Assignment 3 (ML for TS) - MVA 2022/2023

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in your report (one per pair of students) by Friday 7$^{\text{th}}$ April 11:59 PM.
- Rename your report and notebook as follows:
  `FirstnameLastname1_FirstnameLastname1.pdf` and
  `FirstnameLastname2_FirstnameLastname2.ipynb`.
  For instance, `LaurentOudre_CharlesTruong.pdf`.
- Upload your report (PDF file) and notebook (IPYNB file) using this link: https://www.dropbox.com/request/rmETjrLAH9Li3pf8JvOt.

2 Dual-tone multi-frequency signaling (DTMF)

In the last tutorial, we started designing an algorithm to infer from a sound signal the sequence of symbols encoded with DTMF.

Question 1

Finalize this procedure–in particular, find the best hyperparameters. Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values).

Answer 1

The method follows these steps:

- Compute the Short-Term Fourier Transform of our signal.
- Use the penalized L2 model to compute the breakpoints.
- Decode the signal using again the STFT. In each segment, we compute the two leading frequencies thanks to the STFT. We then match this pair of frequencies with some pair in the tones dictionary : we consider that two pairs match if their L1 distance is lower than some threshold. If there is at least one match we take the match with the lowest L1 distance and decode the associated symbol. Otherwise we decode a silence.

Hence there are two hyperparameters in our method: the penalization paramater and the 'silence' threshold. To calibrate them, we used the 'generate signal' method that randomly generates a encoded signal with the associated decoded symbols. With trial and error we kept the values that were the more robust:

- pen = 3.0
- threshold = 100

Question 2

What are the two symbolic sequences encoded in the provided signals?

Answer 2

- Sequence 1: ['B', '9', '4', 'B', '3', '8', 'B','#' ,'1']
- Sequence 2:['C', 'D', '1', '1', '2', '6', '3', '9']

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

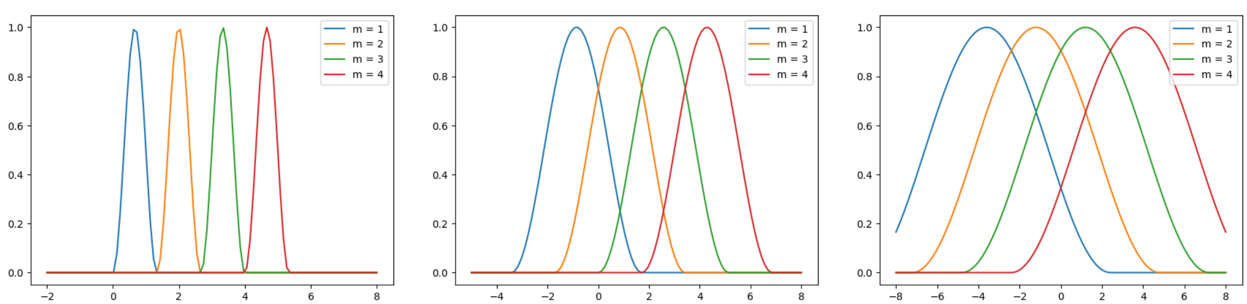
$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R?

Answer 3



(a) $R = 1$ (b) $R = 3$ (c) $R = 5$

Figure 1: The SAGW kernels functions

The parameter R influences the smoothness of the peak of the curve.

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are ['ARZAL', 'BATZ', 'BEG-MEIL', 'BREST-GUIPAVAS', 'BRIGNOGAN', 'CAMARET', 'LANDIVISIAU', 'LANNAERO', 'LANVEOC', 'OUESSANT-STIFF', 'PLOUAY-SA', 'PLOUDALMEZEAU', 'PLOUGONVELIN', 'QUIMPER', 'RIEC SUR BELON', 'SIZUN', 'ST NAZAIRE-MONTOIR', 'VANNES-MEUCON'].

The threshold is equal to 0.83.

The signal is the least smooth at 2014-01-10 09:00:00.

The signal is the smoothest at 2014-01-24 19:00:00.

Question 5

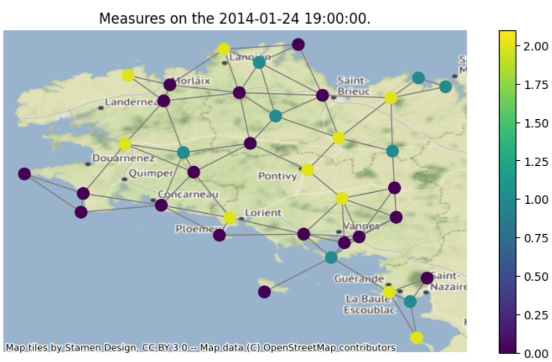
(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v, the vector $[W_f(1, v), W_f(2, v), \dots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low/medium/high frequency:

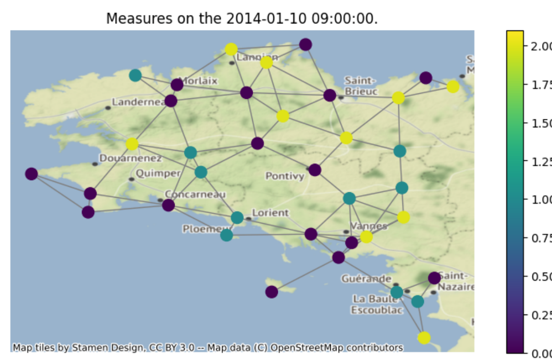
- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

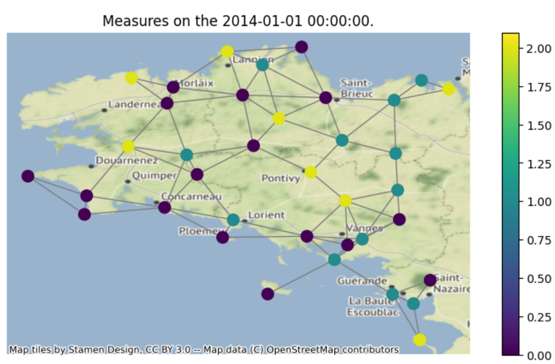
Answer 5



(a) Least smooth signal



(b) Smoothest signal



(c) First available timestamp

Figure 2: Classification of nodes into low/medium/high frequency

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

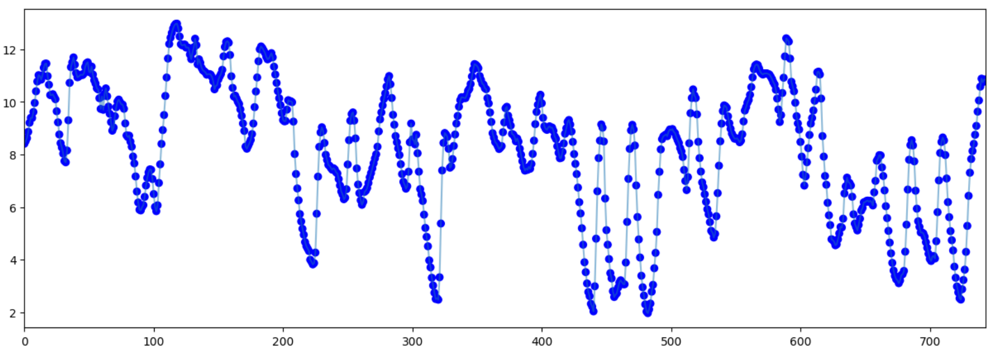


Figure 3: Average temperature. Markers' colours depend on the majority class.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G'.
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

- Denote $L_G \in \mathbb{R}^{n_1 \times n_1}$ and $L_{G'} \in \mathbb{R}^{n_2 \times n_2}$ the laplacians of G and G′. The assignment subject reminds that $H = G \otimes G'$.

  Then $L_H = D_H - A_H$ where D_H and A_H are the degree and ajacency matrices of H.

  The degrees of H are then $D_H = D_G \otimes I_{n_2} + I_{n_1} \otimes D_{G'}$ and its adjacency matrix $A_H = A_G \otimes I_{n_2} + I_{n_1} \otimes A_{G'} = (D_G - L_G) \otimes I_{n_2} + I_{n_1} \otimes (D_{G'} - L_{G'})$.

  Thus $L_H = D_G \otimes I_{n_2} + I_{n_1} \otimes D_{G'} - (D_G - L_G) \otimes I_{n_2} + I_{n_1} \otimes (D_{G'} - L_{G'}) = L_G \otimes I_{n_2} + I_{n_1} \otimes L_{G'}$.

- Let's denote by $\lambda_1 \leq ... \leq \lambda_n$ and $u_1, ..., u_n$ the eigenvalues and the eigenvectors of L_G. Let's denote by $\mu_1 \leq ... \leq \mu_n$ and $v_1, ..., v_n$ the eigenvalues and the eigenvectors of $L_{G'}$.
  $\boxed{\text{The eigenvalues of} L_H \text{ are then the } (\lambda_i + \mu_j)_{i,j} \text{ and the eigenvectors associated are the } (u_i \otimes v_j)_{i,j}.}$
  In fact :

  $$L_H u_i \otimes v_j = L_G \otimes I_{n_2} u_i \otimes v_j + I_{n_1} \otimes L_{G'} u_i \otimes v_j = L_G u_i \otimes I_{n_2} v_j + I_{n_1} u_i \otimes L_{G'} v_j = (\lambda_i + \mu_j) u_i \otimes v_j$$

  The first equality is the distributivity of the Kroenecker product w.r.t. the sum, the second equality is a common rule for Kroenecker products.

- We tried to compute the wavelet transform w.r.t. the cartesian product graph, however certainly due to an error in our code the computation was too long. Hence we weren't able to display then the plot with the classes.

Figure 4: Average temperature. Markers' colours depend on the majority class.