# APM466 A1

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1/26/2022

## Loading data and generate date measure

```r
library(readxl)
library(tidyverse)
library(SciViews)
library(nleqslv)
library(reshape)
library(NLRoot)
library(janitor)
library(WriteXLS)

#load in data and save in R
APM466_A1_data <- read_excel("~/Desktop/APM466_A1_data.xlsx")
WriteXLS(APM466_A1_data, ExcelFileName = "APM466_A1_data.xlsx")
data <- APM466_A1_data
```

```r
#convert the date to class date formatting
data[,3] = as.Date(data$`Issue date`,format='%Y/%m/%d')
data[,5] = as.Date(data$`Maturity date`,format='%Y/%m/%d')

#generate new variables to identify date of observation
data <- data %>%
  mutate(date_1 = as.Date('2022/1/10',format='%Y/%m/%d')) %>%
  mutate(date_2 = as.Date('2022/1/11',format='%Y/%m/%d')) %>%
  mutate(date_3 = as.Date('2022/1/12',format='%Y/%m/%d')) %>%
  mutate(date_4 = as.Date('2022/1/13',format='%Y/%m/%d')) %>%
  mutate(date_5 = as.Date('2022/1/13',format='%Y/%m/%d')) %>%
  mutate(date_6 = as.Date('2022/1/17',format='%Y/%m/%d')) %>%
  mutate(date_7 = as.Date('2022/1/18',format='%Y/%m/%d')) %>%
  mutate(date_8 = as.Date('2022/1/19',format='%Y/%m/%d')) %>%
  mutate(date_9 = as.Date('2022/1/20',format='%Y/%m/%d')) %>%
  mutate(date_10 = as.Date('2022/1/21',format='%Y/%m/%d'))

#generate time to maturity according to observation date
data$date_1_TTM <- as.numeric((data$`Maturity date`- data$date_1), units="days")/365
data$date_2_TTM <- as.numeric((data$`Maturity date`- data$date_2), units="days")/365
data$date_3_TTM <- as.numeric((data$`Maturity date`- data$date_3), units="days")/365
data$date_4_TTM <- as.numeric((data$`Maturity date`- data$date_4), units="days")/365
data$date_5_TTM <- as.numeric((data$`Maturity date`- data$date_5), units="days")/365
data$date_6_TTM <- as.numeric((data$`Maturity date`- data$date_6), units="days")/365
```

```
data$date_7_TTM <- as.numeric((data$`Maturity date`- data$date_7), units="days")/365
data$date_8_TTM <- as.numeric((data$`Maturity date`- data$date_8), units="days")/365
data$date_9_TTM <- as.numeric((data$`Maturity date`- data$date_9), units="days")/365
data$date_10_TTM <- as.numeric((data$`Maturity date`- data$date_10), units="days")/365

#notional: the payment occur at maturity
N=100
#change the coupon rate to per 100 dollars instead of percentage
data$`Coupon rate` = (data$`Coupon rate`)*100

view(data)
```

### Question 4:

```
#define function to calculate ytm from non-linear equation by Newton's method
NW <- function (f, f1, x0 = 0, num = 100, eps = 1e-05, eps1 = 1e-05)
{   a = x0
    b = a - f(a)/f1(a)
    i = 0
    while ((abs(b - a) > eps) & (i < num)) {
        a = b
        b = a - f(a)/f1(a)
        i = i + 1}
    if (abs(f(b)) < eps1) {
        return(a)}
    else print("finding root is fail")
}
```

```
date_order <- data.frame(data$date_1[1], data$date_2[1], data$date_3[1], data$date_4[1], data$date_5[1]
```

```
#calculate the yield to maturity for each bond on each day. (YTM curve)
rep(NA, 10) -> ytm_bond_1 -> ytm_bond_2 -> ytm_bond_3 -> ytm_bond_4 -> ytm_bond_5 -> ytm_bond_6 -> ytm_

#calculate the yield curve/spot curve for each day. (Yield/Spot curve)
#Bootstrapping Method for finding the spot rate and plot the yield curve
#the Bootstrapping method allows us to use coupon-payment bond to recover yield curve,
rep(NA, 10) -> yield_1 -> yield_2 -> yield_3 -> yield_4 -> yield_5 -> yield_6 -> yield_7 -> yield_8 -> 


for (i in 1:10){#iteration over 10 days of observation
  #find yield for each bond
  #use 180 days as 6 months count, and 365 as year count.
  #b represent bond number
  #c is half of the coupon payment
  #ai is accrued interest

  b=1; c = data[b,4]/2
  #ytm
  ai_1 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
  ytm_bond_1[i]= as.numeric(-ln(((data[b,5+i]+ai_1))/(N+c))/(data[b,25+i]))
  #spot rate
```

```r
yield_1[i] = as.numeric(-ln((data[b,5+i]+ai_1)/(N+c))/(data[b,25+i]))

b=2;c=data[b,4]/2
ai_2 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
#time for each coupon and principle payment
for (j in 1:2){assign(paste("t2", j, sep="_"), data[b,25+i]-(2-j)*0.5)}
#ytm
f <- NW(function(x) (-1)*(data[b,5+i]+ai_2)+c*exp(-t2_1*x)+(100+c)*exp(-t2_2*x),
        function(x) c*exp(-t2_1*x)*(-t2_1)+(100+c)*exp(-t2_2*x)*(-t2_2), x0 = 0)
ytm_bond_2[i]= as.numeric(f)
#spot rate
yield_2[i] = as.numeric((-ln(((data[b,5+i])+ ai_2 -c*exp(-t2_1*yield_1[i]))/(100+c)))/t2_2)

b=3;c=data[b,4]/2
ai_3 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:3){assign(paste("t3", j, sep="_"), data[b,25+i]-(3-j)*0.5)}
#ytm
f <- NW(function(x) (-1)*(data[b,5+i]+ai_3)+c*exp(-t3_1*x)+c*exp(-t3_2*x)+(100+c)*exp(-t3_3*x),
        function(x) (c*exp(-t3_1*x)*(-t3_1)+c*exp(-t3_2*x)*(-t3_2)
                    +(100+c)*exp(-t3_3*x)*(-t3_3)), x0 = 0)
ytm_bond_3[i]= as.numeric(f)
#spot rate
yield_3[i] = as.numeric((-ln(((data[b,5+i]+ai_3)-c*exp(-t3_1*yield_1[i])-c*exp(-t3_2*yield_2[i]))
                     /(100+c)))/t3_3)

b=4;c=data[b,4]/2
ai_4 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:4){assign(paste("t4", j, sep="_"), data[b,25+i]-(4-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*(data[b,5+i]+ai_4)+c*exp(-t4_1*x)+c*exp(-t4_2*x)
                     +c*exp(-t4_3*x)+(100+c)*exp(-t4_4*x)),
        function(x) (c*exp(-t4_1*x)*(-t4_1)+c*exp(-t4_2*x)*(-t4_2)+
                     c*exp(-t4_3*x)*(-t4_3)+(100+c)*exp(-t4_4*x)*(-t4_4)), x0 = 0)
ytm_bond_4[i] = as.numeric(f)
#spot rate
yield_4[i] = as.numeric((-ln(((data[b,5+i]+ai_4)-c*exp(-t4_1*yield_1[i])
                   -c*exp(-t4_2*yield_2[i])
                   -c*exp(-t4_3*yield_3[i]))/(100+c)))/t4_4)

b=5;c=data[b,4]/2
ai_5 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:5){assign(paste("t5", j, sep="_"), data[b,25+i]-(5-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*(data[b,5+i]+ai_5)+c*exp(-t5_1*x)+c*exp(-t5_2*x)
                     +c*exp(-t5_3*x)+c*exp(-t5_4*x)+(100+c)*exp(-t5_5*x)),
        function(x) (c*exp(-t5_1*x)*(-t5_1)+c*exp(-t5_2*x)*(-t5_2)+
                     c*exp(-t5_3*x)*(-t5_3)+c*exp(-t5_4*x)*(-t5_4)+
                       (100+c)*exp(-t5_5*x)*(-t5_5)), x0 = 0)
ytm_bond_5[i]= as.numeric(f)

#linear interpolate spot rate for missing values
x<- c(0, t4_1, t4_2, t4_3, t4_4)
y<- c(0, yield_1[i], yield_2[i], yield_3[i], yield_4[i])
```

```r
r_1 = approx(x,y, xout=t5_1)$y
r_2 = approx(x,y, xout=t5_2)$y
r_3 = approx(x,y, xout=t5_3)$y
r_4 = approx(x,y, xout=t5_4)$y

#spot rate
yield_5[i] = as.numeric((-ln(((data[b,5+i]+ai_5)-c*exp(-t5_1*r_1)
                      -c*exp(-t5_2*r_2)
                      -c*exp(-t5_3*r_3)
                      -c*exp(-t5_3*r_4))/
                       (100+c)))/t5_5)

b=6; c=data[b,4]/2
ai_6 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:6){assign(paste("t6", j, sep="_"), data[b,25+i]-(6-j)*0.5)}
#ytm
f <- NW(function(x) (-1)*((data[b,5+i]+ai_6))+c*exp(-t6_1*x)+c*exp(-t6_2*x)
                     +c*exp(-t6_3*x)+c*exp(-t6_4*x)+c*exp(-t6_5*x)+(100+c)*exp(-t6_6*x),
        function(x) (c*exp(-t6_1*x)*(-t6_1)+c*exp(-t6_2*x)*(-t6_2)+
                     c*exp(-t6_3*x)*(-t6_3)+c*exp(-t6_4*x)*(-t6_4)+
                     c*exp(-t6_5*x)*(-t6_5)+(100+c)*exp(-t6_6*x)*(-t6_6)), x0 = 0)
ytm_bond_6[i] = as.numeric(f)
#spot rate
yield_6[i] = as.numeric((-ln(((data[b,5+i]+ai_6)-c*exp(-t6_1*r_1)
                      -c*exp(-t6_2*r_2)
                      -c*exp(-t6_3*r_3)
                      -c*exp(-t6_4*r_4)
                      -c*exp(-t6_5*yield_5[i]))/
                       (100+c)))/t6_6)

b=7; c=data[b,4]/2
ai_7 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:7){assign(paste("t7", j, sep="_"), data[b,25+i]-(7-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*((data[b,5+i]+ai_7))+c*exp(-t7_1*x)+c*exp(-t7_2*x)
                     +c*exp(-t7_3*x)+c*exp(-t7_4*x)+c*exp(-t7_5*x)
                     +c*exp(-t7_6*x)+(100+c)*exp(-t7_7*x)),
        function(x) (c*exp(-t7_1*x)*(-t7_1)+c*exp(-t7_2*x)*(-t7_2)+
                     c*exp(-t7_3*x)*(-t7_3)+c*exp(-t7_4*x)*(-t7_4)+
                     c*exp(-t7_5*x)*(-t7_5)+c*exp(-t7_6*x)*(-t7_6)+
                     (100+c)*exp(-t7_7*x)*(-t7_7)), x0 = 0)
ytm_bond_7[i]= as.numeric(f)
#spot rate
yield_7[i] = as.numeric((-ln(((data[b,5+i]+ai_7)-c*exp(-t7_1*r_1)
                      -c*exp(-t7_2*r_2)
                      -c*exp(-t7_3*r_3)
                      -c*exp(-t7_4*r_4)
                      -c*exp(-t7_5*yield_5[i])
                      -c*exp(-t7_6*yield_6[i]))/
                       (100+c)))/t7_7)

b=8;c=data[b,4]/2
ai_8 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
```

```
for (j in 1:8){assign(paste("t8", j, sep="_"), data[b,25+i]-(8-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*(data[b,5+i]+ai_8)+c*exp(-t8_1*x)+c*exp(-t8_2*x)
                    +c*exp(-t8_3*x)+c*exp(-t8_4*x)+c*exp(-t8_5*x)
                    +c*exp(-t8_6*x)+c*exp(-t8_7*x)+(100+c)*exp(-t8_8*x)),
        function(x) (c*exp(-t8_1*x)*(-t8_1)+c*exp(-t8_2*x)*(-t8_2)+
                    c*exp(-t8_3*x)*(-t8_3)+c*exp(-t8_4*x)*(-t8_4)+
                    c*exp(-t8_5*x)*(-t8_5)+c*exp(-t8_6*x)*(-t8_6)+
                    c*exp(-t8_7*x)*(-t8_7)+(100+c)*exp(-t8_8*x)*(-t8_8)), x0 = 0)
ytm_bond_8[i]= as.numeric(f)
#spot rate
yield_8[i] = as.numeric((-ln(((data[b,5+i]+ai_8)-c*exp(-t8_1*r_1)
                     -c*exp(-t8_2*r_2)
                     -c*exp(-t8_3*r_3)
                     -c*exp(-t8_4*r_4)
                     -c*exp(-t8_5*yield_5[i])
                     -c*exp(-t8_6*yield_6[i])
                     -c*exp(-t8_7*yield_7[i]))/
                      (100+c)))/t8_8)

b=9;c=data[b,4]/2
ai_9 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:9){assign(paste("t9", j, sep="_"), data[b,25+i]-(9-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*(data[b,5+i]+ai_9)+c*exp(-t9_1*x)+c*exp(-t9_2*x)
                    +c*exp(-t9_3*x)+c*exp(-t9_4*x)+c*exp(-t9_5*x)
                    +c*exp(-t9_6*x)+c*exp(-t9_7*x)+c*exp(-t9_8*x)+
                    (100+c)*exp(-t9_9*x)),
        function(x) (c*exp(-t9_1*x)*(-t9_1)+c*exp(-t9_2*x)*(-t9_2)+
                    c*exp(-t9_3*x)*(-t9_3)+c*exp(-t9_4*x)*(-t9_4)+
                    c*exp(-t9_5*x)*(-t9_5)+c*exp(-t9_6*x)*(-t9_6)+
                    c*exp(-t9_7*x)*(-t9_7)+c*exp(-t9_8*x)*(-t9_8)+
                    (100+c)*exp(-t9_9*x)*(-t9_9)), x0 = 0)
ytm_bond_9[i]= as.numeric(f)
#spot rate
yield_9[i] = as.numeric((-ln(((data[b,5+i]+ai_9)-c*exp(-t9_1*r_1)
                     -c*exp(-t9_2*r_2)
                     -c*exp(-t9_3*r_3)
                     -c*exp(-t9_4*r_4)
                     -c*exp(-t9_5*yield_5[i])
                     -c*exp(-t9_6*yield_6[i])
                     -c*exp(-t9_7*yield_7[i])
                     -c*exp(-t9_8*yield_8[i]))/
                      (100+c)))/t9_9)

b=10;c=data[b,4]/2
ai_10 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
for (j in 1:10){assign(paste("t10", j, sep="_"), data[b,25+i]-(10-j)*0.5)}
#ytm
f <- NW(function(x) ((-1)*(data[b,5+i]+ai_10)+c*exp(-t10_1*x)+c*exp(-t10_2*x)
                    +c*exp(-t10_3*x)+c*exp(-t10_4*x)+c*exp(-t10_5*x)
                    +c*exp(-t10_6*x)+c*exp(-t10_7*x)+c*exp(-t10_8*x)
                    +c*exp(-t10_9*x)+(100+c)*exp(-t10_10*x)),
```

```
        function(x) (c*exp(-t10_1*x)*(-t10_1)+c*exp(-t10_2*x)*(-t10_2)+
                     c*exp(-t10_3*x)*(-t10_3)+c*exp(-t10_4*x)*(-t10_4)+
                     c*exp(-t10_5*x)*(-t10_5)+c*exp(-t10_6*x)*(-t10_6)+
                     c*exp(-t10_7*x)*(-t10_7)+c*exp(-t10_8*x)*(-t10_8)+
                     c*exp(-t10_9*x)*(-t10_9)+(100+c)*exp(-t10_10*x)*(-t10_10)), x0 = 0)
  ytm_bond_10[i]= as.numeric(f)
  #spot rate
  yield_10[i] = as.numeric((-ln(((data[b,5+i]+ai_10)-c*exp(-t10_1*r_1)
                      -c*exp(-t10_2*r_2)
                      -c*exp(-t10_3*r_3)
                      -c*exp(-t10_4*r_4)
                      -c*exp(-t10_5*yield_5[i])
                      -c*exp(-t10_6*yield_6[i])
                      -c*exp(-t10_7*yield_7[i])
                      -c*exp(-t10_8*yield_8[i])
                      -c*exp(-t10_9*yield_9[i]))/
                       (100+c)))/t10_10)

  b=11;c=data[b,4]/2
  ai_11 = ((as.numeric(date_order[1,i] - data$`Maturity date`[b]) + 180*b)/365) * (2*c)
  for (j in 1:11){assign(paste("t11", j, sep="_"), data[b,25+i]-(11-j)*0.5)}
  #ytm
  f <- NW(function(x) ((-1)*(data[b,5+i]+ai_11)+c*exp(-t11_1*x)+c*exp(-t11_2*x)
                     +c*exp(-t11_3*x)+c*exp(-t11_4*x)+c*exp(-t11_5*x)
                     +c*exp(-t11_6*x)+c*exp(-t11_7*x)+c*exp(-t11_8*x)
                     +c*exp(-t11_9*x)+c*exp(-t11_10*x)+(100+c)*exp(-t11_11*x)),
        function(x) (c*exp(-t11_1*x)*(-t11_1)+c*exp(-t11_2*x)*(-t11_2)+
                     c*exp(-t11_3*x)*(-t11_3)+c*exp(-t11_4*x)*(-t11_4)+
                     c*exp(-t11_5*x)*(-t11_5)+c*exp(-t11_6*x)*(-t11_6)+
                     c*exp(-t11_7*x)*(-t11_7)+c*exp(-t11_8*x)*(-t11_8)+
                     c*exp(-t11_9*x)*(-t11_9)+c*exp(-t11_10*x)*(-t11_10)+
                     (100+c)*exp(-t11_11*x)*(-t11_11)), x0 = 0)
  ytm_bond_11[i]= as.numeric(f)
  #spot rate
  yield_11[i] = as.numeric((-ln(((data[b,5+i]+ai_11)-c*exp(-t11_1*r_1)
                      -c*exp(-t11_2*r_2)
                      -c*exp(-t11_3*r_3)
                      -c*exp(-t11_4*r_4)
                      -c*exp(-t11_5*yield_5[i])
                      -c*exp(-t11_6*yield_6[i])
                      -c*exp(-t11_7*yield_7[i])
                      -c*exp(-t11_8*yield_8[i])
                      -c*exp(-t11_9*yield_9[i])
                      -c*exp(-t11_10*yield_10[i]))/
                       (100+c)))/t11_11)
}
```

**Part a: Yield to Maturity Curve**

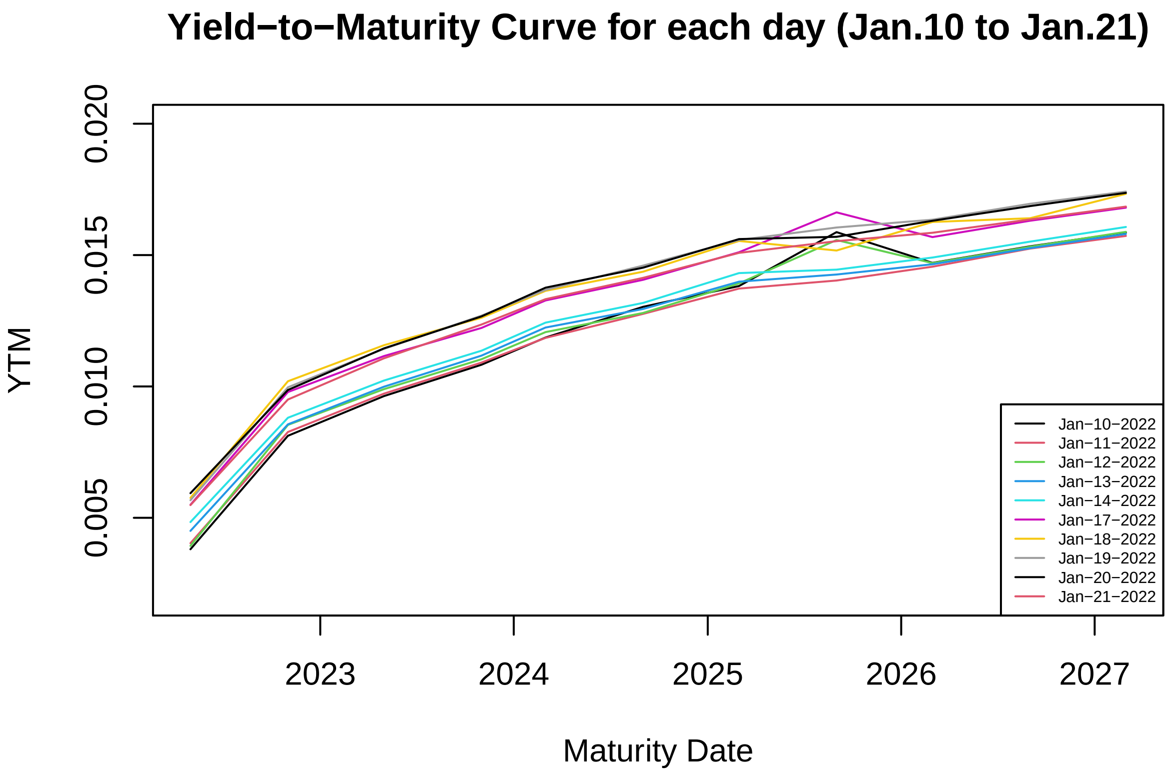
```
#plot ytm curve
ytm_table <-cbind(ytm_bond_1, ytm_bond_2, ytm_bond_3, ytm_bond_4,
                   ytm_bond_5, ytm_bond_6, ytm_bond_7, ytm_bond_8,
```

```
                    ytm_bond_9, ytm_bond_10, ytm_bond_11)
#view(ytm_table)
for (i in 1:10){assign(paste("ytm_day", i, sep="_"), ytm_table[i,])}

ytm_day_table <-data.frame(bond_order =c(1:11), ytm_day_1, ytm_day_2, ytm_day_3, ytm_day_4, ytm_day_5, y

maturity_time <- data$`Maturity date`

plot(x=maturity_time, y= ytm_day_1, type = "l", ylim = c(0.002,0.02), col = c(1),
     main = "Yield-to-Maturity Curve for each day (Jan.10 to Jan.21)",
     xlab = "Maturity Date", ylab="YTM")
lines(x=maturity_time, y= ytm_day_2, col = c(2))
lines(x=maturity_time, y= ytm_day_3, col = c(3))
lines(x=maturity_time, y= ytm_day_4, col = c(4))
lines(x=maturity_time, y= ytm_day_5, col = c(5))
lines(x=maturity_time, y= ytm_day_6, col = c(6))
lines(x=maturity_time, y= ytm_day_7, col = c(7))
lines(x=maturity_time, y= ytm_day_8, col = c(8))
lines(x=maturity_time, y= ytm_day_9, col = c(9))
lines(x=maturity_time, y= ytm_day_10, col = c(10))
legend("bottomright", legend =c("Jan-10-2022", "Jan-11-2022", "Jan-12-2022", "Jan-13-2022", "Jan-14-202
```



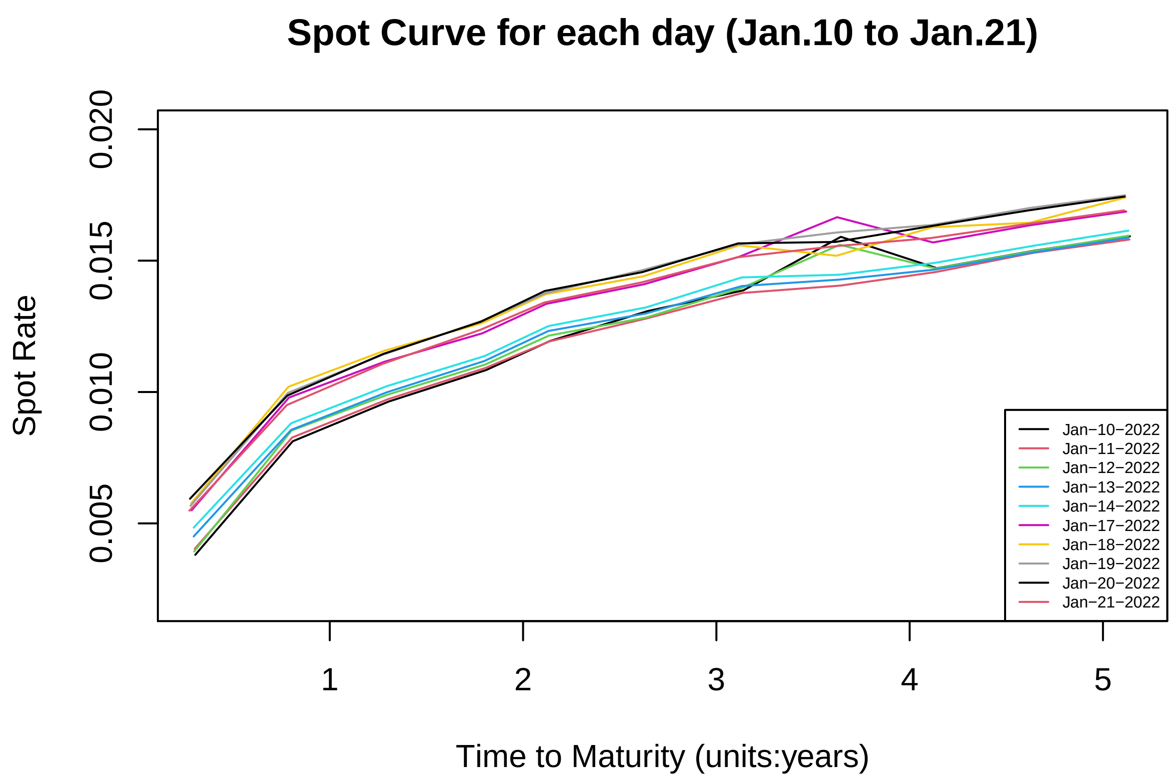

Interpolation technique: We applied the linear interpolation for the curve, such that we connect each consecutive points with straight line. The points on the line will be the value for those maturity date as linear combination of calculated consecutive points.

**Part b: Spot Curve**

```
#plot spot curve
spot_table <-data.frame(yield_1, yield_2, yield_3, yield_4, yield_5,
            yield_6, yield_7, yield_8, yield_9, yield_10, yield_11)

for (i in 1:10){
  assign(paste("spot_day", i, sep="_"), spot_table[i,])
}


plot(x=data$date_1_TTM, y= spot_day_1, type = "l", ylim = c(0.002,0.02), col = c(1),
     main = "Spot Curve for each day (Jan.10 to Jan.21)",
     xlab = "Time to Maturity (units:years)", ylab="Spot Rate")
lines(x=data$date_2_TTM, y= spot_day_2, col = c(2))
lines(x=data$date_3_TTM, y= spot_day_3, col = c(3))
lines(x=data$date_4_TTM, y= spot_day_4, col = c(4))
lines(x=data$date_5_TTM, y= spot_day_5, col = c(5))
lines(x=data$date_6_TTM, y= spot_day_6, col = c(6))
lines(x=data$date_7_TTM, y= spot_day_7, col = c(7))
lines(x=data$date_8_TTM, y= spot_day_8, col = c(8))
lines(x=data$date_9_TTM, y= spot_day_9, col = c(9))
lines(x=data$date_10_TTM, y= spot_day_10, col = c(10))
legend("bottomright", legend =c("Jan-10-2022", "Jan-11-2022", "Jan-12-2022", "Jan-13-2022", "Jan-14-202
```



**Part c: Forward Rate Curve**

```
#generate empty list for each date forward rate of size 4:
# 1yr-1yr, 1yr-2yr, 1yr-3yr, 1yr-4yr
#meaning future one year rate in 2,3,4 year from now
#we need to use linear interpolation to get each exact year's spot rate R_1, R_2, R_3, R_4, R_5.

for (j in 1:10){assign(paste("forward", j, sep="_"), rep(NA, 4))}

R<- matrix(NA, nrow = 10, ncol = 5)

for (j in 1:5){
    x<- c(as.list(data$date_1_TTM))
    y<- c(as.list(spot_day_1))
    R[1,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_2_TTM))
    y<- c(as.list(spot_day_2))
    R[2,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_3_TTM))
    y<- c(as.list(spot_day_3))
    R[3,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_4_TTM))
    y<- c(as.list(spot_day_4))
    R[4,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_5_TTM))
    y<- c(as.list(spot_day_5))
    R[5,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_6_TTM))
    y<- c(as.list(spot_day_6))
    R[6,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_7_TTM))
    y<- c(as.list(spot_day_7))
    R[7,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_8_TTM))
    y<- c(as.list(spot_day_8))
    R[8,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_9_TTM))
```

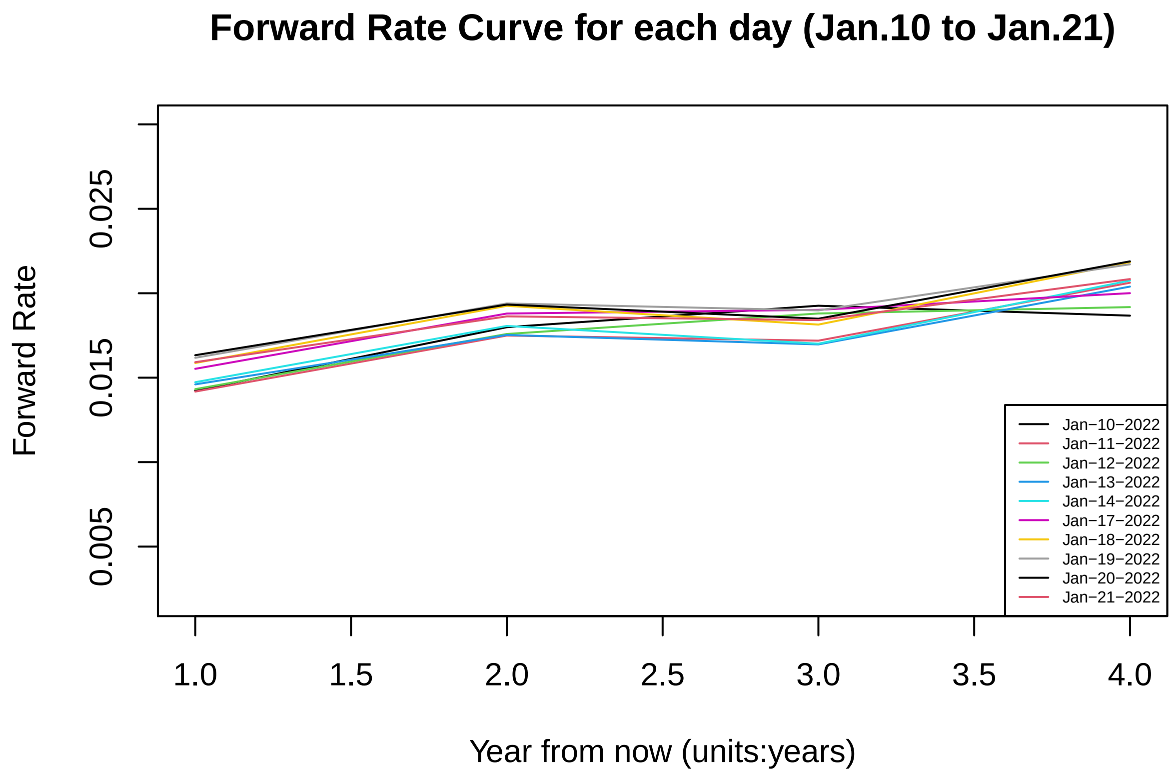
```
    y<- c(as.list(spot_day_9))
    R[9,j] = approx(x,y, xout=j)$y
}
for (j in 1:5){
    x<- c(as.list(data$date_10_TTM))
    y<- c(as.list(spot_day_10))
    R[10,j] = approx(x,y, xout=j)$y
}

Forward <- matrix(NA, nrow = 10, ncol = 4)

for (i in 1:10){
  for (j in 1:4){
    Forward[i,j] = (R[i,j+1]*(j+1) - R[i,j]*j)
  }}

#for (i in 1:10){
  #Forward[i,1] = R[i,1]
  #for (j in 2:5){
    #Forward[i,j] = (R[i,j+1]*(j+1) - R[i,j]*j)
  #}}

plot(x=c(1:4), y= Forward[1,], type = "l", ylim = c(0.002,0.03), col = c(1),
     main = "Forward Rate Curve for each day (Jan.10 to Jan.21)",
     xlab = "Year from now (units:years)", ylab="Forward Rate")
lines(x=c(1:4), y= Forward[2,], col = c(2))
lines(x=c(1:4), y= Forward[3,], col = c(3))
lines(x=c(1:4), y= Forward[4,], col = c(4))
lines(x=c(1:4), y= Forward[5,], col = c(5))
lines(x=c(1:4), y= Forward[6,], col = c(6))
lines(x=c(1:4), y= Forward[7,], col = c(7))
lines(x=c(1:4), y= Forward[8,], col = c(8))
lines(x=c(1:4), y= Forward[9,], col = c(9))
lines(x=c(1:4), y= Forward[10,], col = c(10))
legend("bottomright", legend =c("Jan-10-2022", "Jan-11-2022", "Jan-12-2022", "Jan-13-2022", "Jan-14-2022
```




**Question 5:**

**Part a: Covariance Matrix**

```
#find the covariance matrix for the log-returns of yield
rep(NA, 9) -> X_1 -> X_2 -> X_3 -> X_4 -> X_5
rep(NA, 10) -> r_1 -> r_2 -> r_3 -> r_4 -> r_5
maturity_time <- as.numeric(data$`Maturity date` - data$date_1)/365

for (j in 1:10){
  slope_1 = (ytm_table[j,3] - ytm_table[j,2])/(maturity_time[3] - maturity_time[2])
  r_1[j]= slope_1*1 + ytm_table[j,2]- slope_1*maturity_time[2]
  slope_2 = (ytm_table[j,5] - ytm_table[j,4])/(maturity_time[5] - maturity_time[4])
  r_2[j]= slope_2*2 + ytm_table[j,4]- slope_2*maturity_time[4]
  slope_3 = (ytm_table[j,7] - ytm_table[j,6])/(maturity_time[7] - maturity_time[6])
  r_3[j]= slope_3*3 + ytm_table[j,6]- slope_3*maturity_time[6]
  slope_4 = (ytm_table[j,9] - ytm_table[j,8])/(maturity_time[9] - maturity_time[8])
  r_4[j]=slope_4*4 + ytm_table[j,8]- slope_4*maturity_time[8]
  slope_5 = (ytm_table[j,11] - ytm_table[j,10])/(maturity_time[11] - maturity_time[10])
  r_5[j]=slope_5*5 + ytm_table[j,10]- slope_5*maturity_time[10]
}

for (j in 1:9){
  X_1[j]= log(r_1[j+1]/r_1[j])
```

```r
  X_2[j]= log(r_2[j+1]/r_2[j])
  X_3[j]= log(r_3[j+1]/r_3[j])
  X_4[j]= log(r_4[j+1]/r_4[j])
  X_5[j]= log(r_5[j+1]/r_5[j])
}

X = cbind(X_1, X_2, X_3, X_4, X_5)
cov_X = cov(X)
```

```r
#find the covariance matrix for forward rate
F = cbind(Forward[,1], Forward[,2], Forward[,3], Forward[,4])
cov_F = cov(Forward)
cov_F
```

```
##              [,1]         [,2]          [,3]          [,4]
## [1,] 7.345422e-07 6.221186e-07  2.503390e-07  7.196834e-07
## [2,] 6.221186e-07 5.900404e-07  3.148823e-07  5.817621e-07
## [3,] 2.503390e-07 3.148823e-07  7.597228e-07 -2.334156e-07
## [4,] 7.196834e-07 5.817621e-07 -2.334156e-07  1.182929e-06
```

### Part b: Eigenvalue and Eigenvector of Covariance Matrix

```r
#eigenvalue and eigenvector of Covariance Matrix for log yield
eigen(cov_X)
```

```
## eigen() decomposition
## $values
## [1] 4.004387e-03 4.615035e-04 1.030588e-04 1.710784e-05 6.992398e-06
##
## $vectors
##            [,1]         [,2]       [,3]        [,4]         [,5]
## [1,] -0.5757297 -0.507692480  0.5758973 -0.26225435 -0.101727829
## [2,] -0.4009054 -0.231554436 -0.2013670  0.67987018  0.531869625
## [3,] -0.3815640 -0.082801842 -0.5031178  0.18358712 -0.748813062
## [4,] -0.5031854  0.825687909  0.2497307  0.05082346  0.009769529
## [5,] -0.3301871  0.003770962 -0.5588386 -0.65780843  0.382033644
```

```r
#eigenvalue and eigenvector of Covariance Matrix for log forward rate
eigen(cov_F)
```

```
## eigen() decomposition
## $values
## [1] 2.174358e-06 1.026348e-06 4.360027e-08 2.292747e-08
##
## $vectors
##            [,1]       [,2]        [,3]       [,4]
## [1,] 0.55894120 -0.1659666 -0.75665576  0.2958240
## [2,] 0.48348346 -0.2526439  0.08665414 -0.8336101
## [3,] 0.09652163 -0.8432726  0.39408766  0.3525192
## [4,] 0.66671736  0.4444294  0.51444925  0.3054709
```