NUMERICAL SIMULATION OF THE SHORT FLEXIBLE WHEEL OF THE DOUBLE HARMONIC GEAR TRANSMISSION

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ABSTRACT

The paper presents the results of a research regarding the dynamic behaviour of the short flexible wheel of the double harmonic gear transmission, by emphasizing the stress status and the strains of its wall, in case the wheel is deformed using a mechanical wave generator with two rollers. The dynamic analysis involves modeling and the numerical simulation of flexible toothed wheel, by using the finite element method, with the help of SolidWorks Simulation program in elastic range.

Keywords: *flexible wheel, double harmonic gear transmission, simulation, stress, displacement.*

1. INTRODUCTION

The continuous modernization of the current industry led to the improvement of existing actuator systems, as well as the emergence of new systems, which contain in their structure more efficient gear transmissions capable of achieving very high kinematic accuracy, at smaller dimensions and mass.

The category of these modern transmissions also includes the harmonic gear transmissions, which were imposed by the most diverse applications in all of the top technological areas: the of construction ships and cosmic rockets, airplanes, helicopters, nuclear reactors, industrial robots, radar antennas, naval mechanisms, servo-mechanisms, motor reducers, machine tools, precision dividing heads, drives in sealed areas of chemical and petroleum industries etc. [1, 2, 3, 4].

The functioning principle of the harmonic gear transmission is essentially different from the classic gear, because the transmission of the rotary movement is accomplished by means of elastic deformation, which is propagated by the harmonic law, in the periphery of one of its elements called flexible toothed wheel.

The structural scheme of a double harmonic gear transmission (Figure 1) is made of: a wave generator (1) as input element, a short flexible toothed wheel (2) with the respective external and internal toothed crowns, the fixed rigid gear (3) and the mobile rigid gear (4) as output element, [3].

The short flexible toothed wheel has the form of a circular tube with thin wall, open at both ends and having at each end a toothed crown (external z_2 = 200 teeth and internal z'_2 = 192 teeth).

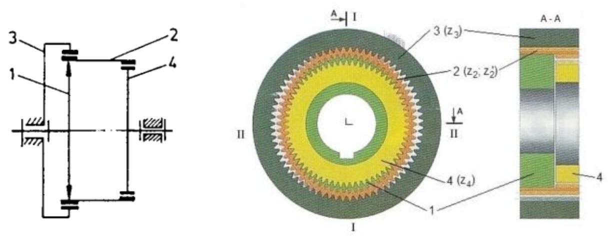


Figure 1 Structural scheme of a double harmonic gear transmission

The wave generator being in sliding contact along the entire periphery of the flexible wheel, it deforms this toothed wheel so that it will have four equidistant driving zones: two with the fixed rigid gear having internal teeth (z_3 / z_2 - first step, I - I) and two with the mobile rigid gear having external teeth (z'_2 / z_4 - second step, II - II). Between the two pairs of opposing driving zones (I - I and II - II respectively) there is a 90^o angle.

By studying the research conducted on the harmonic gear transmission it was found out that the functional performance and durability are greatly influenced by the dynamic behavior and durability of the flexible toothed wheel. This fact lead to the necessity of researching the stress status and the tension of its wall.

2. NUMERICAL SIMULATION OF SHORT FLEXIBLE TOOTHED WHEEL

The flexible toothed wheel of the harmonic gear transmission is in a complex state of tension and elastic deformation, depending on many factors such as: the type of wave generator, the geometric shape of the flexible gear, the torque transmitted and the coupling of the flexible gear with the output shaft.

In order to investigate the state of stress and deformation of the flexible toothed wheel of the double harmonic gear transmission and to achieve numerical simulation using the finite element method, with the help of SolidWorks Simulation module, the following steps were required, [5]:

- 3D geometry modeling of the flexible toothed wheel and wave generator with 2 rollers in SolidWorks CAD software;
- defining case analysis;
- defining material from the library of materials;
- defining restrictions and applying the loading;
- finite element mesh of the model tested;
- calculating the strains and tensions by using SolidWorks Simulation module;
- view and analyze the results.

In numerical calculus, flexible wheel of the double harmonic gear transmission was modeled by a cylinder opened at both ends, defined by the radius, r = 29,3 mm, length l = 30 mm and the constant wall thickness, s = 0.6 mm, which is provided with two teeth (outer respectively inner) of width 12 mm.

The analysis of the dynamic behavior of flexible toothed wheel was made in the case of the geometric model of wave generator with 2 rollers (Figure 2).

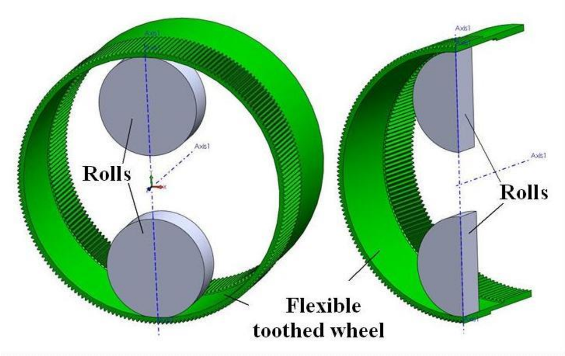


Figure 2 Analysis model

The analysis model is composed of a flexible toothed wheel of double harmonic gear transmission and of a wave generator with 2 rollers. The two rolls were modeled by the two identical circular cylinders, characterized by: cylinder diameter d_r = 22 mm and height of the cylinder b_r = 8 mm.

SolidWorks Simulation operates with the concept of "Study", pointing out specific characteristics of the analysis: analysis type and associated options, materials, set load and boundary conditions and meshing the model analyzed.

The geometry of the flexible toothed wheel of the double harmonic gear transmission was modeled in "solid" and its numerical simulation consisted of a linear static analysis, meshing was performed using spatial finite elements. The selected material for the flexible wheel is steel (Alloy Steel), having the following characteristics: Young′s modulus, E = $2{,}1\cdot10^{11}$ N/m^2; Poisson's ratio, ν = 0,28; yield strength, σ_c = 620,4 MPa.

The geometry of the flexible toothed wheel has the origin in the point O of its symmetry, the Oz - axis being oriented along the generatrix, the xOy plane being oriented NSVE, the Ox - axis being positively oriented from V to E and the Oy - axis from S to N (Figure 3).

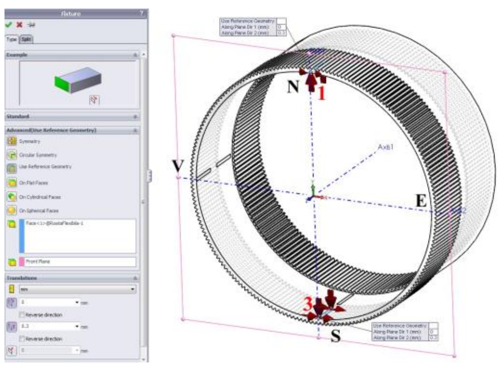


Figure 3 Restrictions applied to the flexible wheel

For the wave generator with 2 rollers, contact between the flexible toothed wheel and the generator, is produced on the inner side of the wheel, in the areas N1 and S3.

In these two areas of contact the following restrictions will be applied to the flexible toothed wheel: two restrictions of value 0, which will cancel the movement of the contact zones N1 respectively S3, in the direction Ox, and two restrictions of value 0.3 mm applied to the exterior of the wheel, which materialize the strain/deformation in the direction Oy of the wheel due to the two rollers of the wave generator.

Also, in simulation will be applied a restriction type Roller/Slider, to the side which is parallel and opposed to the NSVE side. For this type of restriction, the points belonging to this plane side can move freely in their plane, but are not able to move perpendicular to this plane.

The loads applied to the flexible wheel consisted in the only two components of the normal forces (tangential force F_t and radial force F_r), developed in the first stage of harmonic gear (Figure 4). This is because the effect of the interaction between the wave generator and flexible wheel has already been taken into account by imposing elastic deformation produced by the generator inside the flexible wheel.

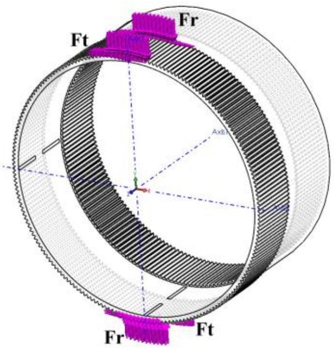


Figure 4 Uploads applied to the flexible toothed wheel

The maximum values of tangential and radial forces from the harmonic gearing were determined for the following steps of the torque of the transmission, M_{t4} (0, 100, 200, 300, 400, 500) N·m, using the relationship:

$$F_{t\,\max} = q_{t\,\max}\cdot b_d = \pi\cdot M_{t4}\cdot p/(2\varphi_2\cdot d_2^2) \qquad (1)$$

$$F_{r\,\max} = F_{t\,\max}\cdot tg\,\alpha \qquad (2)$$

where: q_{tmax} – is tangential force reported to the unit of length of the teeth;
b_d – the length of the toothed crowns;
p – circular pitch;
d_2 – diameter pitch circle of flexible gear;
φ_2 – positioning angle of the gearing area;
α – angle of the tooth profile;
M_{t4} – torque to the output shaft.

In order to simulate the flexible toothed wheel, meshing to use solid type (Figure 5) was used, which generated a total number of 63.687 finite elements and a number of 123.511 nodes.

After following the above mentioned stages, the analyses calculus was performed by the simulation of the deformation of the flexible toothed wheel, in order to determine and display graphically the status of stress and strains.

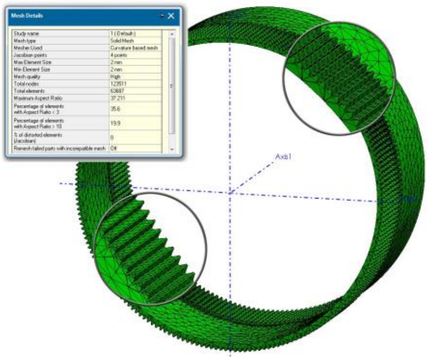


Figure 5 Meshing of the flexible toothed wheel

After the numerical processing of the simulation of the behavior of flexible toothed wheel, can view the results, that can be viewed graphically (charts and color maps) or analytically (numerical values for von Mises stress and displacements).

The tensions presented in diagrams, represent the equivalent stress in the case of composite stresses, which are calculated using the von Mises formula:

$$\sigma_{ech} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \cdot \sigma_3 - \sigma_2 \cdot \sigma_3 - \sigma_2 \cdot \sigma_1} \qquad (3)$$

where: σ_1, σ_2 and σ_3 – are the normal stresses after main directions.

3. NUMERICAL SIMULATION RESULTS

In the case of numerical analysis that was performed using SolidWorks Simulation program, there were studied the variations of the displacements and stresses (von Mises), in the body of the flexible toothed wheel, according to the loading moment of the double harmonic gear transmission.

Thus, by successive runs of the numerical analysis program, maximum values of displacements and von Mises stress were recorded for all six steps of charging (M_{t4} = 0, 100, 200, 300, 400, 500 N·m).

The results obtained after applying numerical simulations are summarized in Table 1 - for maximum stress von Mises, respectively Table 2 – for the resultant displacement, and Figures 6 and 7 present their variation diagrams, [3].

Table 1. Maximum stress von Mises

Torque M_{t4} [N·m]	Tangential force F_t [N]	Radial force F_r [N]	Stress $\sigma_{v\ Mises\ max}$ [MPa]
0	0	0	369.1
100	52,33	19,04	371.8
200	104,66	38,09	374.4
300	156,99	57,13	377.2
400	209,33	76,18	389.8
500	261,66	95,23	415.4

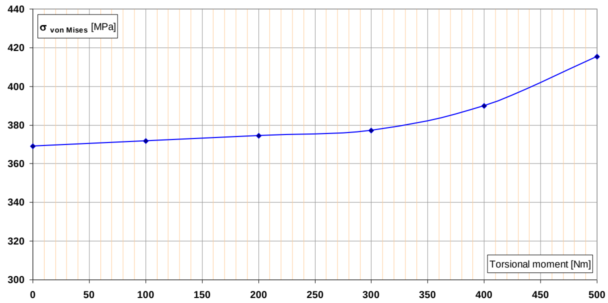


Figure 6 Maximum stress von Mises diagram, $\sigma_{vM} = \sigma_{Vm}(M_{t4})$

Table 2. Resultant displacement of nodes on the generator N

No. node	Positioning share z_{nod} [mm]	Resultant displacement Δ [mm]
1	0	0,3
2	-3,75	0,3
3	-8,25	0,299
4	-12	0,296
5	-16,725	0,294
6	-21,15	0,292
7	-25,575	0,292
8	-29,635	0,291

We can observe that the values of the coordinate z, by which are established the positions of the nodes from a certain flexible wheel generator, give negative results because the Oz-axis has positive orientation, and is in the opposite direction to running through the generatrix.

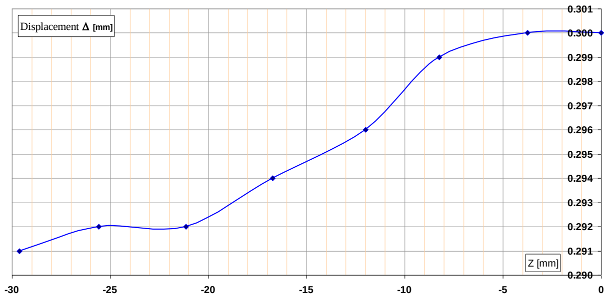
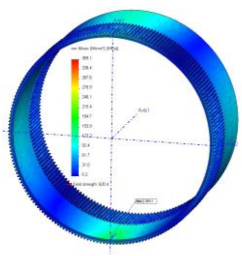
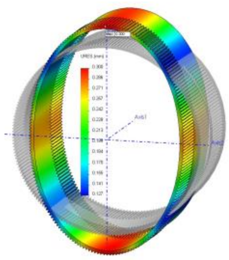


Figure 7 Resultant displacement diagram, Δ = Δ(z)

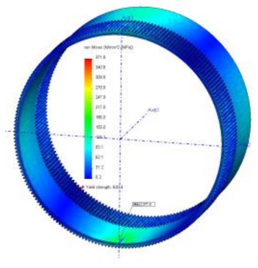
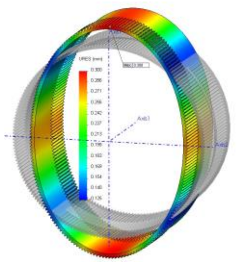
Figure 8 shows, under the form of color maps, the von Mises stress distribution, respectively the resulting displacement Δ, in the case of deformation of the flexible wheel with the wave generator where the 2 rolls.



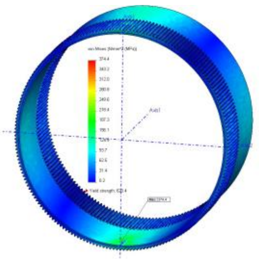
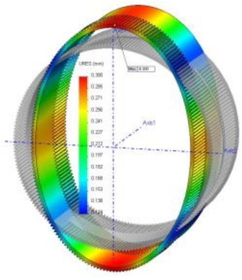
$\sigma_{von\ Mises}$ - M_{t4} = 0 Nm



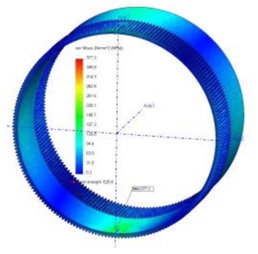
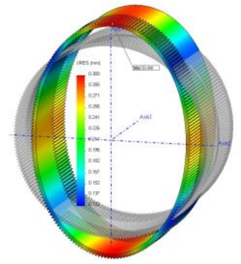
Δ - M_{t4} = 0 Nm

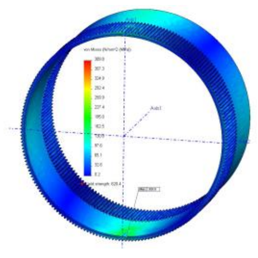
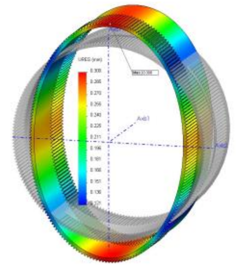
$\sigma_{von\ Mises}$ - M_{t4} = 100 Nm Δ - M_{t4} = 100 Nm

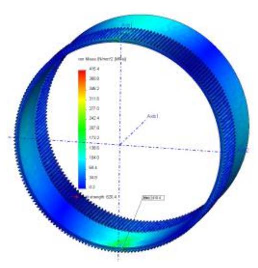
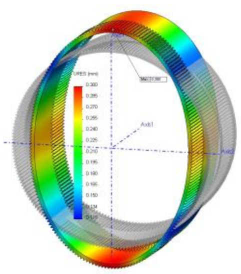
$\sigma_{von\ Mises}$ - M_{t4} = 200 Nm Δ - M_{t4} = 200 Nm

$\sigma_{von\ Mises}$ - M_{t4} = 300 Nm Δ - M_{t4} = 300 Nm

$\sigma_{von\ Mises}$ - M_{t4} = 400 Nm Δ - M_{t4} = 400 Nm

$\sigma_{von\ Mises}$ - M_{t4} = 500 Nm Δ - M_{t4} = 500 Nm

Figure 8 The von Mises stress (σ_{vMmax}) and the resultant displacement (Δ)

In order to research and appreciate more correctly the deformation mode of the flexible toothed wheel there were visualized the resulting displacements Δ of the characteristics nodes, from the finite elements located in the direction of the generatrix N of the wheel, at a load of M_{t4} = 100 N·m (Figure 9).

From the analysis of the variation diagram of the resultant displacements $\Delta = \Delta$ (z), one can see that in the proximity of the point where one applies the deflection force on the flexible toothed wheel (corresponding to cote z = 0 on the N generator), it is found for the value of the resultant displacement the exact value of the maximum radial elastic deformation of the flexible wheel (Δ (0) = w_0 = 0.3 mm).

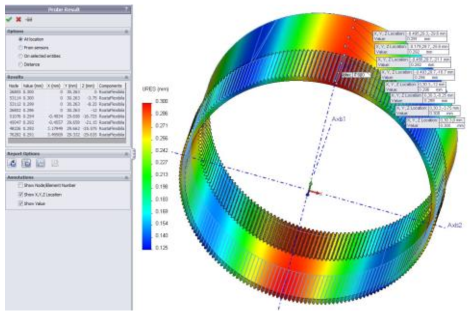


Figure 9 Variation of the resultant displacement Δ

4. CONCLUSIONS

The paper presents the results of numerical simulation of the short flexible toothed wheel, of the double harmonic gear transmission, with the wave generator with 2 rollers, allowing to evaluate the dynamic behavior of the wheel.

The analysis of these results reveals the following conclusions:

- the stress forming in the wall of the flexible toothed wheel are dependent on the load of the transmission, presenting a slightly increasing character (σ_{vMmax} ϵ [369,1; 415,4] MPa) with increasing torque (M_{t4} ϵ [0; 500] N·m);
- the maximum value of the stress occurs in the immediate proximity of the point where we apply the force of elastic deformation on the flexible toothed wheel, in the area of the rolls of wave generator, and this stress is well below the yield strength of the material of the wheel (σ_c);
- maximum stress variation is insignificant at low torque of the transmission ($M_{t4} \leq 100$ N·m);
- the resultant displacement of the nodes located on the generator N has a slightly decreasing character, once we remove the nodes_from the NSVE side of the flexible, a fact that was confirmed by the results of experimental research conducted.

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