《计算机辅助几何设计》作业

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1. Represent a unit sphere using a biquadratic rational B´ezier surface and draw it.

2. Represent the ellipsoid $3x^2+2y^2+z^2=1$ using a bicubic rational B´ezier surface and draw it.

The control points of one-eighth ellipsoid is:

$$p_{11}=(0,b,0),p_{12}=(a,b,0),p_{13}=(a,0,0)$$
$$p_{21}=(0,b,c),p_{22}=(a,b,c),p_{23}=(a,0,c)$$
$$p_{31}=(0,0,c),p_{32}=(0,0,c),p_{33}=(0,0,c)$$

weights of control points is:

$$W=\begin{pmatrix}1 & \frac{\sqrt{2}}{2} & 1\\ 1 & \frac{\sqrt{2}}{2} & 1\\ 1 & \frac{\sqrt{2}}{2} & 1\end{pmatrix}$$

$$x(u,v)=\frac{\begin{pmatrix}B_1^2(u) & B_2^2(u) & B_3^2(u)\end{pmatrix}\begin{pmatrix}W_{11}p_{11} & W_{12}p_{12} & W_{13}p_{13}\\ W_{21}p_{21} & W_{22}p_{22} & W_{23}p_{23}\\ W_{31}p_{31} & W_{32}p_{32} & W_{33}p_{33}\end{pmatrix}\begin{pmatrix}B_1^2(v)\\ B_2^2(v)\\ B_3^2(v)\end{pmatrix}}{\begin{pmatrix}B_1^2(u) & B_2^2(u) & B_3^2(u)\end{pmatrix}W\begin{pmatrix}B_1^2(v)\\ B_2^2(v)\\ B_3^2(v)\end{pmatrix}}$$

use the dual weights:

$$W = \begin{pmatrix} 1 & -\frac{\sqrt{2}}{2} & 1 \\ 1 & -\frac{\sqrt{2}}{2} & 1 \\ 1 & -\frac{\sqrt{2}}{2} & 1 \end{pmatrix}$$

You can get the three-eighth ellsoid,then make half an ellipsoid center symmetrical.

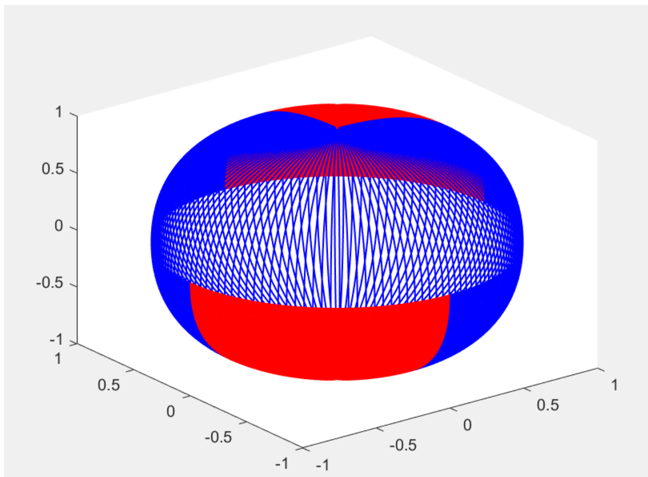

图 1: $x^2 + y^2 + z^2 = 1$

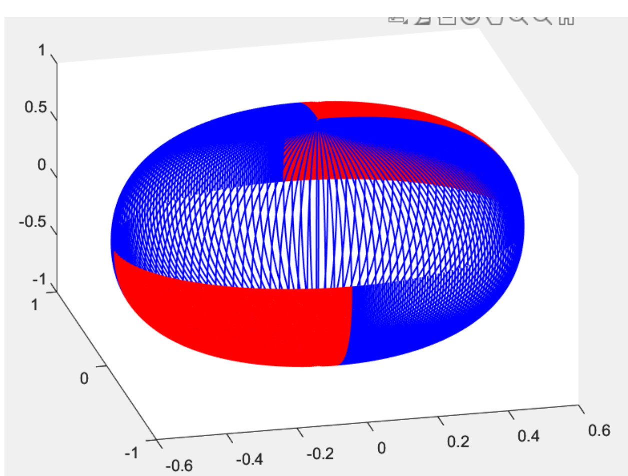

图 2: $3x^2 + 2y^2 + z^2 = 1$

3. A quadratic B´ezier triangle has vertex parameter coordinates a = (0, 0), b = (1, 0), c = (0.5, 1) and the following control points:

$$F(a,a) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, F(a,b) = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, F(a,c) = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$$

$$F(b,b) = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}, F(a,b) = \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix}, F(c,c) = \begin{pmatrix} 6 \\ -4 \\ 4 \end{pmatrix}$$

Among the three parameters $p_1 = (0.25, 0.5), p_2 = (0.3, 0.75), p_3 = (0.5, 0.5)$, which parameter is outside the triangle? For the parameters inside the triangle, calculate the coordinates of the surface F(p, p) at these parameters using the de Casteljau algorithm. 解:

p_2 is out the triangle.

$$\begin{aligned} F(p_1, p_1) =& F(\frac{a+c}{2}, \frac{a+c}{2}) \\ =& \frac{F(a,a)}{4} + \frac{F(a,c)}{2} + \frac{F(c,c)}{4} \\ =& \begin{pmatrix} \frac{7}{2} \\ -2 \\ 4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} F(p_3, p_3) =& F(\frac{a+b+2c}{4}, \frac{a+b+2c}{4}) \\ =& \frac{F(a,a)}{16} + \frac{F(a,b)}{8} + \frac{F(b,b)}{16} + \frac{F(a,c)}{4} + \frac{F(c,c)}{4} + \frac{F(b,c)}{4} \\ =& \begin{pmatrix} 5 \\ -1 \\ 4 \end{pmatrix} \end{aligned}$$