

**Problem Chosen**

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**MCM/ICM  
Summary Sheet**

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**2428775**

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# Harmonizing Waterways, Thriving Humanity: Sustainable Development of Dam Benefits

## Summary

The Great Lakes, with many stakeholders closely tied to it, is a critical lifeline in the United States. The seasonal fluctuations in water levels here are influenced by multiple natural factors, and dams play a crucial role to control the outflow. This paper evaluates the interests of various stakeholders and develops dam regulation strategies suitable for different environmental conditions.

Two main models are established: Model I: a multi-objective programming model for maximizing overall benefits; Model II: an optimization model for dam regulation.

Before building the models, an analysis of the natural environment and stakeholders is conducted. This analysis encompass the connectivity of lakes and rivers, the factors influencing water levels, the relationship between water levels, and the interests of different parties.

For Model I: To obtain the optimal water levels, we establish a **multi-objective programming model(MOP)** that maximizes comprehensive benefits of all parties. The variables are the water levels of Great Lakes. The objective function is the overall interests of related stakeholders. The constraints are the water level requirements from various stakeholders.

Through the analysis of the water level requirements of various stakeholders for the Great Lakes, we formulate the constraint conditions. In order to maintain a balance among all parties, we optimize the objective function and establish the MOP. Then by utilizing the **Particle Swarm Optimization (PSO)** algorithm, we find the global maximum, which represents the optimal water levels that achieve a balance and maximize overall benefits. The optimal water levels for each month in the 4 regions are presented in **Table 2**.

For Model II: This model is an improvement over Model I, aiming to achieve a solution for controlling water levels. The variables are passage rates. The objective function aims to make the predicted values most in line with the actual ones. Constraints include water levels and new constraints on flow and velocity.

Similar to Model One, a MOP is established based on the new constraints and objective function. Additionally, we implement an **LSTM neural network model** to predict monthly precipitation, evaporation, and flow rates. These predictions are then used in the optimization problem allows us to determine the optimal control parameters. And they are used for the simulation of the next month.

Finally, we conduct sensitivity analysis. We simulate the water level and compare it with the original data for 2017. In situations with relatively large water volume, we can still control the water level well, as detailed in **Section 7.2**. Additionally, we investigate the water level control under different environmental conditions, as discussed in **Section 7.3**.

**Keywords:** MOP (Multi-Objective Programming); PSO (Particle Swarm Optimization Algorithm); LSTM neural network; Dam Regulation

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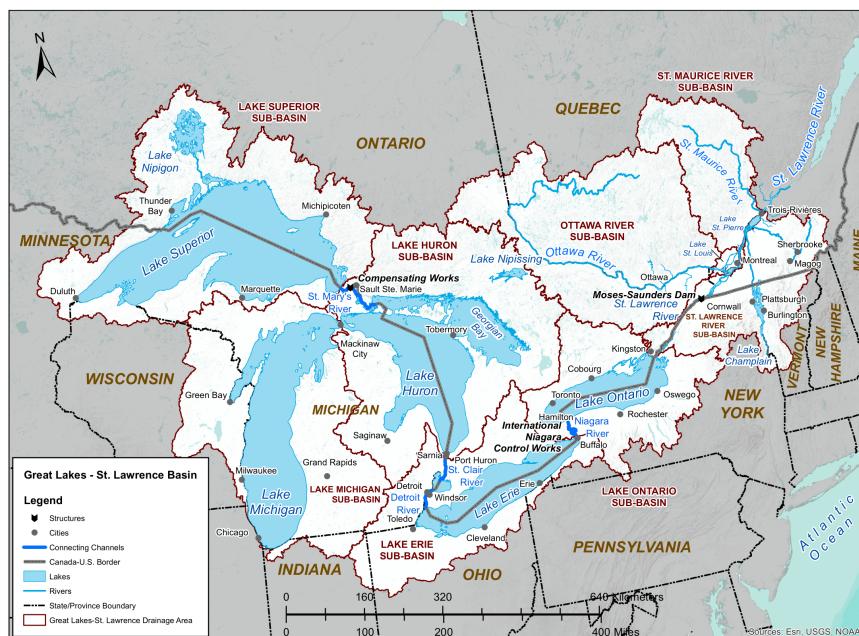
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# 1 Introduction

## 1.1 Background

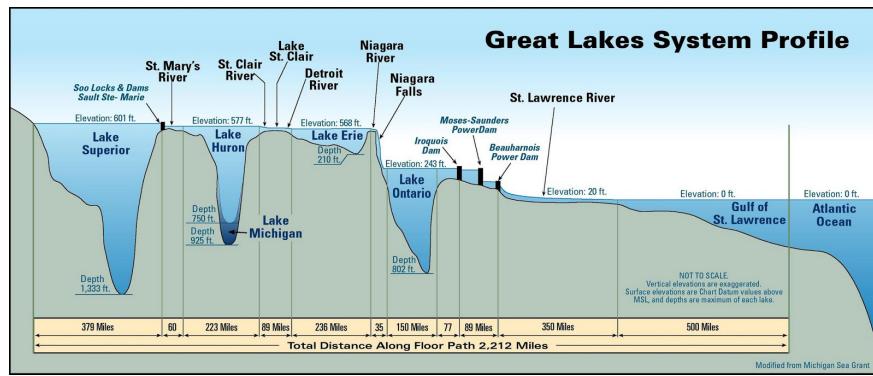
Considering that the cities around the lakes, like Duluth, are struggling with the combination of extreme weather and high water levels, “Climate-driven change to Great Lakes water levels is a prospect we take seriously,” Jeff Johnson, a public information officer with Michigan’s Department of Environment, Great Lakes, and Energy said.[1]

The Great Lakes of North America, with a total surface area of 94,250 square miles (244,106 square kilometers) and united by various lakes, rivers and other waterways, are the largest group of freshwater lakes in the world which can be seen in **Figure1**. The five Great Lakes are Superior, Huron, Michigan, Erie, and Ontario. They account for 21% of the total surface water.[2] Between these lakes, there are rivers connecting them and eventually flowing into the Atlantic Ocean. The rivers and the relation between the lakes like the comparison of depths can be seen in **Figure2**.



**Figure 1:** Map of great lakes[3]

The water level is controlled by multiple influencing factors, such as temperature, wind, tides, precipitation, upstream flows, groundwater, surface water runoff, evaporation, lake water diversions, water level regulations, seasonal cycles, and long-term climate changes.[5] Roughly, lake water levels usually drop in winter due to increased evaporation. Spring water levels usually rise as snow melts, spring rain, and runoff occur. Like Jay Austin, a professor at the University of Minnesota Duluth said “We don’t have that sort of control over nature.”[1] Thus it is difficult to predict and regulate the changeable natural changes, so people can only artificially control the control mechanisms(dams – Compensating Works and Moses-Saunders Dam) to adjust the water level. At the same time, attention must be paid to the cost and benefits of stakeholders, and try to balance.



**Figure 2: Great Lakes System Profile[2]**

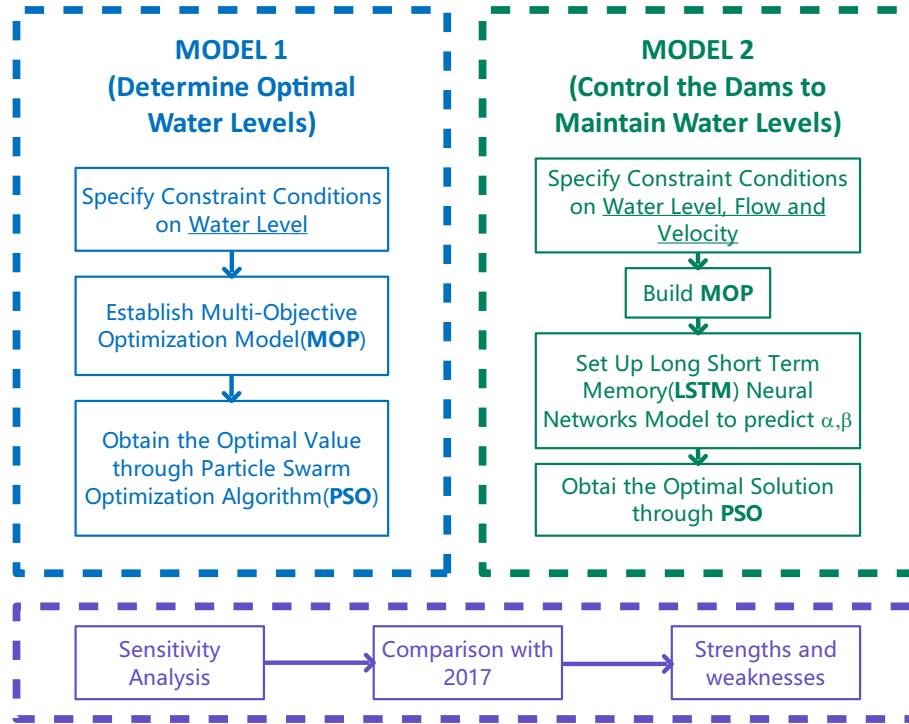
## 1.2 Restatement of the Problem

The Great Lakes's water level affects all stakeholders' interests, including residents, shipping, the environment and other parties. Through the analysis and investigation of the background, some indications from the ICM supervisor, as well as the exploration and discussion of the problem, the reproduction of the problem can be expressed as follows:

- Build a mathematical model to find out the optimal water levels of the Great Lakes at any time(by month) of the year.
  - Establish a constraint optimization model considering water levels according to the needs of stakeholders and the historical data for the Great Lakes.
  - Calculate the optimal water level for each lake in each month of the year.
- Establish a model to maintain optimal water levels.
  - Set up a constraint optimization model considering water flow and velocities additionally based on the first problem's model and the outflow and inflow of the Great Lakes.
  - Build an LSTM neural network model to calculate the coefficients required.
- Perform sensitivity analyses and compare them with the data from 2017.
  - Conduct a sensitivity analysis and check the sensitivity of the model to the outflow of the two control dams.
  - Compare new control results of dams with actual record water levels for the various stakeholders for 2017.
  - Check the sensitivity of the model to the changes in environmental conditions such as precipitation, winter snow-pack, and ice jams.
- Summarize the results and provide the IJC leadership with a one-page memo to explain the model we built is worth choosing.

### 1.3 Our Work

Our work can be roughly divided into three parts, which is generally shown as the figure following.



**Figure 3:** The Framework of Our Work

## 2 Assumptions and Justifications

- It can be assumed that the volume of the lake can be represented by the triangular cone, i.e  $V = \frac{1}{3}Sh$ , V represents the volume, S represents the surface area and h represents the depth of the lake.
  - According to the data from **Great Lakes Water Level Data**[6], this conclusion can be obtained.
- The shape of a section of fresh water on the surface of the lake can be regarded as the same.
  - The overall shape of the lake shows an inverted triangular cone, which can be seen in **Figure 2**, and the surface area and depth of these lakes are very large and deep. In addition, the change in water level is negligible compared with the depth of the whole lake, so the area of the surface section of the lake and below can be considered the same.
- Assume the flow direction is one-way, from Lake Superior to the Atlantic Ocean.
  - From Lake Superior to the Atlantic Ocean, their altitudes descend generally. It can be certified by **Figure2** and **Figure6(b)**.

- Use the months as the time units.
  - The data on lakes in the data set is in months, and the data will change in different months of the year, so the month is the most appropriate.
- Consider the Lake Huron and Lake Michigan combined.
  - The data in the data set is given by combining the two sets of these 2 lakes.
- Mainly consider the needs of shipping companies and people who manage shipping docks or live near Montreal harbor.
  - Although all stakeholders' needs are considered, the impacts of water levels on them are deepest.
- The inflow and outflow of rivers are assumed to be constant and unaffected by evaporation and precipitation.
  - Rivers have smaller surface areas than lakes. Affected a lot will lead to the model being too complex, so it is unnecessary to consider it.
- Suppose that there is no dike break in the whole process.
  - If the dam is about to burst, the dam should be opened before it breaks to flow out. Otherwise, it may cause more serious consequences.
- Taking the sea level as the designated datum, the water level is the height of the surface of the lake from the sea level.
  - The water level is usually defined above.

### 3 Notations

The important symbols commonly used in the article and their meanings are given in the **Table 1** below.

**Table 1:** Important Symbols and Descriptions

Symbol	Description
<b><i>Indices</i></b>	
<i>I</i>	number of Great Lakes ( $i \in \{1, 2, 3, 4\}$ , from West to East)
<i>R</i>	number of rivers ( $r \in \{1, 2, 3, 4\}$ , from West to East)
<i>M</i>	number of months ( $m \in \{1, \dots, 12\}$ )
<i>J</i>	number of stakeholders ( $j \in \{1, \dots, J\}$ )
<i>K</i>	number of control mechanisms ( $k \in \{1, 2\}$ )

Symbol	Description
<b><u>Parameters</u></b>	
$h_i$	water level of the $i$ th lake
$h_{im}$	water level of $i$ th lake in month $m$
$p_j(h_{im})$	the gain of the stakeholder $j$ at the water level $h_{im}$
$P_j$	average monthly total income of the stakeholder $j$ (normalization coefficient)
upper bound $^{j}_{im}$	the upper boundary required by stakeholders $j$ for the water level of the $i$ th lake in the $m$ month
lower bound $^{j}_{im}$	the lower boundary required by stakeholders $j$ for the water level of the $i$ th lake in the $m$ month
$\alpha_{im}$	changes in lake $i$ (precipitation-evaporation) in month $m$
$\beta_{im}$	net outflow of lake $i$ at month $m$ (outflow of river)
$\omega_i$	the fluctuation error of the $i$ th lake
$S_i$	the area of the $i$ th lake
$S_r$	the area of the $r$ th river
$\gamma_k$	proportion of water discharge from the sluice gate $k$ ( $\gamma_k \in \{0, 1\}$ )
$Q_r$	river flow of the $r$ th river
$v_r$	water flow velocity of the $r$ th river

## 4 Model Preparation

### 4.1 Data Collection and Preliminary processing

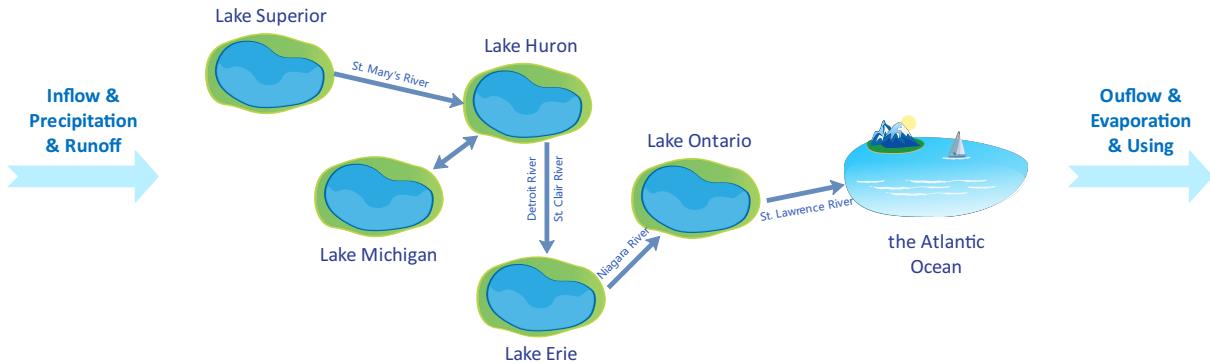
To build the mathematical model and solve the problems, we collected some data including geographical data on the lake, water levels of lakes and the rivers connecting them, river flow, evaporation, precipitation, industrial and agricultural water consumption, freezing conditions, etc. Since the problem requires a comparison with the data for 2017 and the data varies widely between different years, this paper only considers the data and circumstances for 2016-2020. In addition, since Lake Huron and Lake Michigan are basically put together in the data set, the two are considered together in this article.

After collecting the data, we initially process it to remove outliers in the first 2% and last 2% to prepare for the following operations.

The sources of the detailed data that we mainly adopted are shown in the table below.

### 4.2 Connectivity between lakes and rivers

According to the background and data, we construct the general water flow trend diagram as follows. The current in the Great Lakes flows from east to west because of the terrain, through their interconnected rivers, and eventually into the Atlantic Ocean. The amount of water obtained is mainly affected by the inflow, precipitation, and runoff, and the amount of water lost is mainly affected by the outflow, evaporation and use. The amount of water circulating between the lakes may also be affected by the water freezing caused by low temperatures and gated dams.

**Figure 4:** General trend

### 4.3 Stakeholders

There are several kinds of stakeholders mentioned in the water level control problem of Lake Ontario, which are shipping companies(transportation practitioners), residents, environmentalists, property owners, fishermen, and hydro-power generation companies. Now, we not only consider stakeholders of Lake Ontario but also take into account stakeholders of all water bodies. Therefore, we expand the stakeholders of Lake Ontario to the ones of all lakes. Stakeholders and their requirements considered in this article are shown below.

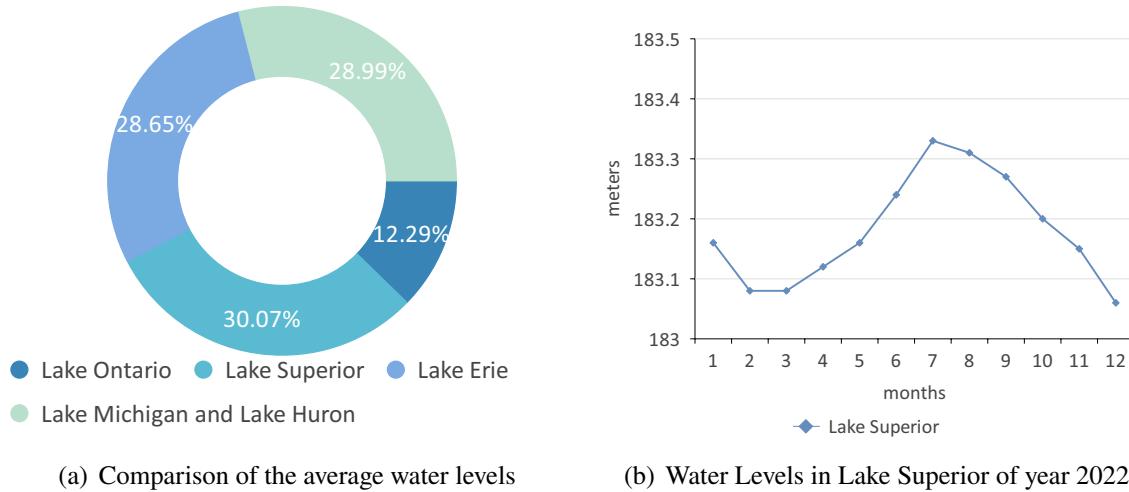
Internal Needs	Water Level Needs	Water velocity Needs
	waterway is enough to carry the transport	high static
	suitable for life	low steady
	help maintain the habitat for species to thrive and clean out static bays and tributaries	seasonal high and low /
	suitable for life	mid-level steady
	Suitable for using facilities such as docks and boat launching ramps	mid-level steady
	use high-level water as a storage system to maximize flows during high energy usage periods	limited high /

**Figure 5:** Stakeholders' requirements

#### 4.4 Water level analysis of the lake

The mean water level data of great lakes are visualized as **Figure6(a)**. From **Figure2** we can see that Lake Superior has the highest water level and Lake Ontario has the lowest water level. In addition, from the proportion, we can also confirm that the water level from the east to the west is constantly decreasing.

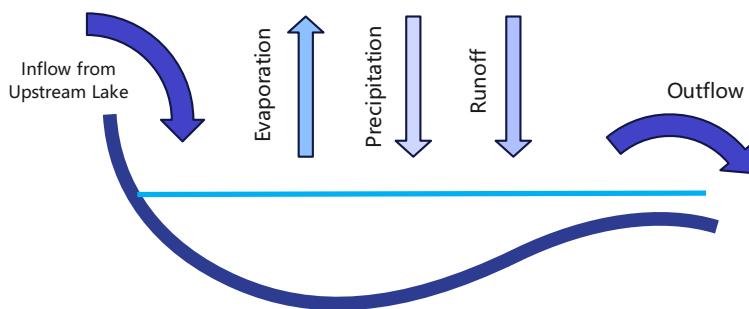
Besides, we observe the changing trend of water level in different months of the year. It can be seen that the water level in different months fluctuates, and the water level in different months needs to be considered separately. Therefore, the time unit considered in this paper is the month. **Figure6(b)** is shown as an example.



**Figure 6:** Water levels comparison

#### 4.5 Factors affecting the water level

The changes in lake water levels considered in this article can be approximately shown by **Figure7**.



**Figure 7:** Schematic diagram of the water level influence

## 5 Model I : Emulational Lakes and Rivers Constraint Model

### 5.1 Objective Function

Due to the interests of shipping companies(transportation practitioners) and hydro-power generation companies can be quantified[7][8], and the interests of the other stakeholders are difficult to quantify and are only needed to reach the scope of their requirements, which do not need clear interest data, the interests of the transportation and hydro-power generation companies are considered to determine target function in this article.

#### 5.1.1 Interests of all stakeholders

- **Shipping companies:**

In different seasons, navigable ships are affected by water levels and freezing conditions affected by temperatures. If the water level is not high enough, the weight of the cargo will be limited. If the water is non-flowing, the ship cannot sail normally.[7] Therefore, we assume that the shipping companies' earnings are proportional to the water levels during the peak sailing season when the freezing rate coverage is less than the threshold. Otherwise, the shipping companies' earnings are considered a smaller constant value in the slow shipping season.

$$p_{\text{shipping}} = \begin{cases} t^l (h - h_{\text{base}})^a \beta^n & \text{if rate} < \text{threshold} \\ \max(P(\text{constant})) & \text{otherwise} \end{cases} \quad (1)$$

where  $a$  is an odd number,  $h$  is the water level,  $h_{\text{base}}$  is the normalization constant of water level,  $t$  is the temperature of the month,  $\beta$  is the net outflow, icy rate is the freezing rate and fixed profit is the fixed income for the freezing season.

- **Hydro-power generation companies:**

Hydroelectric power is generated by converting the potential energy contained in a large amount of stored water into the kinetic energy of the turbine, which then pushes the generator to generate electricity. As a result, as water levels rise, hydro-power generation companies will also generate more electricity. But at the same time, maintenance costs and construction costs will also increase, so a balance is needed to find to make the most profitable.

$$P_{\text{hydro-power}} = c \cdot \beta t g h \cdot \eta - (h - h_0)^k \quad (2)$$

where  $c$  is the net profit per unit of electricity,  $h$  is the water level,  $h_0$  is the lowest construction cost of the hydro-power station,  $\beta$  is the net outflow,  $t$  shows time(one month),  $g$  is the gravitational acceleration,  $\eta$  is the conversion rate of the potential energy of water into electric energy.

#### 5.1.2 Trade-off coefficient

We need to maximize the sum of the interests of all categories of stakeholders but also need to minimize the interests of any two of them. Since the benefits that different stakeholders can obtain vary greatly in specific values, normalization needs to be used to compare them together more fairly.

Here  $P_j$  represents the average monthly total income of the stakeholder  $j$ . We use it to normalize. Normalize and subtract the benefits of each two stakeholder so that the benefits gotten by different stakeholders eventually can be more balanced. As the differences between any two of them are calculated twice, we divide them by 2.

The objective function is:

$$\max \left( \sum_{i=1}^4 \sum_{j \in J} p_j(h_i) - \frac{1}{2} \sum_{j_1, j_2 \in J, j_1 \neq j_2} \left| \frac{\sum_{i=1}^4 p_{j_1}(h_{im})}{P_{j_1}} - \frac{\sum_{i=1}^4 p_{j_2}(h_{im})}{P_{j_2}} \right| \right) \quad (3)$$

## 5.2 Constraints

According to the requirements of the question, we only need to consider the water levels, not the velocities of water flow in the first problem, so we determine the following constraints.

### 5.2.1 Constraints from the various stakeholders

- **Constraints from residents:**

People who manage shipping docks or live near Montreal harbor want low and steady water in the river. Thus we have constraints shown below.

$$h_{im} \leq \text{upper bound}_{im}^1 \quad (4)$$

- **Constraints from environmentalists:**

Environmentalists want seasonal high high-water levels and low low-water levels which water levels are high in summer and low in winter. Thus constraints are:

$$h_{im} = \begin{cases} \geq \text{lower bound}_{im}^2 & m \in \text{summer} \\ \leq \text{upper bound}_{im}^2 & m \in \text{winter} \end{cases} \quad (5)$$

- **Constraints from property owners and fishermen:**

Property owners on the shores, recreational boaters and fishing boats want mid-level, steady water levels. So we have:

$$\text{lower bound}_{im}^3 \leq h_{im} \leq \text{upper bound}_{im}^3 \quad (6)$$

$$|h_{im} - h_{i(m+1)}| \leq \text{max change} \quad (7)$$

## 5.3 Summary of the Multiple Objective Planning Model (MOP)

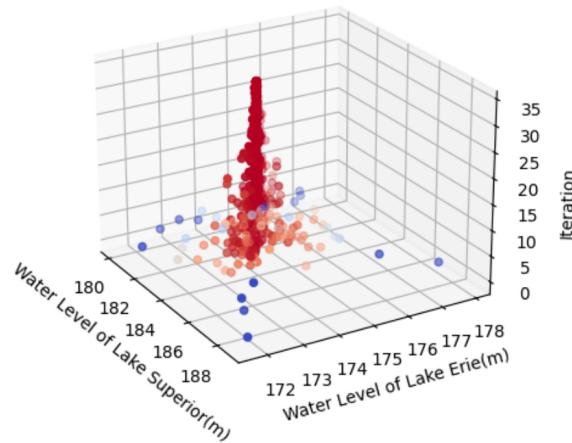
We need to maximize the benefits and maintain the balance between the various stakeholders, and we need to meet certain constraints. Then the Multiple Objective Planning Model of the  $m$  th month is set up. Our goal is to get the results of the MOP shown below:

$$\begin{aligned}
 & \max \left( \sum_{i=1}^4 \sum_{j \in J} p_j(h_i) - \frac{1}{2} \sum_{\substack{j_1, j_2 \in J \\ j_1 \neq j_2}} \left| \frac{\sum_{i=1}^4 p_{j_1}(h_{im})}{P_{j_1}} - \frac{\sum_{i=1}^4 p_{j_2}(h_{im})}{P_{j_2}} \right| \right) \\
 & \text{s.t.} \\
 & \left\{ \begin{array}{l} \text{lower bound}_{im}^j \leq h_{im} \leq \text{upper bound}_{im}^j \\ \frac{1}{3} S_1 \frac{dh_{1m}}{dt} = \alpha_1 - \beta_1 + w_1 \\ \frac{1}{3} S_2 \frac{dh_{2m}}{dt} = \alpha_2 + \beta_1 - \beta_2 + w_2 \\ \frac{1}{3} S_3 \frac{dh_{3m}}{dt} = \alpha_3 + \beta_2 - \beta_3 + w_3 \\ \frac{1}{3} S_4 \frac{dh_{4m}}{dt} = \alpha_4 + \beta_3 - \beta_4 + w_4 \\ h_{1m} \geq h_{2m} \geq h_{3m} \geq h_{4m} \\ \omega_i \sim \mathcal{N}(\mu_i, \sigma^2) \end{array} \right. \quad (8)
 \end{aligned}$$

## 5.4 Calculation Results

To solve the constrained optimization problem, first, we need to combine the original objective function with penalty functions to construct the Lagrangian function:

$$\begin{aligned}
 L(h_{1m}, h_{2m}, h_{3m}, h_{4m}) &= \text{profit} \\
 &+ \sum_{h \in \text{HARD constraints}} \lambda_h \cdot \text{Violation} \\
 &+ \sum_{s \in \text{SOFT constraints}} \lambda_s \cdot \text{ReLU}(\text{Violation}) \quad (9)
 \end{aligned}$$



**Figure 8:** Particle Swarm Optimization (PSO) Effect Graph

In this paper, we categorize constraints into hard constraints and soft constraints. Hard constraints must not be violated strictly and must be satisfied at all times, while soft constraints are intended to be satisfied as much as possible.

Next, we apply the PSO algorithm to the Lagrangian function to obtain the extremum.

The example in **Figure8** shows the particle swarm for Lake Superior and Lake Erie after multiple iterations. It is easy to observe that with an increase in the number of iterations, the particles transition from blue points to red points, demonstrating an overall convergence and ascent process. Eventually, they converge at the top, representing the optimal extremum of the Lagrangian function.

After solving the extremum of the Lagrangian function, the optimal water levels for each month of the five Great Lakes are obtained in **Table2**.

The pseudocode of PSO is as follows:

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**Algorithm 1** Particle Swarm Optimization (PSO)

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**Input:** comprehensive profit, constraints

**for** each particle  $i$  **do**

    Initialize position  $\mathbf{x}_i$  randomly within permissible range

    Initialize velocity  $\mathbf{v}_i$  randomly within permissible range

**while** termination criterion not met **do**

**for** each particle  $i$  **do**

        Calculate fitness value

**if** the fitness value is better than  $p_{i,pbest}^k$  in history **then**

            Set current fitness value as the  $p_{i,pbest}^k$

    Choose the particle having the best fitness value as the  $p_{gbest}^k$

**for** each particle  $i$  **do**

        Calculate velocity according to the equation

$$v_i^{k+1} = \omega v_i^k + c_1 r_1 (p_{i,pbest}^k - x_i^k) + c_2 r_2 (p_{gbest}^k - x_i^k)$$

        Update particle position according to the equation

$$x_i^{k+1} = x_i^k + v_i^{k+1}$$


---

**Table 2:** Optimal water level of the lake for each month

Lake(m)	Month					
	Jan	Feb	Mar	Apr	May	Jun
Superior	183.413	183.24425	183.18975	183.2525	183.23575	183.35433
Michigan and Huron	176.579	176.46825	176.47775	176.6285	176.65775	176.76833
Erie	174.279	174.22425	174.31575	174.5125	174.53775	174.59233
Ontario	74.605	74.62625	74.69375	74.9225	75.08375	75.12833

Lake(m)	Jul	Aug	Sep	Oct	Nov	Dec
Superior	183.481	183.563	183.63925	183.65675	183.553	183.49425
Michigan and Huron	176.865	176.903	176.92125	176.87275	176.775	176.73625
Erie	174.609	174.569	174.53725	174.44675	174.355	174.34225
Ontario	75.075	74.955	74.79125	74.62875	74.545	74.54625

## 6 Model II : Optimum Water Level Algorithm Model for Maintaining the Great Lakes

### 6.1 Optimization objective

In order to improve the fitting degree and accuracy of the model, we minimize the residual difference. We square the difference to make it easy to find the derivatives. The expression is as follows.

$$\min \frac{1}{2} \sum_1^4 (h_i - h_{besti})^2 \quad (10)$$

### 6.2 Added constraints

Since the solution is needed to achieve according to the problem, we add a new parameter  $\gamma$  to represent the situation of the sluice gate discharge ratio. Furthermore, based on the consideration of the inflow and outflow data, which can reflect the velocity, we add new parameters  $Q_r$  to show the amount of river flow and  $v_r$  to show the water flow velocity. Their specific calculation formulas are as follows.

$$\begin{cases} Q_1 = \gamma_1 \beta_1 \\ Q_2 = \beta_2 \\ Q_3 = \beta_3 \\ Q_4 = \gamma_2 \beta_4 \end{cases} \quad (11)$$

$$v_r = \frac{Q_r}{S_r} \quad (12)$$

- **Constraints from water flow velocity:**

As the St Lawrence River is wanted be static for transportation, there exists an upper limit.

$$v_4 \leq v_{\max 4} \quad (13)$$

- **Constraints from river flow:**

There are flow restrictions in hydro-power stations. The Robert Moses-Robert H. Saunders Power Dam(Moses-Saunders) is one of 2 mainstem hydroelectric generating stations from Lake Ontario to St. Lawrence and almost all rivers' flow passes through the Moses-Saunders dam.[9] So we consider its upper-bound river flow mainly. The limitations of the river flow of Moses-Saunders Power Dam are shown below.

$$\frac{1}{3} S_4 \frac{dh_4}{dt} \leq 10704(m^3/s) \quad (14)$$

### 6.3 Summary of the New MOP

Based on the objective of minimizing the residual difference and relative constraints, we have this new MOP following.

$$\begin{aligned}
 & \min \frac{1}{2} \sum_1^4 (h_i - h_{besti})^2 \\
 & \text{s.t.} \\
 & \left\{ \begin{array}{l}
 \frac{1}{3}S_1(h_1 - h_{origin1}) = \alpha_1 - \beta_1\gamma_1 + w_1 \\
 \frac{1}{3}S_2(h_2 - h_{origin2}) = \alpha_2 + \beta_1\gamma_1 - \beta_2 + w_2 \\
 \frac{1}{3}S_3(h_3 - h_{origin3}) = \alpha_3 + \beta_2 - \beta_3 + w_3 \\
 \frac{1}{3}S_4(h_4 - h_{origin4}) = \alpha_4 + \beta_3 - \beta_4\gamma_2 + w_4 \\
 0 \leq r_1 \leq 1 \\
 0 \leq r_2 \leq 1 \\
 v_4 \leq v_{max} \\
 \frac{1}{3}S_4 \frac{dh_4}{dt} \leq Q_{max} \\
 h_1 \geq h_2 \geq h_3 \geq h_4 > 0
 \end{array} \right. \quad (15)
 \end{aligned}$$

### 6.4 LSTM neural network model

#### 6.4.1 Predict $\alpha$ and $\beta$ by LSTM

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##### Algorithm 2 LSTM Get Alpha

**Input:**  $\alpha_t, E_t, P_t \triangleright$  which are other inflows at past times, evaporation at past times, precipitation at past times

Randomly initialize model parameters  $\theta_{alpha}$

**for** each training epoch  $\in [1, epochs]$  **do**

**for** each training sample  $(\alpha_t, E_t, P_t) \in D$  **do**  
 $i_t = \sigma(W_{ii} \cdot \alpha_t + W_{ie} \cdot E_t + W_{ip} \cdot P_t + b_{ii})$   
 $f_t = \sigma(W_{fi} \cdot \alpha_t + W_{fe} \cdot E_t + W_{fp} \cdot P_t + b_{fi})$   
 $\tilde{C}_t = \tanh(W_{ci} \cdot \alpha_t + W_{ce} \cdot E_t + W_{cp} \cdot P_t + b_{ci})$   
 $C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t$   
 $o_t = \sigma(W_{oi} \cdot \alpha_t + W_{oe} \cdot E_t + W_{op} \cdot P_t + b_{oi})$   
 $t+1 = o_t \cdot \tanh(C_t)$   
Compute loss function  $L: L = \frac{1}{2}(t+1 - \text{ground\_truth})^2$   
Compute gradient  $\nabla_{\theta} L$ , update parameters  $\theta_{alpha}: \theta_{alpha} = \theta_{alpha} - \eta \nabla_{\theta} L$

$\hat{\alpha}_{t+1} = \text{model}(\alpha_t, E_t, P_t, \theta_{alpha})$

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Since the weather is uncertain but has been similar at the same time of year in the past, We use the LSTM to predict alpha. We also use the LSTM to predict beta. The reason for being able to

predict beta instead of adjusting it based on the dam's passage rate is that the flow of the river is an intrinsic property of its nature, and is not something that can be altered by the opening or closing of a dam.

$$\alpha_{t+1} = \text{LSTM}(\alpha_t, E_t, P_t; \theta_{alpha}) \quad (16)$$

$$\beta_{t+1} = \text{LSTM}(\beta_t; \theta_{beta}) \quad (17)$$

The pseudocode for establishing an LSTM model to solve for  $\alpha$  is above.

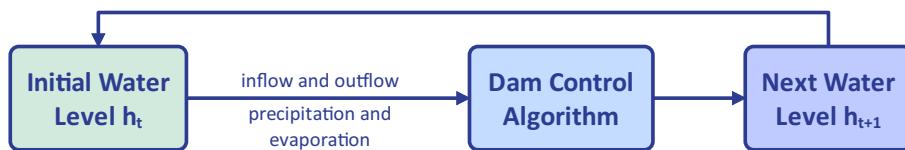
#### 6.4.2 Solve the constraint problem

Based on the predicted  $\alpha$  and  $\beta$  obtained from our predictions, we substitute them into the new system of constraint equations(15) for solving.

## 7 Sensitivity Analysis

### 7.1 Simulation method

Our approach is based on the following flowchart:



**Figure 9:** The general structure of the simulation

### 7.2 Sensitivity for the outflow of 2 control dams

We plan to show the sensitivity of the outflow by comparing it with the original data for 2017.

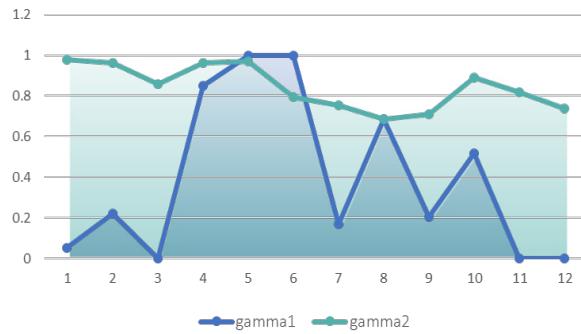
- **Comparison with the data for 2017**

Based on the designed algorithm and model, we predict the monthly water levels for the year 2017 using data from December 2016 and relevant parameters regarding dam control. The figures below illustrate the openness of the dam for each month and compare the expected, actual, and predicted water levels for 2017.

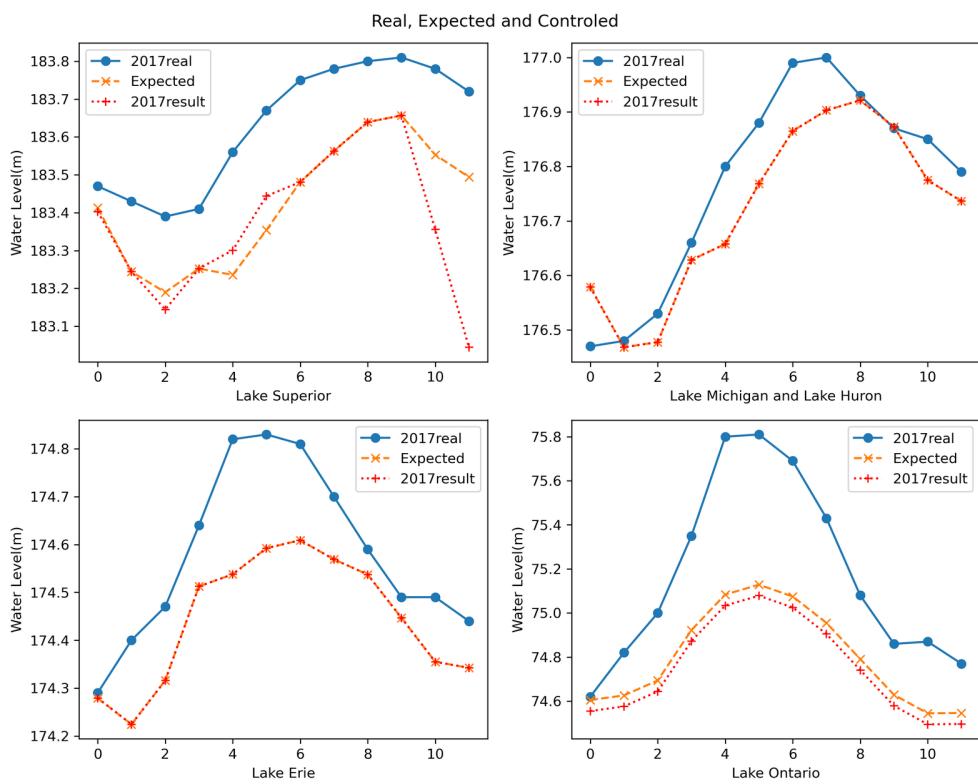
From **Figure 10**, we can find that the proportions of water discharge from the sluice gate are obviously different and Compensating Works's openness is generally lower than Moses-Saunders Dam's. In addition, compensating works have more seasonal changes while Moses-Saunders Dam's distributes more evenly throughout the year 2017.

The four charts in **Figure 11** respectively represent four lakes(actually 5, but 2 of them are combined) results of the comparison. For Lake Michigan Huron and Lake Erie, our predicted water levels closely align with the expected values throughout the entire year. For Lake Ontario, our predicted results are slightly lower than the expected values for the whole year. For Lake Superior,

our predicted results are also close to the expected ones except there is a significant decrease at the end of the year. Overall, all results predicted are generally lower than the original water levels in 2017.



**Figure 10:**  $\gamma$  results for 2017



**Figure 11:** Comparison results of water levels in 2017

### 7.3 Sensitivity for changes in environmental conditions

- **Precipitation**

If precipitation is too scarce, it will lead to drought. However, if precipitation is too abundant,

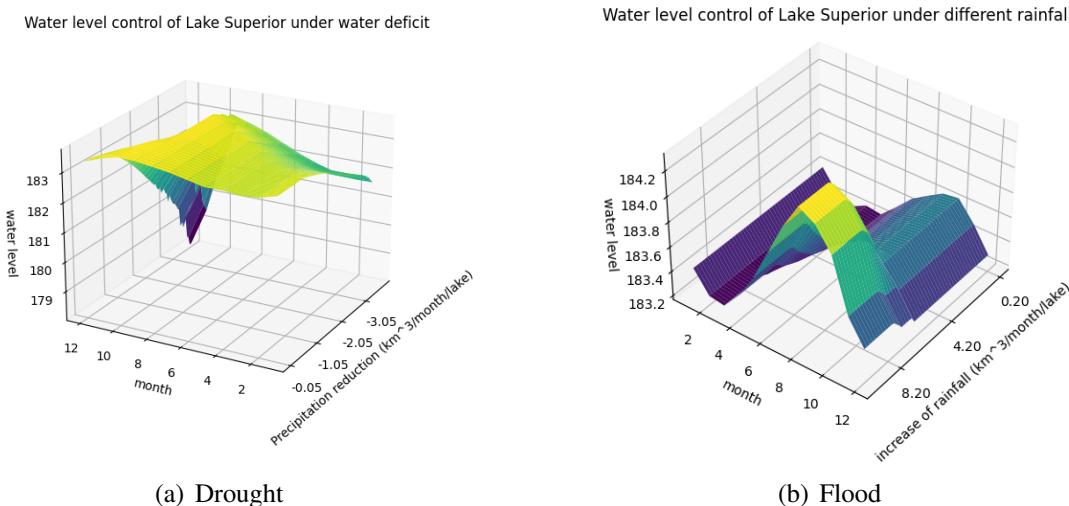
it will result in the occurrence of floods. As the precipitation increases, the  $\alpha$  will gradually become larger.

- Drought(Lack of Water)

From **Figure12(a)**, conclusions can be drawn that as the precipitation decreases to a certain extent, the model cannot maintain water levels well, resulting in a significant decrease in water levels. The situation is most severe from October to January of the following year.

- Flood (Overflow of Water)

From **Figure12(b)**, it can be observed that when precipitation reaches a certain level, the model may not provide effective strategies for controlling water levels, leading to a significant increase in water levels from March to August.



**Figure 12:** Sensitivity for Precipitation

- **Snow pack**

If there is more snow accumulation, it implies that there is less liquid water in the lake, and consequently, the  $\alpha$  will be smaller. When there is no snowpack, all the obtained water will become part of the lake, which is the general scenario.

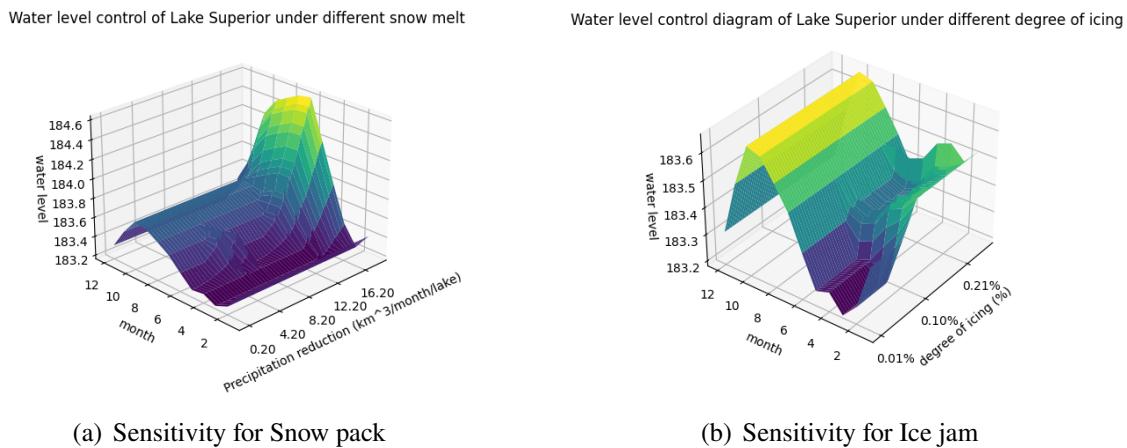
As we can see from **Figure13(a)**, with the gradual increase in snow accumulation beyond a certain critical point, it will exceed the range that the model can effectively control. The water level experiences a sharp rise in February to May due to snow melt.

- **Ice jams**

The more ice jams that occur, the higher the freezing rate of water will be and the smaller the  $\beta$  will be. If there are no ice jams, it also falls under the general scenario.

From **Figure13(b)**, it can be observed that when the freezing extent is low, the water level follows the characteristics of seasonal variation. However, as the freezing rate increases to

a certain extent, the model's ability to maintain seasonal variations in water level will be weakened. Ice formation occurs from January to March, leading to a decrease in  $\beta$ .



**Figure 13:** Sensitivity for snow pack and ice jam

## 8 Strengths and weaknesses

### 8.1 Strengths

- The two main models used in this article are universal, and our model can be applied to other water bodies.
- Detailed and complete needs of stakeholders and situations like environmental changes are considered based on the extensive literature reviewed to ensure the rigor of the model.
- A large amount of visualization has been employed, making the overall structure and results more clear and intuitive.
- While pursuing the maximization of interests for all parties, we also balanced the interests among different stakeholders to maintain a relative equilibrium in the benefits they can gain.
- Using the particle swarm optimization algorithm to improve the efficiency of obtaining extremum.

### 8.2 Weaknesses

- The model has limited capability in regulating water levels. This is primarily evident during sensitivity analysis, whereas the water level rises to a certain extent, the effectiveness of regulation is constrained due to the impact of the upper limit imposed by the dam itself. Consequently, the water level cannot be further effectively adjusted.

- Our models assume that the river is not influenced by evaporation and precipitation, with equal inflow and outflow. However, in reality, this may not hold true. Due to difficulties in data aggregation and the relatively minor impact compared to lakes, we simplify this aspect.
- The water flow is assumed to be unidirectional in the model established in this paper. However, the actual situation is complex and variable, involving bidirectional and multi-directional changes. Due to the topography of the Great Lakes, we simplify the model to a certain extent.
- Due to challenges in collecting certain data, the model does not account for the impacts associated with water usage, which could potentially introduce some influences on the results.

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- [12] Great Lakes Ice Concentration from NOAA <https://coastwatch.glerl.noaa.gov/statistics/great-lakes-ice-concentration/>
- [13] Average Surface Water Temperature (GLSEA) from NOAA <https://coastwatch.glerl.noaa.gov/statistics/average-surface-water-temperature-glsea/>

# Memo

To: the International Joint Commission

From: Team #2428775

Date: February 6, 2024

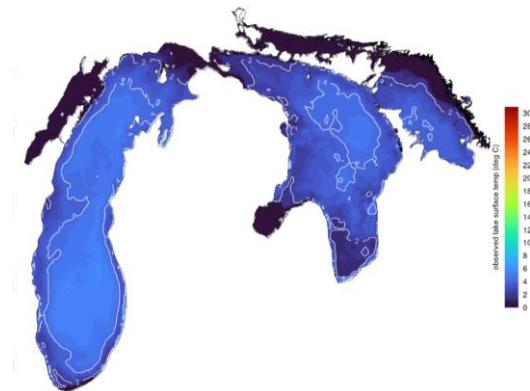
Subject:

Harmonizing Waterways, Thriving Humanity :Sustainable Development of Dam Benefits

The Great Lakes, with many stakeholders closely tied to it, is a critical lifeline in the United States. The seasonal fluctuations in water levels here are influenced by **multiple natural factors**, and dams play a crucial role in controlling the outflow. Our strategy evaluates the interests of **various stakeholders** and develops **dam regulation** strategies suitable for different environmental conditions.

## The data we take into account:

- Historical water levels of the Great Lakes
- Evaporation and rainfall
- Icy rate
- Snow melt
- Animal migration
- Hydropower plant facility data
- Shipping industry data

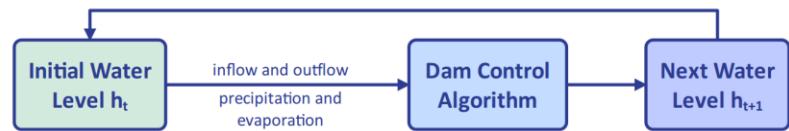


## Our win-win strategy:

MIN: the difference in standardization interests between any two parties

MAX: the sum of the total profits of the parties

- ✓ Ecological habitat
- ✓ Hydraulic power plant
- ✓ Shipping company
- ✓ Fishery industry
- ✓ Resident



## Advantage:

### a. Adapting to Ever-Changing Conditions:

This model is designed to adapt to continuously changing environmental conditions by seamlessly integrating real-time data. This integration ensures the model's relevance and accuracy within dynamic ecosystems, allowing it to evolve in tandem with the fluctuating parameters of the environment.

### b. Robust Sensitivity:

The model excels in sensitivity analysis, effectively managing water levels in response to varying factors. Notably, it adeptly navigates scenarios with increased 2017 precipitation, showcasing its adaptability and strength in optimizing outcomes amidst changing environmental dynamics.

### c. Fast convergence

Particle swarm optimization algorithm is used to find the global optimal solution, which converges quickly and consumes less computing resources.