CS589 ASSIGNMENT 4

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Question 1

- In theory, increasing the numbers of clusters should retain more of the original data and therefore cost more but be more accurate. However, a lot of clusters are consequently prone to overfitting the data. The idea behind the 'elbow' rule is to find the the point where adding another cluster is not longer worth the extra information in comparison to the cost. The "elbow" is that cutoff point representing the optimal number of clusters to choose.
- K-means doe not guarantee to find optimal solution due to its dependence on initialization. K-means++ aims to give you some guaranteed approximation factor by cleverly choosing initial cluster centers. The cluster centers are based on initial points and K-means++ tries to maximize the distance between the cluster centers. It turns out that this initialization step gives you the guarantee.

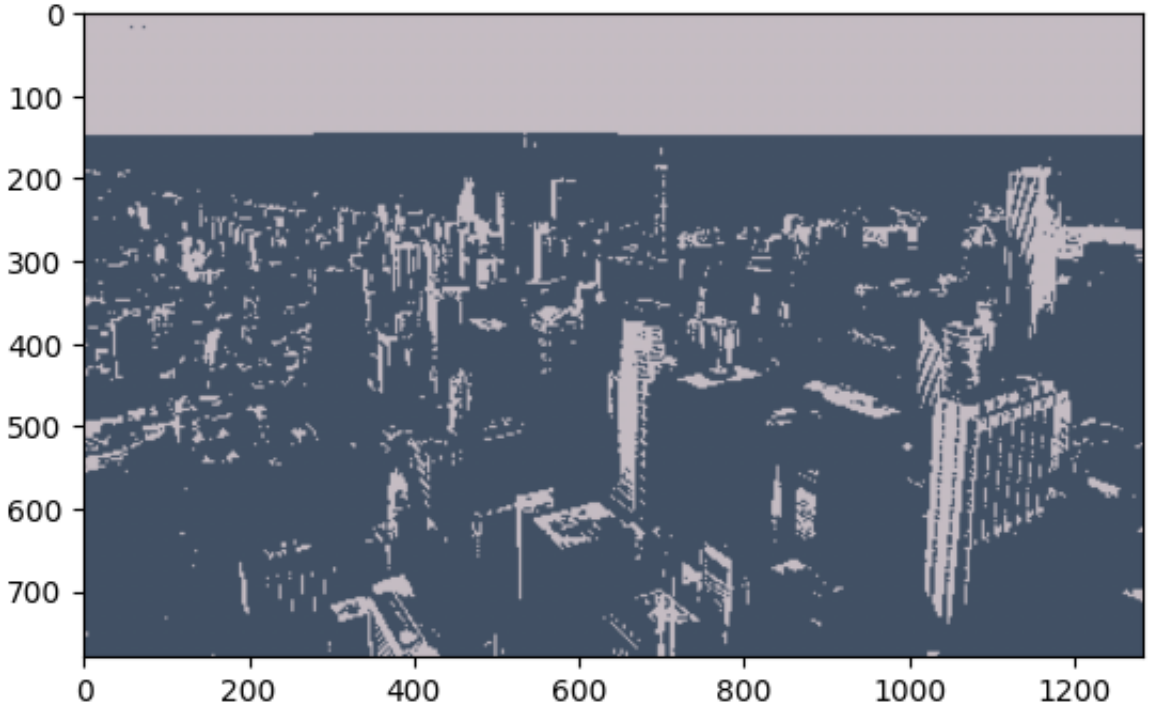
Question 2

In [52]:
```
import numpy as np
import sklearn
import matplotlib.image as mpimg
import matplotlib.pyplot as plt
```

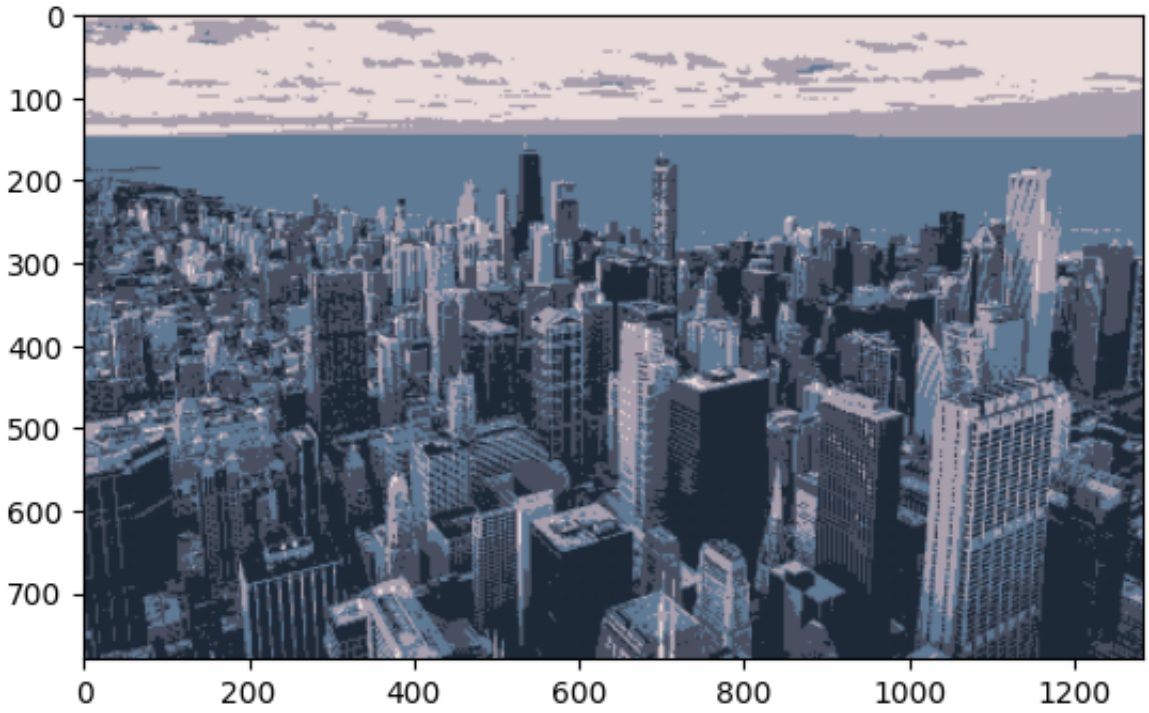
In [53]:
```
import sklearn.cluster

def find_chunk(y,x, img_x):
    return (y//3)*(img_x//3) + x//3

def flat_chunk(chunk):
    res = []
    for row in chunk:
        for pixel in row:
            for c in pixel:
                res.append(c.astype(np.float32))
    return res

k_list = [2, 5, 10, 50, 200, 500, 1000, 2000]
```

```
img = mpimg.imread('cityview.png') # colors between 0 and 1

# split image into 3 by 3 chunks
index_chunk_dict = {}
chunks = np.array([img[i:i+3, j:j+3] for i in range(0, img.shape[0], 3) for

# transform each chunk into a vector
matrix = np.array([flat_chunk(chunk) for chunk in chunks])


reconstructed_images = []
reconstructed_img = img.copy()

for k in k_list:

    reconstructed_img = img.copy()

    # apply k-means clustering
    centroids, label, inertia = sklearn.cluster.k_means(matrix, k)

    # reconstruct image
    for y in range(0, img.shape[0], 3):
        for x in range(0, img.shape[1], 3):
            chunk_index = find_chunk(y,x,img.shape[1])
            cluster_number = label[chunk_index]
            cluster_vector = centroids[cluster_number]
            reshaped_vector = cluster_vector.reshape(3, 3, 3)
            reconstructed_img[y:y+3, x:x+3] = reshaped_vector

    reconstructed_img = reconstructed_img.astype(np.float32)
    print("k = ", k)
    plt.imshow(reconstructed_img)
    plt.show()
    reconstructed_images.append(reconstructed_img.copy())
```

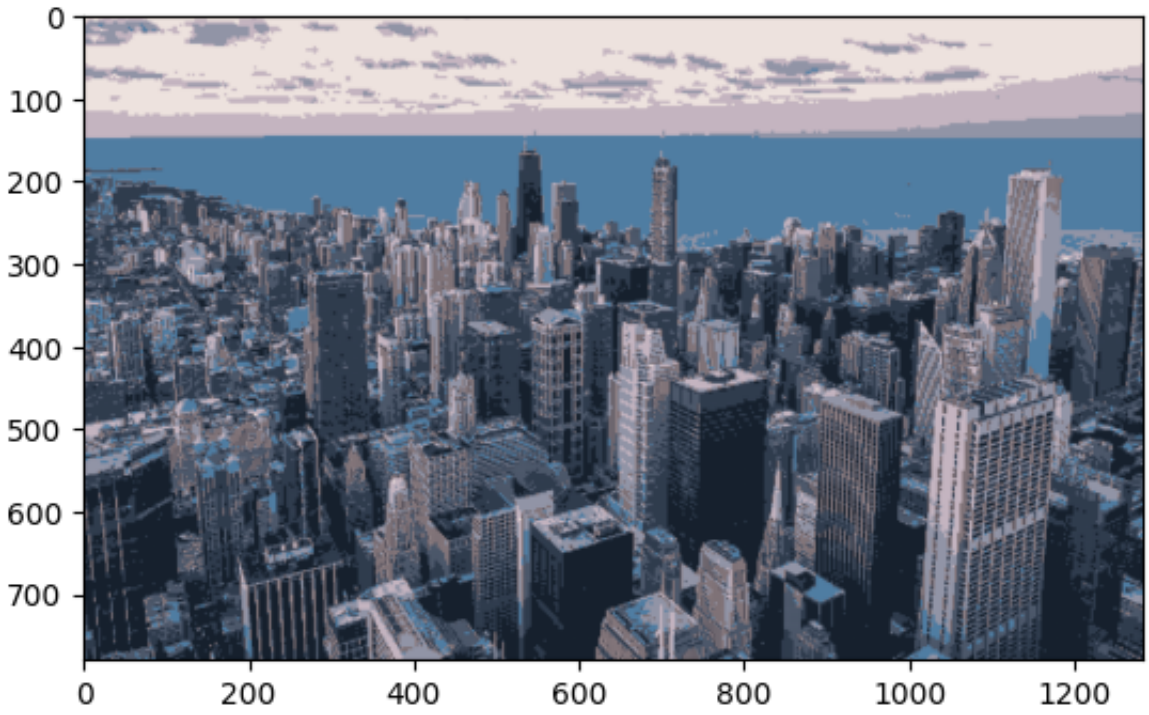
```
k =  2
```


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

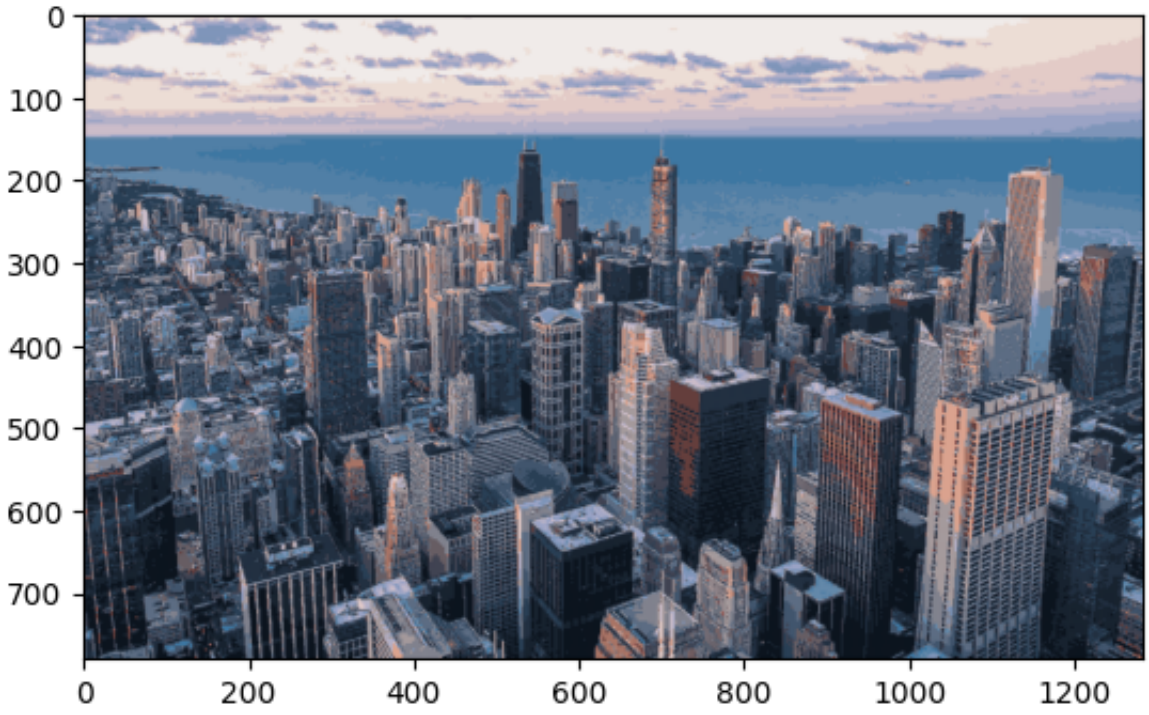
k = 5


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

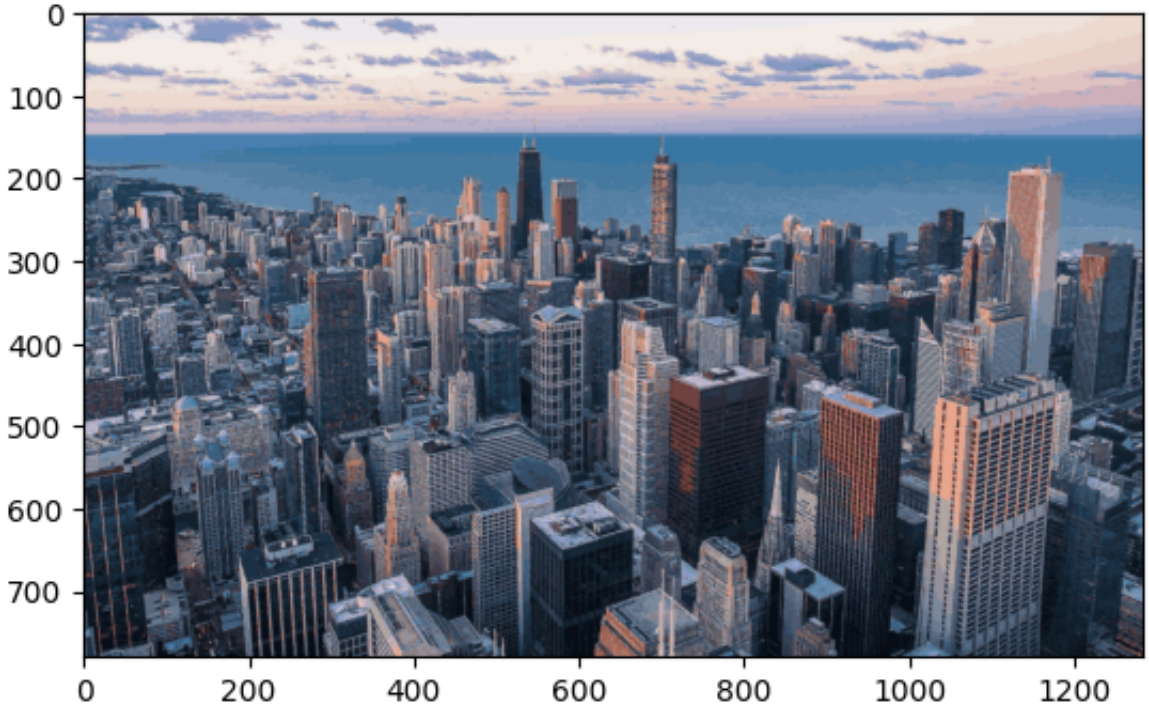
k = 10


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

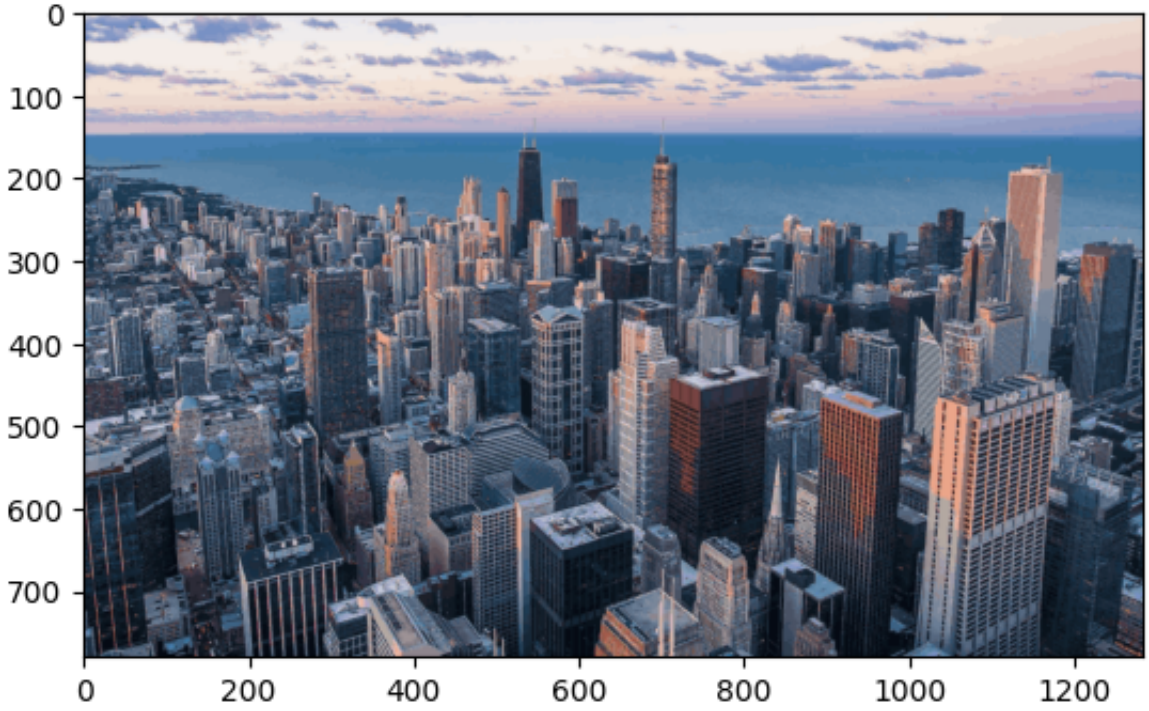
```
k =  50
```


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

```
k =  200
```


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

```
k =  500
```


0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200

```
k =  1000
```

k = 2000

Question 3

In [54]:
```
recon_errors = {}
k_list = [2, 5, 10, 50, 200, 500, 1000, 2000]
for i, recon_img in enumerate(reconstructed_images):
    e = (img - recon_img)**2
```

```
    k = k_list[i]
    recon_errors[k] = e.mean()

print('k , mean squared error\n')
for item in recon_errors.items():
    print(item, '\n')
```

```
k , mean squared error

(2, np.float32(0.028688248))

(5, np.float32(0.012967184))

(10, np.float32(0.009750629))

(50, np.float32(0.0054576797))

(200, np.float32(0.0036261887))

(500, np.float32(0.002817617))

(1000, np.float32(0.0023217162))

(2000, np.float32(0.0018872514))
```

Question 4

In [55]:
```
# need chunk array - 111280
# need label array - 111280 ??
# need array centroids of size k * 27

# original size is 111,280 (chunks) elements of size 27
# new size is 111,280 (chunks) elements matched to one of k clusters of size
# before, each chunk had 27 unique numbers associated with it
# now, multiple chunks are matched to a single cluster vector of size 27 out

# now, k * 27 + 111280 * 1 (now each chunk has a single cluter index associa



compressed_sizes = {}
print("k : total numbers\n")
for k in k_list:
    compressed_sizes[k] = k*27 + 1 * 111280
    print(k, ':', k * 27 + 1 * 111280, '\n')
```

```
k : total numbers

2 : 111334

5 : 111415

10 : 111550

50 : 112630

200 : 116680

500 : 124780

1000 : 138280

2000 : 165280
```

Question 5

In [56]:
```
# Compression rate = compressed size / original size

original_size = 1284 * 780 * 3
print("k : compression rate\n")
for k in k_list:
    print(k, ':', compressed_sizes[k] / original_size, '\n')
```

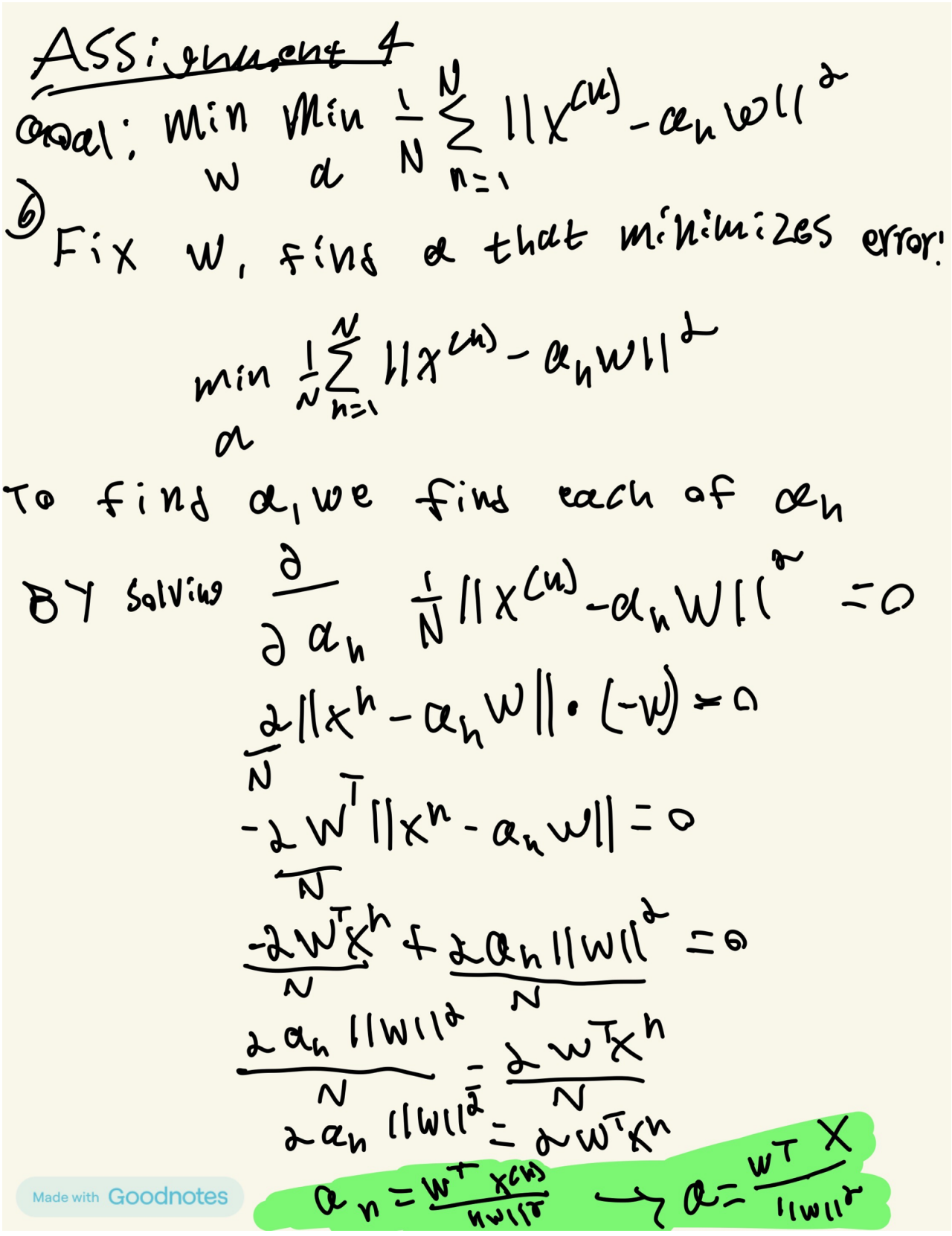
```
k : compression rate

2 : 0.03705500971856112

5 : 0.037081968740847245

10 : 0.037126900444657454

50 : 0.037486354075139124

200 : 0.038834305189445376

500 : 0.041530207418057886

1000 : 0.04602337779907873

2000 : 0.05500971856112043
```

Question 6

In [57]:
```
from IPython.display import Image
```

```
Image(filename='IMG_0043.jpg', width=800)
```

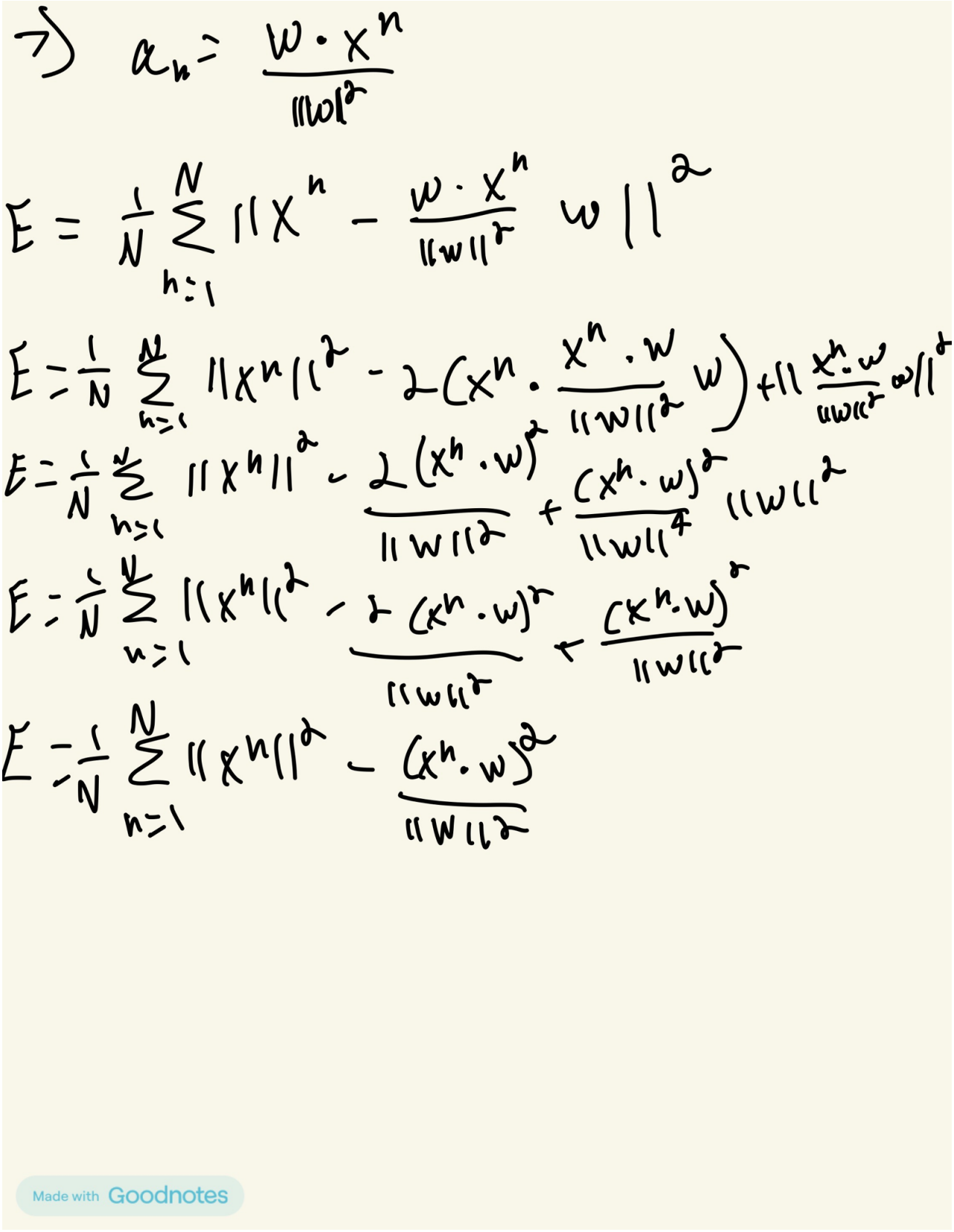
Out[57]:



Question 7

In [58]:
```
Image(filename='IMG_0044.jpg', width=800)
```

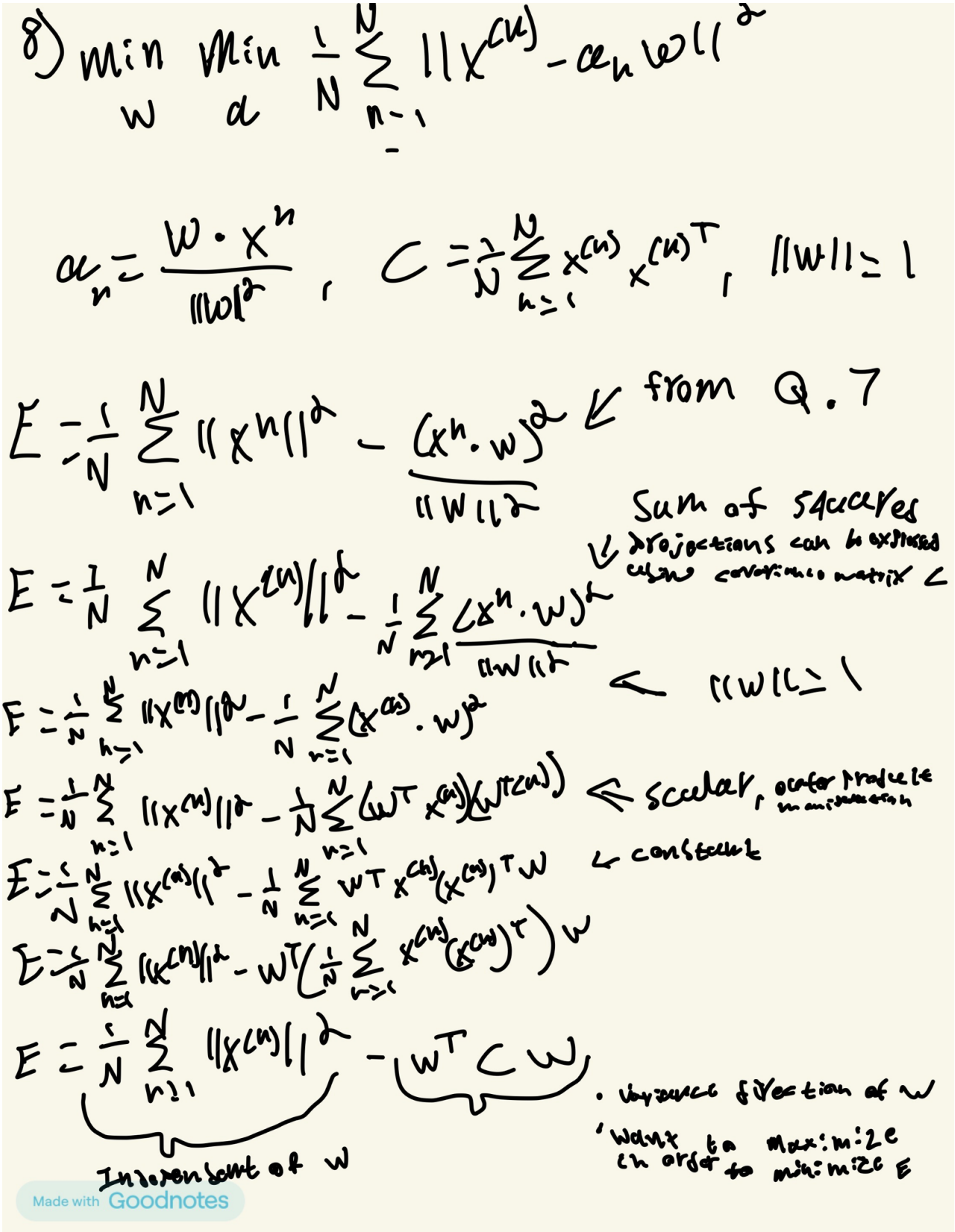
Out[58]:



Question 8

In [59]:
```
Image(filename='IMG_0045.jpg', width=800)
```

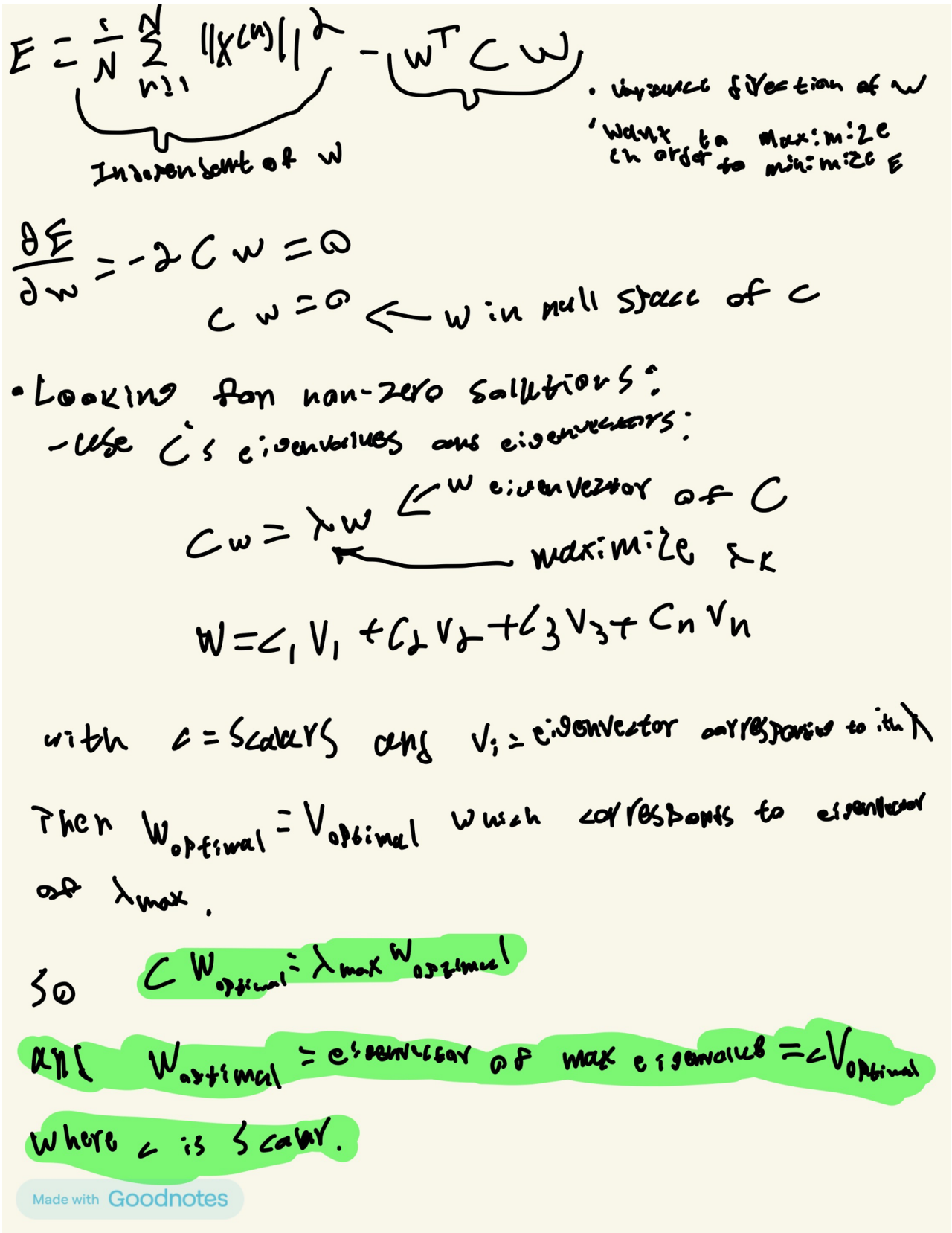
Out[59]:



Made with Goodnotes

In [60]:
```
Image(filename='IMG_0046.jpg', width=800)
```

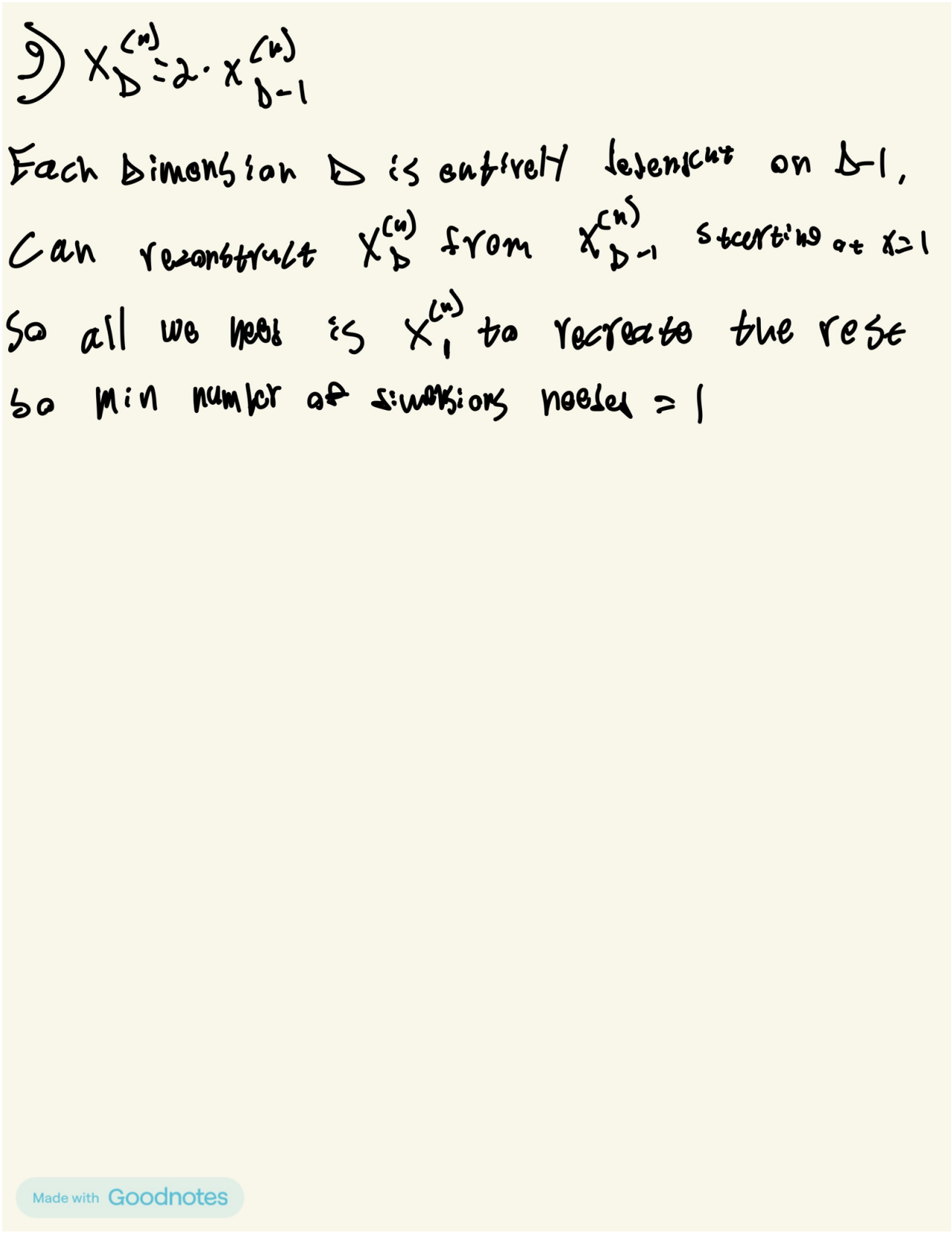
Out[60]:



Question 9

In [61]:
```
Image(filename='IMG_0047.jpg', width=800)
```

Out[61]:

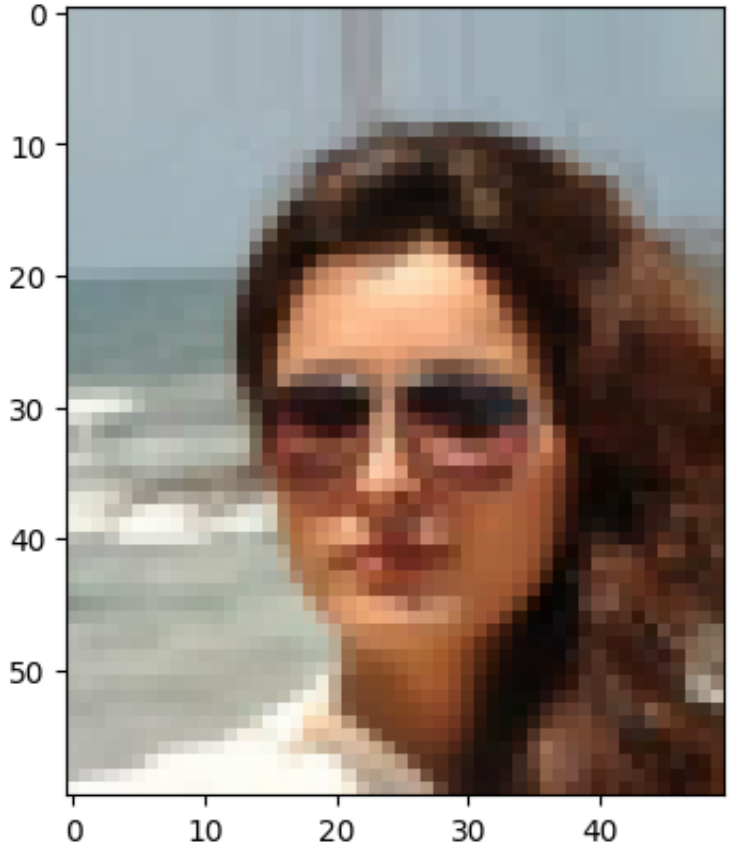


Question 10

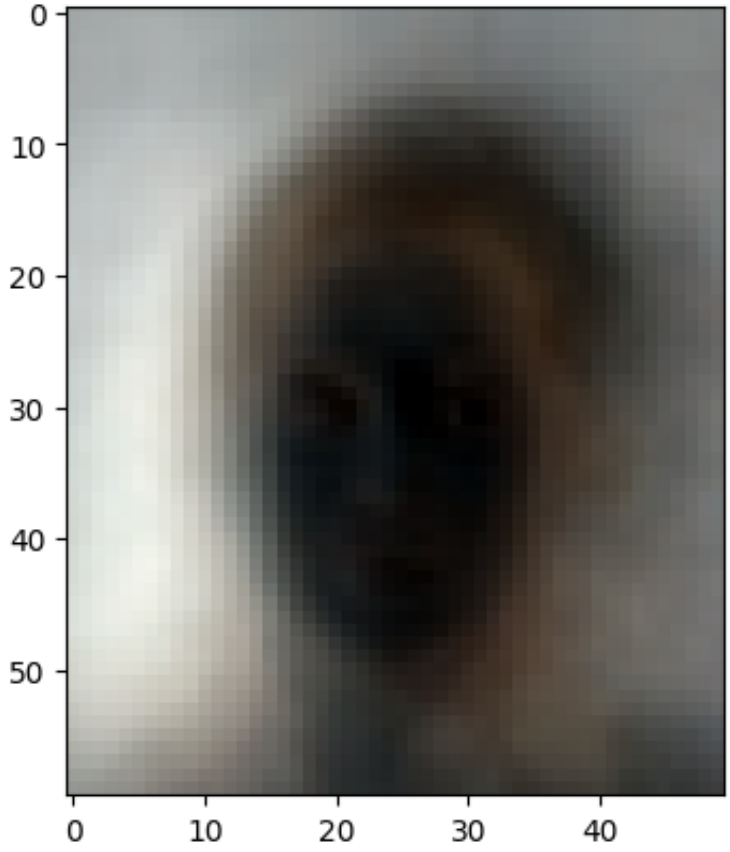
In [62]:
```
original_img = mpimg.imread('portrait.png') # load img
```

```
print("original image")
plt.imshow(original_img)
plt.show()
original_img_vector = original_img.flatten() # turn image to vector of 9000
k_list = [3, 5, 10, 30, 50, 100, 200, 500, 1000]
# get mean of all images in portraits folder
image_vectors= []
for i in range(1000):
    img = mpimg.imread('portraits/portrait_' + str(i) + '.png')
    img_vector = img.flatten()
    image_vectors.append(img_vector)


# get cov matrix
image_vectors = np.array(image_vectors)
cov_matrix = np.cov(image_vectors, rowvar=False)

# get eigenvectors
eig_vals, eig_vecs = np.linalg.eigh(cov_matrix)

sorted_indices = np.argsort(eig_vals)[::-1]  # Get indices of sorted eigenva
eig_vals = eig_vals[sorted_indices]  # Sort eigenvalues
eig_vecs = eig_vecs[:, sorted_indices]  # Sort eigenvectors according to eig


for k in k_list:

    # project original image onto k eigenvectors
    y = []
    for j in range(k):
        eigen_vector = eig_vecs[:, j]
        y.append(np.dot(eigen_vector.T, original_img_vector))

    # reconstruct original image
    reconstructed_img = [float(0) for i in range(9000)]
    for j in range(len(y)):
        reconstructed_img += (y[j] * eig_vecs[:, j])
    reconstructed_img = np.array(reconstructed_img)
    # scale all numbers to (0-1)
    reconstructed_img = np.clip(reconstructed_img, 0, 1)
    reconstructed_img = reconstructed_img.reshape(60,50,3)
    print("k = ", k)
    plt.imshow(reconstructed_img)
    plt.show()
```

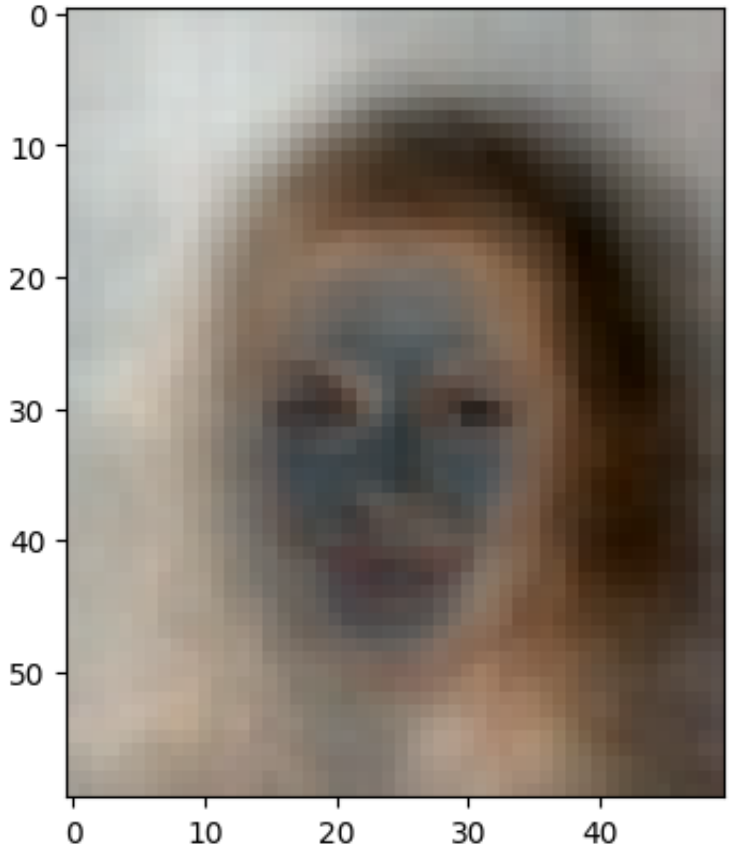
```
original image
```



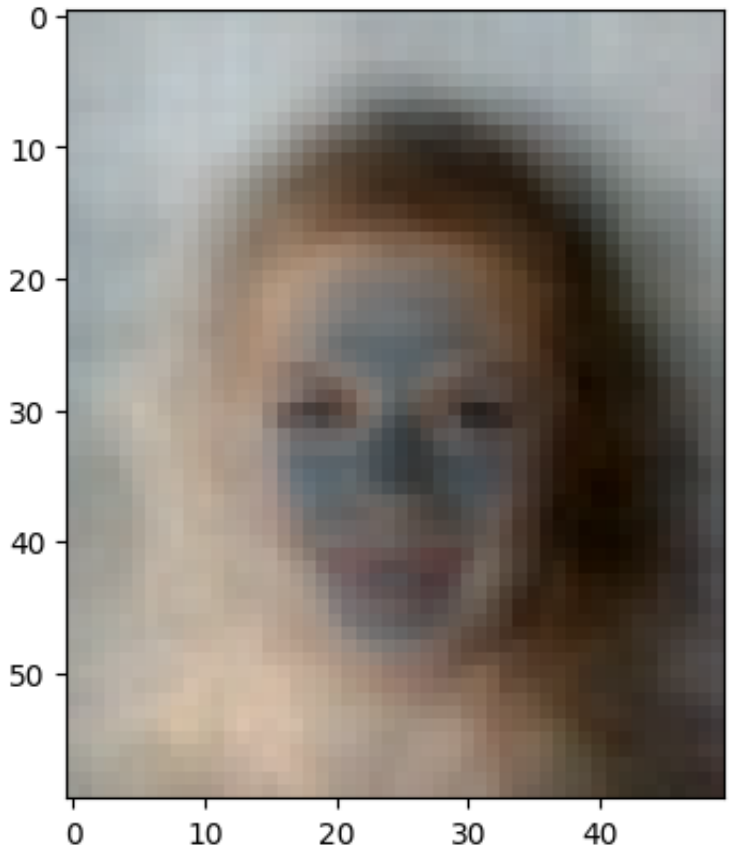
k = 3



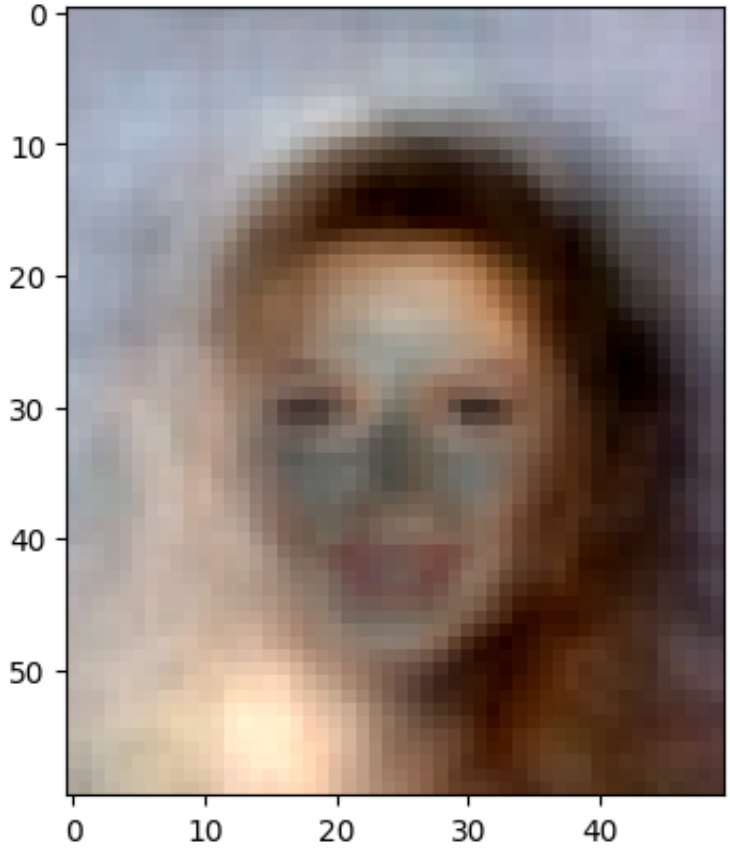
```
k =  5
```



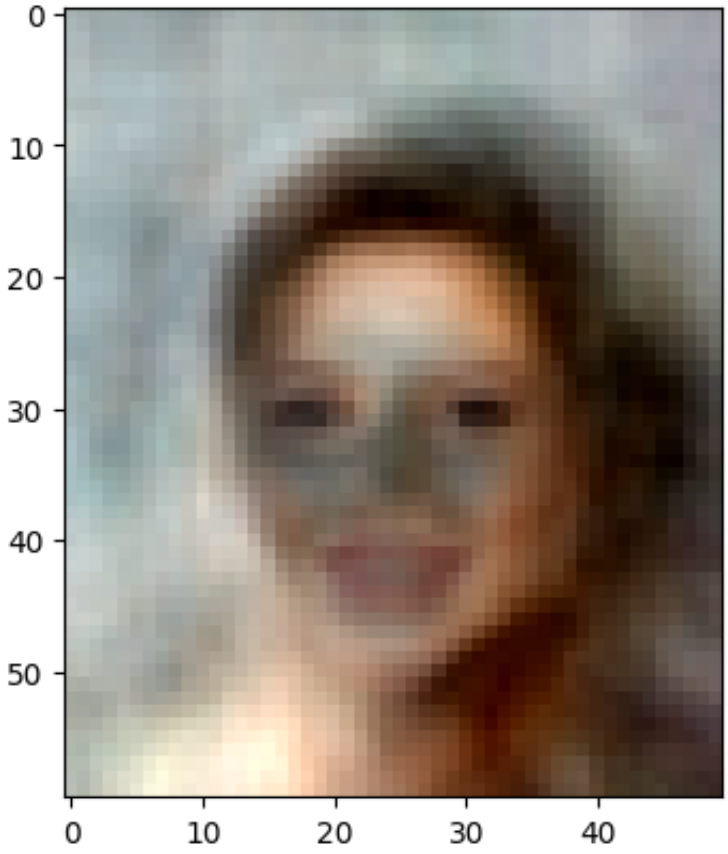

```
k =  10
```



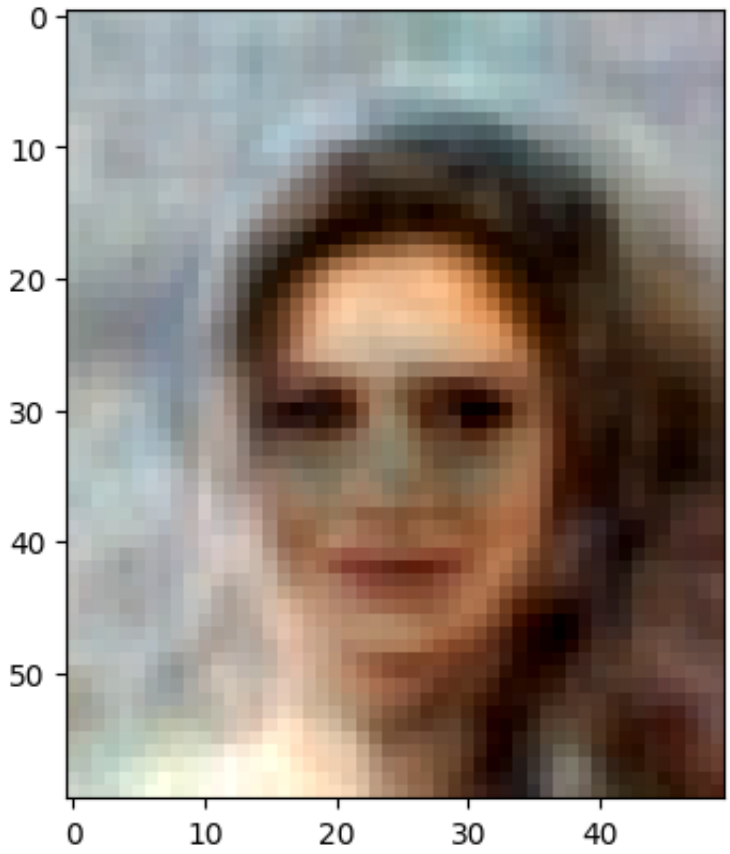

```
k =  30
```



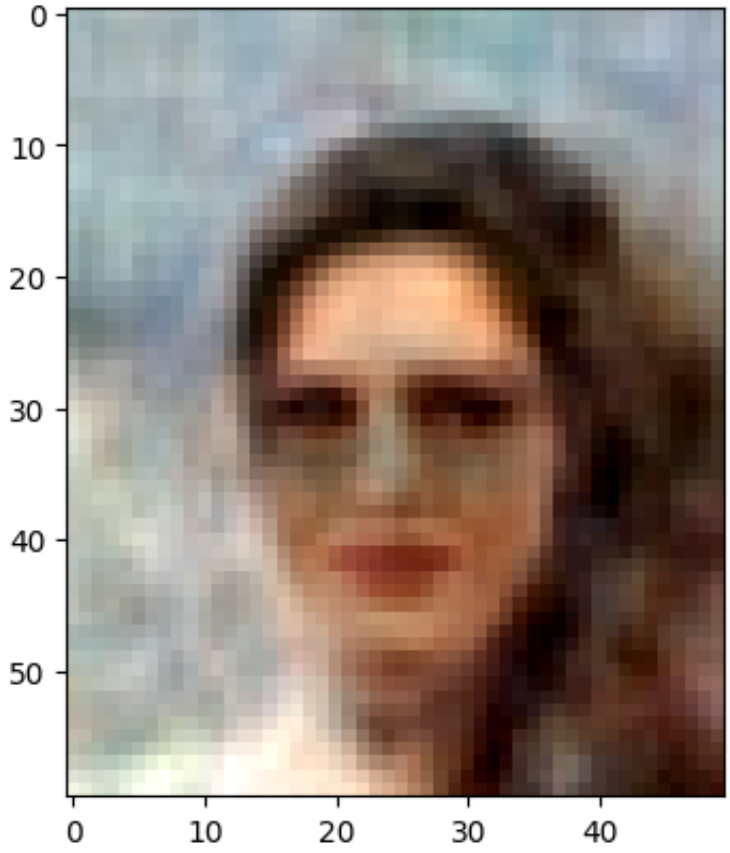
```
k =  50
```



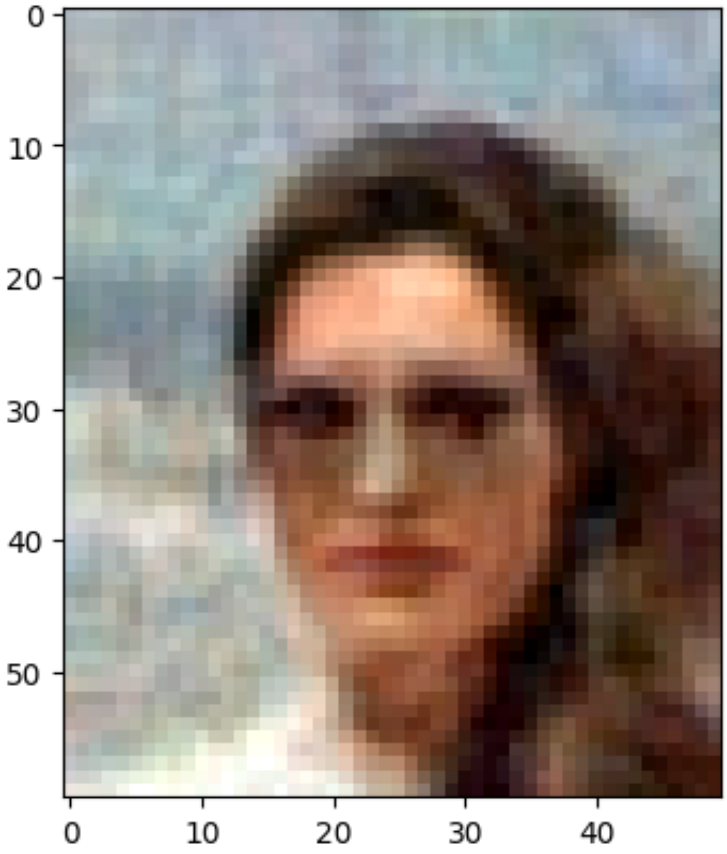

```
k =  100
```



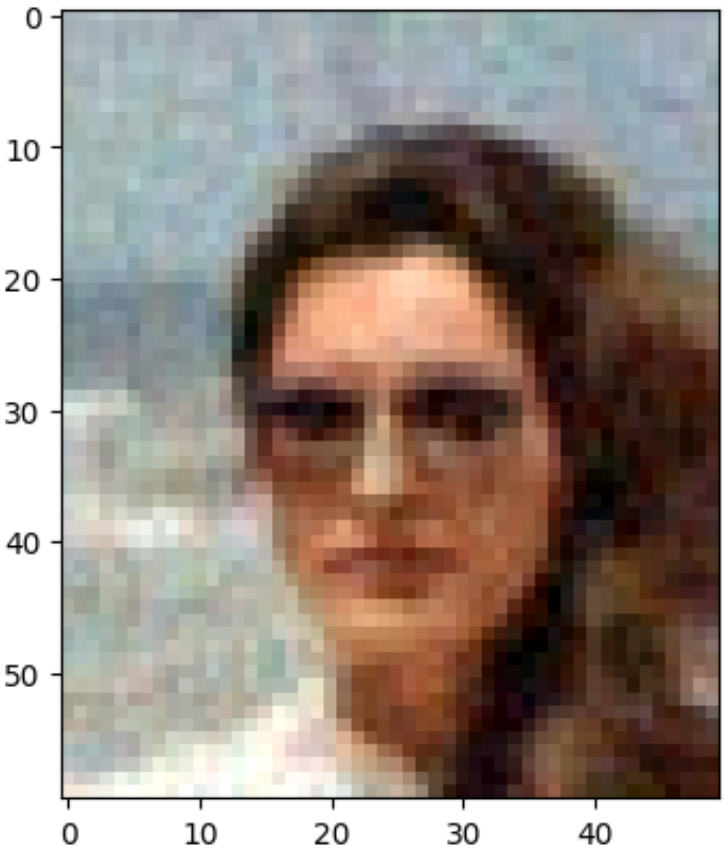

```
k =  200
```



```
k =  500
```




```
k =  1000
```



Question 11

In [63]:
```
compressed_sizes = {}
for k in k_list:
    compressed_sizes[k] = k * 1000  + k * 9000

# Compression rate = compressed size / original size

original_size = 1000 * 60 * 50 * 3 # 1000 images
print("k : compression rate\n")
for k in k_list:
    print(k, ':', compressed_sizes[k] / original_size, '\n')
```

```
k : compression rate

3 : 0.0033333333333333335

5 : 0.005555555555555556

10 : 0.011111111111111112

30 : 0.03333333333333333

50 : 0.05555555555555555

100 : 0.1111111111111111

200 : 0.2222222222222222

500 : 0.5555555555555556

1000 : 1.1111111111111112
```