separated_field** (list of numpy.ndarray): The separated representation. Every array is of shape : (field, shape, ..., nb elem for dim 1, ..., nb elem for dim `npa`)

def agglomerate(`self`, separated_field):

• [View Source](#)

Extract the unpacked representation from a separated representation.

Parameters

- **separated_field** (list of numpy.ndarray): The separated representation. Every array is of shape : (field, shape, ..., nb elem for dim 1, ..., nb elem for dim `npa`)

Returns

- **unpacked_field** (numpy.ndarray): The unpacked representation. Its shape is : (field, shape, ..., `self.nb_nodes`)

def unique_field_indices(`self`, field_shape, representation='separated'):

• [View Source](#)

Get the unique, unpacked or separated representation of a field's unique indices.

Parameters

- **field_shape** (tuple of int): The shape of the field. For example, if it is a vector field, `field_shape` should be (3,). If it is a second order tensor field, it should be (3, 3).
- **representation** (str, optional): The user must choose between "unique" , "unpacked" , and "separated" . It corresponds to the type of representation to get, by default "separated"

Returns

- **unique_field_indices** (numpy.ndarray of int or list of numpy.ndarray of int): The unique, unpacked or separated representation of a field's unique indices. If unique, its shape is (`field_shape` , `self.nb_unique_nodes`). If unpacked, its shape is : (`field_shape` , `self.nb_nodes`). If separated, every array is of shape : (* `field_shape` , nb elem for dim 1, ..., nb elem for dim `npa`).

def get_duplicate_unpacked_nodes_mask(`self`):

• [View Source](#)

Returns a boolean mask indicating which nodes in the unpacked representation are duplicates.

Returns

- **duplicate_nodes_mask** (numpy.ndarray): Boolean mask of shape (nb_nodes,) where True indicates a node is duplicated across multiple patches and False indicates it appears only once.

def extract_exterior_borders(`self`, splines):

• [View Source](#)

Extract exterior borders from B-spline patches.

Parameters

- **splines** (list[BSpline]): Array of B-spline patches to extract borders from.

Returns

- **border_connectivity** (MultiPatchBSplineConnectivity): Connectivity information for the border patches.
- **border_splines** (list[BSpline]): Array of B-spline patches representing the borders.
- **border_unique_to_self_unique_connectivity** (numpy.ndarray of int): Array mapping border unique nodes to original unique nodes.

Raises

- **AssertionError**: If isoparametric space dimension is less than 2.

```
def extract_interior_borders(self, splines):
```

• [View Source](#)

Extract interior borders from B-spline patches where nodes are shared between patches.

Parameters

- **splines** (list[BSpline]): Array of B-spline patches to extract borders from.

Returns

- **border_connectivity** (MultiPatchBSplineConnectivity): Connectivity information for the border patches.
- **border_splines** (list[BSpline]): Array of B-spline patches representing the borders.
- **border_unique_to_self_unique_connectivity** (numpy.ndarray of int): Array mapping border unique nodes to original unique nodes.

Raises

- **AssertionError**: If parametric space dimension is less than 2.

```
def subset(self, splines, patches_to_keep):
```

• [View Source](#)

Create a subset of the multi-patch B-spline connectivity by keeping only selected patches.

Parameters

- **splines** (list[BSpline]): Array of B-spline patches to subset.
- **patches_to_keep** (numpy.array of int): Indices of patches to keep in the subset.

Returns

- **new_connectivity** (MultiPatchBSplineConnectivity): New connectivity object containing only the selected patches.
- **new_splines** (list[BSpline]): Array of B-spline patches for the selected patches.
- **new_unique_to_self_unique_connectivity** (numpy.ndarray of int): Array mapping new unique nodes to original unique nodes.

```
def save_block(
    self,
    splines,
    block,
    separated_ctrl_pts,
    path,
    name,
    n_step,
    n_eval_per_elem,
    separated_fields,
    fiels_on_interior_only
):
```

• [View Source](#)

Process a block of patches, saving the meshes in their corresponding file. Each block has its own progress bar.

```

def save_paraview(
    self,
    splines,
    separated_ctrl_pts,
    path,
    name,
    n_step=1,
    n_eval_per_elem=10,
    unique_fields={},
    separated_fields=None,
    verbose=True,
    fields_on_interior_only=True
):

```

• [View Source](#)

Save the multipatch B-spline data to Paraview format using parallel processing.

Parameters

- **splines** (list[B-spline]): Array of B-spline patches to save
- **separated_ctrl_pts** (list[numpy.ndarray]): Control points for each patch in separated representation
- **path** (str): Directory path where files will be saved
- **name** (str): Base name for the saved files
- **n_step** (int, optional): Number of time steps, by default 1
- **n_eval_per_elem** (int or list[int], optional): Number of evaluation points per element, by default 10
- **unique_fields** (dict, optional): Fields in unique representation to save, by default {}
- **separated_fields** (list[dict], optional): Fields in separated representation to save, by default None
- **verbose** (bool, optional): Whether to show progress bars, by default True
- **fields_on_interior_only** (bool, optional): Whether to save fields on interior only, by default True

Raises

- **NotImplementedError**: If a callable is passed in unique_fields
- **ValueError**: If pool is not running and cannot be restarted

class CouplesBSplineBorder:

• [View Source](#)

```

CouplesBSplineBorder(
    spline1_inds,
    spline2_inds,
    axes1,
    axes2,
    front_sides1,
    front_sides2,
    transpose_2_to_1,
    flip_2_to_1,
    NPa
)

```

• [View Source](#)

spline1_inds

spline2_inds

axes1

axes2

front_sides1

front_sides2

transpose_2_to_1

flip_2_to_1

NPa

nb_couples

@classmethod

def extract_border_pts(cls, field, axis, front_side, field_dim=1, offset=0):

• [View Source](#)

@classmethod

def extract_border_spline(cls, spline, axis, front_side):

• [View Source](#)

@classmethod

def transpose_and_flip(cls, field, transpose, flip, field_dim=1):

• [View Source](#)

@classmethod

def transpose_and_flip_knots(cls, knots, spans, transpose, flip):

• [View Source](#)

@classmethod

def transpose_and_flip_back_knots(cls, knots, spans, transpose, flip):

• [View Source](#)

@classmethod

def transpose_and_flip_spline(cls, spline, transpose, flip):

• [View Source](#)

@classmethod

def from_splines(cls, separated_ctrl_pts, splines):

• [View Source](#)

def append(self, other):

• [View Source](#)

def get_operator_allxi1_to_allxi2(self, spans1, spans2, couple_ind):

• [View Source](#)

def get_connectivity(self, shape_by_patch):

• [View Source](#)

def get_borders_couples(self, separated_field, offset=0):

• [View Source](#)

def get_borders_couples_splines(self, splines):

• [View Source](#)

def compute_border_couple_DN(self, couple_ind: int, splines: list, XI1_border: list, k1: list):

• [View Source](#)

bsplyne.b_spline_basis

- [View Source](#)



class BSplineBasis:

- [View Source](#)

BSpline basis in 1D.

Attributes

- **p** (int): Degree of the polynomials composing the basis.
- **knot** (numpy.array of float): Knot vector of the BSpline.
- **m** (int): Last index of the knot vector.
- **n** (int): Last index of the basis : when evaluated, returns an array of size **n** + 1.
- **span** (tuple of 2 float): Interval of definition of the basis.

BSplineBasis(p, knot)

- [View Source](#)

Create a **BSplineBasis** object that can compute its basis, and the derivatives of these functions.

Parameters

- **p** (int): Degree of the BSpline.
- **knot** (numpy.array of float): Knot vector of the BSpline.

Returns

- **BSplineBasis** (BSplineBasis instance): Contains the **BSplineBasis** object created.

Examples

Creation of a **BSplineBasis** instance of degree 2 and knot vector [0, 0, 0, 1, 1, 1] :

```
>>> BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1], dtype='float'))
```

p

knot

m

n

span

def linspace(self, n_eval_per_elem=10):

- [View Source](#)

Generate a set of xi values over the span of the basis.

Parameters

- **n_eval_per_elem** (int, optional): Number of values per element, by default 10

Returns

- **numpy.array of float**: Set of xi values over the span.

def linspace_for_integration(self, n_eval_per_elem=10, bounding_box=None):

- [View Source](#)

Generate a set of xi values over the span of the basis, centered on intervals of returned lengths.

Parameters

- **n_eval_per_elem** (int, optional): Number of values per element, by default 10

- **bounding_box** (numpy.array of float, optional): Lower and upper bounds, by default `self . span`

Returns

- **xi** (numpy.array of float): Set of xi values over the span.
- **dxi** (numpy.array of float): Integration weight of each point.

```
def gauss_legendre_for_integration(self, n_eval_per_elem=None, bounding_box=None):
```

• [View Source](#)

Generate a set of xi values with their corresponding weight according to the Gauss Legendre integration method over a given bounding box.

Parameters

- **n_eval_per_elem** (int, optional): Number of values per element, by default `self.p + 1`
- **bounding_box** (numpy.array of float, optional): Lower and upper bounds, by default `self . span`

Returns

- **xi** (numpy.array of float): Set of xi values over the span.
- **dxi** (numpy.array of float): Integration weight of each point.

```
def normalize_knots(self):
```

• [View Source](#)

Maps the knots vector to [0, 1].

```
def N(self, XI, k=0):
```

• [View Source](#)

Compute the `k`-th derivative of the BSpline basis functions for a set of values in the parametric space.

Parameters

- **XI** (numpy.array of float): Values in the parametric space at which the BSpline is evaluated.
- **k** (int, optional): `k`-th derivative of the BSpline evaluated. The default is 0.

Returns

- **DN** (scipy.sparse.coo_matrix of float): Sparse matrix containing the values of the `k`-th derivative of the BSpline basis functions in the rows for each value of `XI` in the columns.

Examples

Evaluation of the BSpline basis on these `XI` values : [0, 0.5, 1]

```
>>> basis = BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1], dtype='float'))
>>> basis.N(np.array([0, 0.5, 1], dtype='float')).A
array([[1. , 0. , 0. ],
       [0.25, 0.5 , 0.25],
       [0. , 0. , 1. ]])
```

Evaluation of the 1st derivative of the BSpline basis on these `XI` values : [0, 0.5, 1]

```
>>> basis = BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1], dtype='float'))
>>> basis.N(np.array([0, 0.5, 1], dtype='float'), k=1).A
array([[ -2.,  2.,  0.],
       [ -1.,  0.,  1.],
       [ 0., -2.,  2.]])
```

```
def plotN(self, k=0, show=True):
```

• [View Source](#)

Plots the basis functions over the span.

Parameters

- **k** (int, optional): **k** -th derivative of the BSpline plotted. The default is 0.
- **show** (bool, optional): Should the plot be displayed ? The default is True.

Returns

- **None.**

def knotInsertion(self, knots_to_add):

• [View Source](#)

Performs the knot inserion process on the `BSplineBasis` instance and returns the **D** matrix.

Parameters

- **knots_to_add** (numpy.array of float): Array of knots to append to the knot vector.

Returns

- **D** (scipy.sparse.coo_matrix of float): The matrix **D** such that : $\text{newCtrlPtsCoordinate} = \mathbf{D} @ \text{ancientCtrlPtsCoordinate}$.

Examples

Insert the knots [0.5, 0.5] to the `BSplineBasis` instance and return the operator to apply on the control points.

```
>>> basis = BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1], dtype='float'))
>>> basis.knotInsertion(np.array([0.5, 0.5], dtype='float')).A
array([[1. , 0. , 0. ],
       [0.5, 0.5, 0. ],
       [0.25, 0.5, 0.25],
       [0. , 0.5, 0.5 ],
       [0. , 0. , 1. ]])
```

The knot vector is modified (as well as n and m) :

```
>>> basis.knot
array([0. , 0. , 0. , 0.5, 0.5, 1. , 1. , 1. ])
```

def orderElevation(self, t):

• [View Source](#)

Performs the order elevation algorithm on the basis and return a linear transformation to apply on the control points.

Parameters

- **t** (int): New degree of the B-spline basis will be its current degree plus **t** .

Returns

- **STD** (scipy.sparse.coo_matrix of float): The matrix **STD** such that : $\text{newCtrlPtsCoordinate} = \mathbf{STD} @ \text{ancientCtrlPtsCoordinate}$.

Examples

Elevate the orderof the `BSplineBasis` instance by 1 and return the operator to apply on the control points.

```
>>> basis = BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1], dtype='float'))
>>> basis.orderElevation(1).A
array([[1. , 0. , 0. , 0. ],
       [0.33333333, 0.66666667, 0. , 0. ],
       [0. , 0.66666667, 0.33333333],
       [0. , 0. , 1. , 0. ]])
```

The knot vector and the degree are modified (as well as n and m) :

```
>>> basis.knot  
array([0., 0., 0., 0., 1., 1., 1., 1.]
```

```
>>> basis.p  
3
```




- [View Source](#)

class BSpline:

- [View Source](#)

B-Spline class for representing and manipulating B-spline curves, surfaces and volumes.

Provides functionality for evaluating, manipulating and visualizing B-splines of arbitrary dimension. Supports knot insertion, order elevation, and visualization through Paraview and Matplotlib.

Attributes

- **NPa** (int): Dimension of the parametric space.
- **bases** (np.ndarray[BSplineBasis]): Array containing BSplineBasis instances for each parametric dimension.

```
BSpline(
    degrees: Iterable[int],
    knots: Iterable[numpy.ndarray[Any, numpy.dtype[numpy.floating]]]
)
```

- [View Source](#)

Initialize a [BSpline](#) instance with specified degrees and knot vectors.

Creates a [BSpline](#) object by generating basis functions for each isoparametric dimension using the provided polynomial degrees and knot vectors.

Parameters

- **degrees** (Iterable[int]): Collection of polynomial degrees for each isoparametric dimension. The length determines the dimensionality of the parametric space ([NPa](#)). For example:
 - [p] for a curve
 - [p, q] for a surface
 - [p, q, r] for a volume
 - ...
- **knots** (Iterable[npt.NDArray[np.floating]]): Collection of knot vectors for each isoparametric dimension. Each knot vector must be a numpy array of [floats](#) . The number of knot vectors must match the number of degrees. For a degree [p](#) , the knot vector must have size [m + 1](#) where [m >= p](#) .

Examples

Create a 2D B-spline surface:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
```

Create a 1D B-spline curve:

```
>>> degree = [3]
>>> knot = [np.array([0, 0, 0, 0, 1, 1, 1, 1], dtype='float')]
>>> curve = BSpline(degree, knot)
```

Notes

- The number of control points in each dimension will be [m - p](#) where [m](#) is the size of the knot vector minus 1 and [p](#) is the degree
- Each knot vector must be non-decreasing
- The multiplicity of each knot must not exceed [p + 1](#)

NPa: int

bases: numpy.ndarray[[bsplyne.b_spline_basis.BSplineBasis](#)]

@classmethod

def from_bases(

[cls](#),

bases: Iterable[[bsplyne.b_spline_basis.BSplineBasis](#)]

) -> [BSpline](#):

• [View Source](#)

Create a [BSpline](#) instance from an array of [BSplineBasis](#) objects. This is an alternative constructor that allows direct initialization from existing basis functions rather than creating new ones from degrees and knot vectors.

Parameters

- **bases** (Iterable[[BSplineBasis](#)]): An iterable (e.g. list, tuple, array) containing [BSplineBasis](#) instances. Each basis represents one parametric dimension of the resulting B-spline. The number of bases determines the dimensionality of the parametric space.

Returns

- **BSpline**: A new [BSpline](#) instance with the provided basis functions.

Examples

```
>>> basis1 = BSplineBasis(2, np.array([0, 0, 0, 1, 1, 1]))
>>> basis2 = BSplineBasis(2, np.array([0, 0, 0, 0.5, 1, 1, 1]))

>>> spline = BSpline.from_bases([basis1, basis2])
```

def getDegrees([self](#)) -> numpy.ndarray[typing.Any, numpy.dtype[numpy.integer]]:

• [View Source](#)

Returns the polynomial degree of each basis function in the isoparametric space.

Returns

- **degrees** (npt.NDArray[np.integer]): Array containing the polynomial degrees of the B-spline basis functions. The array has length [NPa](#) (dimension of isoparametric space), where each element represents the degree of the corresponding isoparametric dimension.

Examples

```
>>> degrees = np.array([2, 2], dtype='int')
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> spline.getDegrees()
array([2, 2])
```

Notes

- For a curve (1D), returns [degree_xi]
- For a surface (2D), returns [degree_xi, degree_eta]
- For a volume (3D), returns [degree_xi, degree_eta, degree_zeta]
- ...

def getKnots([self](#)) -> list[numpy.ndarray[typing.Any, numpy.dtype[numpy.floating]]]:

• [View Source](#)

Returns the knot vector of each basis function in the isoparametric space.

This method collects all knot vectors from each [BSplineBasis](#) instance stored in the [bases](#) array. The knot vectors define the isoparametric space partitioning and the regularity properties of the B-spline.

Returns

- **knots** (list[npt.NDArray[np.floating]]): List containing the knot vectors of the B-spline basis functions. The list has length `NPa` (dimension of isoparametric space), where each element is a `numpy.ndarray` containing the knots for the corresponding isoparametric dimension.

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> spline.getKnots()
[array([0., 0., 0., 0.5, 1., 1., 1.]),
 array([0., 0., 0., 0.5, 1., 1., 1.])]
```

Notes

- For a curve (1D), returns [`knots_xi`]
- For a surface (2D), returns [`knots_xi` , `knots_eta`]
- For a volume (3D), returns [`knots_xi` , `knots_eta` , `knots_zeta`]
- Each knot vector must be non-decreasing
- The multiplicity of interior knots determines the continuity at that point

def getNbFunc(self) -> int:

• [View Source](#)

Compute the total number of basis functions in the B-spline.

This method calculates the total number of basis functions by multiplying the number of basis functions in each isoparametric dimension (`n + 1` for each dimension).

Returns

- **int:** Total number of basis functions in the B-spline. This is equal to the product of (`n + 1`) for each basis, where `n` is the last index of each basis function.

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> spline.getNbFunc()
16
```

Notes

- For a curve (1D), returns (`n + 1`)
- For a surface (2D), returns (`n1 + 1`) × (`n2 + 1`)
- For a volume (3D), returns (`n1 + 1`) × (`n2 + 1`) × (`n3 + 1`)
- The number of basis functions equals the number of control points needed

def getSpans(self) -> list[tuple[float, float]]:

• [View Source](#)

Returns the span of each basis function in the isoparametric space.

This method collects the spans (intervals of definition) from each `BSplineBasis` instance stored in the `bases` array.

Returns

- **spans** (list[tuple[float, float]]): List containing the spans of the B-spline basis functions. The list has length `NPa` (dimension of isoparametric space), where each element is a tuple (`a` , `b`) containing the lower and upper bounds of the span for the corresponding isoparametric dimension.

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> spline.getSpans()
[(0.0, 1.0), (0.0, 1.0)]
```

Notes

- For a curve (1D), returns [(*xi_min* , *xi_max*)]
- For a surface (2D), returns [(*xi_min* , *xi_max*), (*eta_min* , *eta_max*)]
- For a volume (3D), returns [(*xi_min* , *xi_max*), (*eta_min* , *eta_max*), (*zeta_min* , *zeta_max*)]
- The span represents the interval where the B-spline is defined
- Each span is determined by the *p* -th and (*m* - *p*) -th knots, where *p* is the degree and *m* is the last index of the knot vector

def linspace(

self,

n_eval_per_elem: Union[int, Iterable[int]] = 10

) -> tuple[numpy.ndarray[typing.Any, numpy.dtype[numpy.floating]], ...]:

• [View Source](#)

Generate sets of evaluation points over the span of each basis in the isoparametric space.

This method creates evenly spaced points for each isoparametric dimension by calling `linspace` on each `BSplineBasis` instance stored in the `bases` array.

Parameters

- ***n_eval_per_elem*** (Union[int, Iterable[int]], optional): Number of evaluation points per element for each isoparametric dimension. If an `int` is provided, the same number is used for all dimensions. If an `Iterable` is provided, each value corresponds to a different dimension. By default, 10.

Returns

- ***XI*** (tuple[npt.NDArray[np.floating], ...]): Tuple containing arrays of evaluation points for each isoparametric dimension. The tuple has length `NPa` (dimension of isoparametric space).

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> xi, eta = spline.linspace(n_eval_per_elem=2)
>>> xi
array([0. , 0.25, 0.5 , 0.75, 1.  ])
>>> eta
array([0. , 0.5, 1.  ])
```

Notes

- For a curve (1D), returns (*xi* points,)
- For a surface (2D), returns (*xi* points, *eta* points)
- For a volume (3D), returns (*xi* points, *eta* points, *zeta* points)
- The number of points returned for each dimension depends on the number of elements in that dimension times the value of `n_eval_per_elem`

```
def linspace_for_integration(
    self,
    n_eval_per_elem: Union[int, Iterable[int]] = 10,
    bounding_box: Optional[Iterable] = None
) -> tuple[tuple[numpy.ndarray[typing.Any, numpy.dtype[numpy.float64]], ...], tuple[numpy.ndarray[typing.Any
```

• [View Source](#)

Generate sets of evaluation points and their integration weights over each basis span.

This method creates evenly spaced points and their corresponding integration weights for each isoparametric dimension by calling `linspace_for_integration` on each `BSplineBasis` instance stored in the `bases` array.

Parameters

- **n_eval_per_elem** (Union[int, Iterable[int]], optional): Number of evaluation points per element for each isoparametric dimension. If an `int` is provided, the same number is used for all dimensions. If an `Iterable` is provided, each value corresponds to a different dimension. By default, 10.
- **bounding_box** (Union[Iterable[tuple[float, float]], None], optional): Lower and upper bounds for each isoparametric dimension. If `None`, uses the span of each basis. Format: [(`xi_min`, `xi_max`), (`eta_min`, `eta_max`), ...]. By default, `None`.

Returns

- **XI** (tuple[npt.NDArray[np.float64], ...]): Tuple containing arrays of evaluation points for each isoparametric dimension. The tuple has length `NPa` (dimension of isoparametric space).
- **dXI** (tuple[npt.NDArray[np.float64], ...]): Tuple containing arrays of integration weights for each isoparametric dimension. The tuple has length `NPa` (dimension of isoparametric space).

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> (xi, eta), (dxi, deta) = spline.linspace_for_integration(n_eval_per_elem=2)
>>> xi # xi points
array([0.125, 0.375, 0.625, 0.875])
>>> dxi # xi weights
array([0.25, 0.25, 0.25, 0.25])
```

Notes

- For a curve (1D), returns ((`xi` points), (`xi` weights))
- For a surface (2D), returns ((`xi` points, `eta` points), (`xi` weights, `eta` weights))
- For a volume (3D), returns ((`xi` points, `eta` points, `zeta` points), (`xi` weights, `eta` weights, `zeta` weights))
- The points are centered in their integration intervals
- The weights represent the size of the integration intervals

```
def gauss_legendre_for_integration(
    self,
    n_eval_per_elem: Union[int, Iterable[int], NoneType] = None,
    bounding_box: Optional[Iterable] = None
) -> tuple[tuple[numpy.ndarray[typing.Any, numpy.dtype[numpy.float64]], ...], tuple[numpy.ndarray[typing.Any
```

• [View Source](#)

Generate sets of evaluation points and their Gauss-Legendre integration weights over each basis span.

This method creates Gauss-Legendre quadrature points and their corresponding integration weights for each

isoparametric dimension by calling `gauss_legendre_for_integration` on each `BSplineBasis` instance stored in the `bases` array.

Parameters

- **n_eval_per_elem** (Union[int, Iterable[int], None], optional): Number of evaluation points per element for each isoparametric dimension. If an `int` is provided, the same number is used for all dimensions. If an `Iterable` is provided, each value corresponds to a different dimension. If `None`, uses $(p + 2) // 2$ points per element where `p` is the degree of each basis. This number of points ensures an exact integration of a `p`-th degree polynomial. By default, `None`.
- **bounding_box** (Union[Iterable[tuple[float, float]], None], optional): Lower and upper bounds for each isoparametric dimension. If `None`, uses the span of each basis. Format: `((xi_min, xi_max), (eta_min, eta_max), ...)`. By default, `None`.

Returns

- **XI** (tuple[npt.NDArray[np.floating], ...]): Tuple containing arrays of Gauss-Legendre points for each isoparametric dimension. The tuple has length `NPa` (dimension of isoparametric space).
- **dXI** (tuple[npt.NDArray[np.floating], ...]): Tuple containing arrays of Gauss-Legendre weights for each isoparametric dimension. The tuple has length `NPa` (dimension of isoparametric space).

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> (xi, eta), (dxi, deta) = spline.gauss_legendre_for_integration()

>>> xi # xi points
array([0.10566243, 0.39433757, 0.60566243, 0.89433757])
>>> dxi # xi weights
array([0.25, 0.25, 0.25, 0.25])
```

Notes

- For a curve (1D), returns `((xi points), (xi weights))`
- For a surface (2D), returns `((xi points, eta points), (xi weights, eta weights))`
- For a volume (3D), returns `((xi points, eta points, zeta points), (xi weights, eta weights, zeta weights))`
- The points and weights follow the Gauss-Legendre quadrature rule
- When `n_eval_per_elem` is `None`, uses $(p + 2) // 2$ points per element for exact integration of polynomials up to degree `p`

def normalize_knots(self):

• [View Source](#)

Maps all knot vectors to the interval `[0, 1]` in each isoparametric dimension.

This method normalizes the knot vectors of each `BSplineBasis` instance stored in the `bases` array by applying an affine transformation that maps the span interval to `[0, 1]`.

Examples

```
>>> degrees = [2, 2]
>>> knots = [np.array([-1, -1, -1, 0, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 2, 4, 4, 4], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> spline.getKnots()
[array([-1., -1., -1., 0., 1., 1., 1.]),
 array([0., 0., 0., 2., 4., 4., 4.])]
>>> spline.normalize_knots()
>>> spline.getKnots()
[array([0., 0., 0., 0.5, 1., 1., 1.]),
 array([0., 0., 0., 0.5, 1., 1., 1.])]
```

Notes

- The transformation preserves the relative spacing between knots
- The transformation preserves the multiplicity of knots
- The transformation is applied independently to each isoparametric dimension
- This operation modifies the knot vectors in place

def DN(

self,

XI: Union[numpy.ndarray[Any, numpy.dtype[numpy.floating]], tuple[numpy.ndarray[Any, numpy.dtype[numpy.

k: Union[int, Iterable[int]] = 0

) -> Union[scipy.sparse._base.spmatrix, numpy.ndarray[scipy.sparse._base.spmatrix]]:

• [View Source](#)

Compute the k -th derivative of the B-spline basis at given points in the isoparametric space.

This method evaluates the basis functions or their derivatives at specified points, returning a matrix that can be used to evaluate the B-spline through a dot product with the control points.

Parameters

- **XI** (Union[npt.NDArray[np.floating], tuple[npt.NDArray[np.floating], ...]]): Points in the isoparametric space where to evaluate the basis functions. Two input formats are accepted:
 1. **numpy.ndarray** : Array of coordinates with shape (**NPa**, **n_points**). Each column represents one evaluation point [**xi**, **eta**, ...]. The resulting matrices will have shape (**n_points**, number of functions).
 2. **tuple** : Contains **NPa** arrays of coordinates (**xi**, **eta**, ...). The resulting matrices will have (**n_xi** × **n_eta** × ...) rows.
- **k** (Union[int, Iterable[int]], optional): Derivative orders to compute. Two formats are accepted:
 1. **int** : Same derivative order along all axes. Common values:
 - **k=0** : Evaluate basis functions (default)
 - **k=1** : Compute first derivatives (gradient)
 - **k=2** : Compute second derivatives (hessian)
 2. **list[int]** : Different derivative orders for each axis. Example: **[1, 0]** computes first derivative w.r.t **xi**, no derivative w.r.t **eta**. By default, 0.

Returns

- **DN** (Union[sps.spmatrix, np.ndarray[sps.spmatrix]]): Sparse matrix or array of sparse matrices containing the basis evaluations:
 - If **k** is a **list** or is 0: Returns a single sparse matrix containing the mixed derivative specified by the list.
 - If **k** is an **int** > 0: Returns an array of sparse matrices with shape [**NPa**] * **k**. For example, if **k=1**, returns **NPa** matrices containing derivatives along each axis.

Examples

Create a 2D quadratic B-spline:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
```

Evaluate basis functions at specific points using array input:

```
>>> XI = np.array([[0, 0.5, 1],      # xi coordinates
...               [0, 0.5, 1]])      # eta coordinates
>>> N = spline.DN(XI, k=0)
>>> N.A # Convert sparse matrix to dense for display
array([[1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0., 0.25, 0.25, 0., 0., 0.25, 0.25, 0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1.]])
```

Compute first derivatives using tuple input:

```
>>> xi = np.array([0, 0.5])
>>> eta = np.array([0, 1])
>>> dN = spline.DN((xi, eta), k=1) # Returns [NPa] matrices
>>> len(dN) # Number of derivative matrices
2
>>> dN[0].A # Derivative w.r.t xi
array([[ -4.,  0.,  0.,  0.,  4.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0., -4.,  0.,  0.,  0.,  4.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0., -2.,  0.,  0.,  0.,  2.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -2.,  0.,  0.,  0.,  2.,  0.,  0.,  0.,  0.]])

>>> dN[1].A # Derivative w.r.t eta
array([[ -4.,  4.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0., -4.,  4.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0., -2.,  2.,  0.,  0., -2.,  2.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0., -2.,  2.,  0.,  0., -2.,  2.,  0.,  0.,  0.,  0.]])
```

Compute mixed derivatives:

```
>>> d2N = spline.DN((xi, eta), k=[1, 1]) # Second derivative: d²/dxi·deta
>>> d2N.A
array([[16., -16.,  0.,  0., -16., 16.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0., 16., -16.,  0.,  0., -16., 16.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  8., -8.,  0.,  0., -8.,  8.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  8., -8.,  0., -0., -8.,  8.,  0.,  0.,  0.,  0.]])
```

Notes

- For evaluating the B-spline with control points in NPh -D space: `values = DN @ ctrlPts.reshape((NPh, -1)).T`
- When using tuple input format for XI , points are evaluated at all combinations of coordinates
- When using array input format for XI , each column represents one evaluation point
- The gradient ($k=1$) returns NPa matrices for derivatives along each axis
- Mixed derivatives can be computed using a list of derivative orders

def knotInsertion(

`self,`

`ctrlPts: numpy.ndarray,`

`knots_to_add: Iterable[Union[numpy.ndarray[Any, numpy.dtype[numpy.float64]], int]]`

) -> `numpy.ndarray[typing.Any, numpy.dtype[numpy.floating]]`:

• [View Source](#)

Add knots to the B-spline while preserving its geometry.

This method performs knot insertion by adding new knots to each isoparametric dimension and computing the new control points to maintain the exact same geometry. The method modifies the `BSpline` object by updating

its basis functions with the new knots.

Parameters

- **ctrlPts** (npt.NDArray[np.floating]): Control points defining the B-spline geometry. Shape: (**NPh** , n1, n2, ...) where:
 - **NPh** is the dimension of the physical space
 - ni is the number of control points in the i-th isoparametric dimension
- **knots_to_add** (Iterable[Union[npt.NDArray[np.floating], int]]): Refinement specification for each isoparametric dimension. For each dimension, two formats are accepted:
 1. **numpy.ndarray** : Array of knots to insert. These knots must lie within the span of the existing knot vector.
 2. **int** : Number of equally spaced knots to insert in each element.

Returns

- **new_ctrlPts** (npt.NDArray[np.floating]): New control points after knot insertion. Shape: (**NPh** , m1, m2, ...) where $m_i \geq n_i$ is the new number of control points in the i-th isoparametric dimension.

Examples

Create a 2D quadratic B-spline and insert knots:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...         np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')
... ]
>>> spline = BSpline(degrees, knots)
>>> ctrlPts = np.random.rand(3, 4, 4) # 3D control points
```

Insert specific knots in first dimension only:

```
>>> knots_to_add = [np.array([0.25, 0.75], dtype='float'),
...                 np.array([], dtype='float')]
>>> new_ctrlPts = spline.knotInsertion(ctrlPts, knots_to_add)
>>> new_ctrlPts.shape
(3, 6, 4) # Two new control points added in first dimension
>>> spline.getKnots()[0] # The knot vector is modified
array([0. , 0. , 0. , 0.25, 0.5 , 0.75, 1. , 1. , 1. ])
```

Insert two knots per element in both dimensions:

```
>>> new_ctrlPts = spline.knotInsertion(new_ctrlPts, [1, 1])
>>> new_ctrlPts.shape
(3, 10, 6) # Uniform refinement in both dimensions
>>> spline.getKnots()[0] # The knot vectors are further modified
array([0. , 0. , 0. , 0.125, 0.25 , 0.375, 0.5 , 0.625, 0.75 ,
       0.875, 1. , 1. , 1. , 1. ])
```

Notes

- Knot insertion preserves the geometry and parameterization of the B-spline
- The number of new control points depends on the number and multiplicity of inserted knots
- When using integer input, knots are inserted with uniform spacing in each element
- The method modifies the basis functions but maintains C^{p-m} continuity, where p is the degree and m is the multiplicity of the inserted knot

```
def orderElevation(
    self,
    ctrlPts: numpy.ndarray,
    t: Iterable[int]
) -> numpy.ndarray[typing.Any, numpy.dtype[numpy.floating]]:
```

• [View Source](#)

Elevate the polynomial degree of the B-spline while preserving its geometry.

This method performs order elevation by increasing the polynomial degree of each isoparametric dimension and computing the new control points to maintain the exact same geometry. The method modifies the [BSpline](#) object by updating its basis functions with the new degrees.

Parameters

- **ctrlPts** (npt.NDArray[np.floating]): Control points defining the B-spline geometry. Shape: (**NPh** , n1, n2, ...) where:
 - **NPh** is the dimension of the physical space
 - ni is the number of control points in the i-th isoparametric dimension
- **t** (Iterable[int]): Degree elevation for each isoparametric dimension. For each dimension i, the new degree will be $p_i + t_i$ where p_i is the current degree.

Returns

- **new_ctrlPts** (npt.NDArray[np.floating]): New control points after order elevation. Shape: (**NPh** , m1, m2, ...) where $m_i \geq n_i$ is the new number of control points in the i-th isoparametric dimension.

Examples

Create a 2D quadratic B-spline and elevate its order:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> ctrlPts = np.random.rand(3, 4, 4) # 3D control points
```

Elevate order by 1 in first dimension only:

```
>>> t = [1, 0] # Increase degree by 1 in first dimension
>>> new_ctrlPts = spline.orderElevation(ctrlPts, t)
>>> new_ctrlPts.shape
(3, 6, 4) # Two new control points added in first dimension (one per element)

>>> spline.getDegrees() # The degrees are modified
array([3, 2])
```

Notes

- Order elevation preserves the geometry and parameterization of the B-spline
- The number of new control points depends on the current degree and number of elements
- The method modifies the [BSpline](#) object by updating its basis functions
- This operation is more computationally expensive than knot insertion

```
def greville_abscissa(
    self,
    return_weights: bool = False
) -> Union[list[numpy.ndarray[Any, numpy.dtype[numpy.floating]]], tuple[list[numpy.ndarray[Any, numpy.dtype
```

• [View Source](#)

Compute the Greville abscissa and optionally their weights for each isoparametric dimension.

The Greville abscissa can be interpreted as the "position" of the control points in the isoparametric space. They are often used as interpolation points for B-splines.

Parameters

- **return_weights** (bool, optional): If **True** , also returns the weights (span lengths) of each basis function. By default, False.

Returns

- **greville** (list[npt.NDArray[np.floating]]): List containing the Greville abscissa for each isoparametric dimension. The list has length **NPa** , where each element is an array of size **n + 1** , **n** being the last index of the basis functions in that dimension.
- **weights** (list[npt.NDArray[np.floating]], optional): Only returned if **return_weights** is **True** . List containing the weights for each isoparametric dimension. The list has length **NPa** , where each element is an array containing the span length of each basis function.

Examples

Compute Greville abscissa for a 2D B-spline:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> greville = spline.greville_abscissa()
>>> greville[0] # xi coordinates
array([0. , 0.25, 0.75, 1. ])
>>> greville[1] # eta coordinates
array([0. , 0.5, 1. ])
```

Compute both abscissa and weights:

```
>>> greville, weights = spline.greville_abscissa(return_weights=True)
>>> weights[0] # weights for xi direction
array([0.5, 1. , 1. , 0.5])
```

Notes

- For a curve (1D), returns [**xi** abscissa]
- For a surface (2D), returns [**xi** abscissa, **eta** abscissa]
- For a volume (3D), returns [**xi** abscissa, **eta** abscissa, **zeta** abscissa]
- The Greville abscissa are computed as averages of **p** consecutive knots
- The weights represent the size of the support of each basis function
- The number of abscissa in each dimension equals the number of control points

```
def saveParaview(
    self,
    ctrlPts: numpy.ndarray,
    path: str,
    name: str,
    n_step: int = 1,
    n_eval_per_elem: Union[int, Iterable[int]] = 10,
    fields: Optional[dict] = None,
    groups: Optional[dict[str, dict[str, Union[str, int]]]] = None,
    make_pvd: bool = True,
    verbose: bool = True,
    fiels_on_interior_only: bool = True
) -> dict[str, dict[str, typing.Union[str, int]]]:
```

• [View Source](#)

Save B-spline visualization data as Paraview files.

This method creates three types of visualization files:

- Interior mesh showing the B-spline surface/volume
- Element borders showing the mesh structure
- Control points mesh showing the control structure

All files are saved in VTU format with an optional PVD file to group them.

Parameters

- **ctrlPts** (npt.NDArray[np.floating]): Control points defining the B-spline geometry. Shape: (**NPh** , n1, n2, ...) where:
 - **NPh** is the dimension of the physical space
 - ni is the number of control points in the i-th isoparametric dimension
 - **path** (str): Directory path where the PV files will be saved
 - **name** (str): Base name for the output files
 - **n_step** (int, optional): Number of time steps to save. By default, 1.
 - **n_eval_per_elem** (Union[int, Iterable[int]], optional): Number of evaluation points per element for each isoparametric dimension. By default, 10.
 - If an **int** is provided, the same number is used for all dimensions.
 - If an **Iterable** is provided, each value corresponds to a different dimension.
 - **fields** (Union[dict, None], optional): Fields to visualize at each time step. Dictionary format: { "field_name": **field_value** } where **field_value** can be either:
 1. A numpy array with shape (**n_step** , **size** , ***ctrlPts.shape[1:]**) where:
 - **n_step** : Number of time steps
 - **size** : Size of the field at each point (1 for scalar, 3 for vector)
 - ***ctrlPts.shape[1:]** : Same shape as control points (excluding **NPh**)
 2. A function that computes field values (**npt.NDArray[np.floating]**) at given points from the **BSpline** instance and **xi** , the tuple of arrays containing evaluation points for each dimension (**tuple[npt.NDArray[np.floating], ...]**). The result should be an array of shape (**n_step** , **n_points** , **size**) where:
 - **n_step** : Number of time steps
 - **n_points** : Number of evaluation points (n_xi × n_eta × ...)
 - **size** : Size of the field at each point (1 for scalar, 3 for vector)
- By default, None.
- **groups** (Union[dict[str, dict[str, Union[str, int]]], None], optional): Nested dictionary specifying file groups for PVD organization. Format: { "group_name": { "ext": str, # File extension (e.g., "vtu") "npart": int, # Number of parts in the group "nstep": int # Number of timesteps } } The method automatically creates/updates three groups:
 - "interior": For the B-spline surface/volume mesh
 - "elements_borders": For the element boundary mesh
 - "control_points": For the control point meshIf provided, existing groups are updated; if None, these groups are created. By default, None.
 - **make_pvd** (bool, optional): Whether to create a PVD file grouping all VTU files. By default, True.
 - **verbose** (bool, optional): Whether to print progress information. By default, True.
 - **fiels_on_interior_only** (bool, optional): Whether to save fields only on the interior mesh (**True**) or on all meshes (**False**). By default, True.

Returns

- **groups** (dict[str, dict[str, Union[str, int]]]): Updated groups dictionary with information about saved files.

Examples

Save a 2D B-spline visualization:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float'),

...         np.array([0, 0, 0, 0.5, 1, 1, 1], dtype='float')
]
>>> spline = BSpline(degrees, knots)
>>> ctrlPts = np.random.rand(3, 4, 4) # 3D control points
>>> spline.saveParaview(ctrlPts, "./output", "bspline")
```

Save with a custom field:

```
>>> def displacement(spline, XI):
...     # Compute displacement field
...     return np.random.rand(1, np.prod([x.size for x in XI]), 3)
>>> fields = {"displacement": displacement}
>>> spline.saveParaview(ctrlPts, "./output", "bspline", fields=fields
)
```

Notes

- Creates three types of VTU files for each time step:
 - {name}_interior_{part}_{step}.vtu
 - {name}_elements_borders_{part}_{step}.vtu
 - {name}_control_points_{part}_{step}.vtu
- If `make_pvd=True`, creates a PVD file named {name}.pvd
- Fields can be visualized as scalars or vectors in Paraview
- The method supports time-dependent visualization through `n_step`

def getGeomdl(self, ctrl_pts):

• [View Source](#)

```
def plotMPL(
    self,
    ctrl_pts: numpy.ndarray,
    n_eval_per_elem: Union[int, Iterable[int]] = 10,
    ax: Optional[matplotlib.axes._axes.Axes] = None,
    ctrl_color: str = '#1b9e77',
    interior_color: str = '#7570b3',
    elem_color: str = '#666666',
    border_color: str = '#d95f02'
):
```

• [View Source](#)

Plot the B-spline using Matplotlib.

Creates a visualization of the B-spline geometry showing the control mesh, B-spline surface/curve, element borders, and patch borders. Supports plotting 1D curves and 2D surfaces in 2D space, and 2D surfaces and 3D volumes in 3D space.

Parameters

- **ctrl_pts** (numpy.ndarray of float): Control points defining the B-spline geometry. Shape: (NPh, n1, n2, ...) where:
 - NPh is the dimension of the physical space (2 or 3)
 - ni is the number of control points in the i-th isoparametric dimension
- **n_eval_per_elem** (Union[int, Iterable[int]], optional): Number of evaluation points per element for visualizing the B-spline. Can be specified as:
 - Single integer: Same number for all dimensions
 - Iterable of integers: Different numbers for each dimension By default, 10.
- **ax** (Union[mpl.axes.Axes, None], optional): Matplotlib axes for plotting. If None, creates a new figure and axes. For 3D visualizations, must be a 3D axes if provided (created with `projection='3d'`). Default is None (creates new axes).
- **ctrl_color** (str, optional): Color for the control mesh visualization:

- Applied to control points (markers)
- Applied to control mesh lines Default is '#1b9e77' (green).
- **interior_color** (str, optional): Color for the B-spline geometry:
 - For curves: Line color
 - For surfaces: Face color (with transparency)
 - For volumes: Face color of boundary surfaces (with transparency) Default is '#7570b3' (purple).
- **elem_color** (str, optional): Color for element boundary visualization:
 - Shows internal mesh structure
 - Helps visualize knot locations Default is '#666666' (gray).
- **border_color** (str, optional): Color for patch boundary visualization:
 - Outlines the entire B-spline patch
 - Helps distinguish patch edges Default is '#d95f02' (orange).

Examples

Plot a 2D curve in 2D space:

```
>>> degrees = [2]
>>> knots = [np.array([0, 0, 0, 1, 1, 1], dtype='float')]

>>> spline = BSpline(degrees, knots)
>>> ctrl_pts = np.random.rand(2, 3) # 2D control points
>>> spline.plotMPL(ctrl_pts)
```

Plot a 2D surface in 3D space:

```
>>> degrees = [2, 2]
>>> knots = [np.array([0, 0, 0, 1, 1, 1], dtype='float'),
...          np.array([0, 0, 0, 1, 1, 1], dtype='float')]
>>> spline = BSpline(degrees, knots)
>>> ctrl_pts = np.random.rand(3, 3, 3) # 3D control points

>>> spline.plotMPL(ctrl_pts)
```

Plot on existing axes with custom colors:

```
>>> fig = plt.figure()
>>> ax = fig.add_subplot(projection='3d')
>>> spline.plotMPL(ctrl_pts, ax=ax, ctrl_color='red', interior_color='blue')
)
```

Notes

Visualization components:

- Control mesh: Shows control points and their connections
- B-spline: Shows the actual curve/surface/volume
- Element borders: Shows the boundaries between elements
- Patch borders: Shows the outer boundaries of the B-spline

Supported configurations:

- 1D B-spline in 2D space (curve)
- 2D B-spline in 2D space (surface)
- 2D B-spline in 3D space (surface)
- 3D B-spline in 3D space (volume)

For 3D visualization:

- Surfaces are shown with transparency
- Volume visualization shows the faces with transparency
- View angle is automatically set for surfaces based on surface normal



- [View Source](#)

```
def writePVD(fileName: str, groups: dict):
```

- [View Source](#)

Writes a Paraview Data (PVD) file that references multiple VTK files.

Creates an XML-based PVD file that collects and organizes multiple VTK files into groups, allowing visualization of multi-part, time-series data in Paraview.

Parameters

- **fileName** (str): Base name for the mesh files (without numbers and extension)
- **groups** (dict[str, dict]): Nested dictionary specifying file groups with format: {"group_name": {"ext": "file_extension", "npart": num_parts, "nstep": num_timesteps}}

Notes

VTK files must follow naming pattern: {fileName}_{group}_{part}_{timestep}.{ext} Example: for fileName="mesh", group="fluid", part=1, timestep=5, ext="vtu": mesh_fluid_1_5.vtu

Returns

- **None**

```
def merge_meshes(meshes: Iterable[meshio._mesh.Mesh]) -> meshio._mesh.Mesh:
```

- [View Source](#)

Merges multiple meshio.Mesh objects into a single mesh.

Parameters

- **meshes** (Iterable[io.Mesh]): An iterable of meshio.Mesh objects to merge.

Returns

- **io.Mesh**: A single meshio.Mesh object containing the merged meshes with combined vertices, cells and point data.

```
def merge_saves(
    path: str,
    name: str,
    nb_patches: int,
    nb_steps: int,
    group_names: list
) -> None:
```

- [View Source](#)

Merge multiple mesh files and save the merged results.

This function reads multiple mesh files for each group and time step, merges them into a single mesh, and writes the merged mesh to a new file. It also generates a PVD file to describe the collection of merged meshes.

Parameters

- **path** (str): The directory path where the mesh files are located.
- **name** (str): The base name of the mesh files.
- **nb_patches** (int): The number of patches to merge for each group and time step.
- **nb_steps** (int): The number of time steps for which meshes are available.
- **group_names** (list[str]): A list of group names to process.

Returns

- **None**