数值代数第四次实验报告

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4.1 问题描述

1. 考虑两点边值问题

$$\begin{cases} \varepsilon \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = a, 0 < a < 1, \\ y(0) = 0, y(1) = 1. \end{cases}$$

容易知道它的精确解为

$$y = \frac{1-a}{1-e^{-\frac{1}{2}}}(1-e^{-\frac{x}{2}}) + ax$$

为了把微分方程离散化,把 [0,1] 区间 n 等分,令 $h=1/n,x_i=ih,i=1,\cdots,n-1$,得到差分方程

$$\varepsilon\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}+\frac{y_{i+1}-y_i}{h}=a,$$

简化为

$$(\varepsilon + h)y_{i+1} - (2\varepsilon + h)y_i + \varepsilon y_{i-1} = ah^2$$

离散化后得到线性方程组 Ay = b, 其中

$$A = \begin{pmatrix} -(2\varepsilon + h) & \varepsilon + h \\ \varepsilon & -(2\varepsilon + h) & \varepsilon + h \\ & \varepsilon & -(2\varepsilon + h) & \ddots \\ & \ddots & \ddots & \varepsilon + h \\ & \varepsilon & -(2\varepsilon + h) \end{pmatrix}$$

注意将线性方程组与上述差分方程进行对比得出正确的 b 向量(尤其注意第一行和最后一行)。

对 $\varepsilon=1, a=1/2, n=100$, 分别用 Jacobi 迭代法,G-S 迭代法和 SOR 迭代法求线性方程组的解,要求 4 位有效数字,然后比较迭代次数,运行时间与精确解的误差。迭代法终止条件为 $||x_{k+1}-x_k||<10^{-6}$.

对 $\varepsilon = 0.1, 0.01, 0.0001$, 考虑同样的问题。

4.2 程序运行结果

```
e = 1 a = 0.5 n = 100时三种方法的结果如下:
Jacobi法结果:
解为:
          0.0256
                                0.0508
0.0128
                     0.0382
                                                    0.0758
                                          0.0633
                                                               0.0881
                                                                          0.1004
                                                                                    0.1126
                                                                                              0.1248
          0. 1489
0. 2653
0. 3755
                                                               0. 2079
0. 3211
0. 4284
                                                                                    0. 2310
0. 3430
                                                     0.1962
                                                                          0.2195
0.3321
                                          0.1845
                                                                                               0.2425
                     0.1608
                                0.1727
 0.1369
 0. 2539
0. 3647
                     0. 2766
0. 3862
                                0. 2878
0. 3968
                                          0.2990
                                                     0.3101
                                                                                              0.3539
                                                                          0.4388
                                          0.4074
                                                                                    0.4492
                                                     0.4179
                                                                                              0.4595
                                                    0.5204
0.6180
                                0.5003
                                          0.5104
           0.4800
                     0.4902
                                                               0.5304
                                                                          0.5403
                                                                                    0.5502
 0.4698
                                                                                              0.5600
          0.5795
0.6744
                                                               0.6275
0.7203
                     0.5892
                                                                          0.6370
                                                                                               0.6558
 0.5698
                                0.5988
                                          0.6084
                                                                                    0.6464
                     0.6837
                                0.6929
                                          0.7020
                                                                          0.7293
 0.6651
                                                     0.7112
                                                                                    0.7383
                                                                                              0.7473
                                                                                    0.8263
 0.7562
           0.7651
                     0.7740
                                0.7828
                                          0.7916
                                                     0.8003
                                                               0.8090
                                                                          0.8177
                                                                                              0.8349
                                          0.8774
 0.8435
           0.8520
                     0.8605
                                0.8690
                                                     0.8858
                                                               0.8942
                                                                          0.9025
                                                                                    0.9108
                                                                                               0.9191
0.9273 0.9355 0.9437 0.95
迭代次数为: 13172
运行时间为: 46350ms
与精确解的误差为: 0.00124715
                                0.9518
                                          0.9599
                                                               0.9760
                                                                          0.9841
                                                     0.9680
                                                                                    0.9920
G-S迭代法结果:
解为:
 0.0128
                                                    0.0757
0.1962
          0.0256
                     0.0382
                               0.0508
                                          0.0633
                                                                          0.1004
                                                               0.0881
                                                                                    0.1126
                                                                                              0.1248
                                0. 1727
0. 2878
                                                                                    0. 2310
0. 3430
                     0.1608
                                                                          0.2195
                                                                                              0.2425
 0.1368
          0.1488
                                          0.1845
                                                               0.2079
                                                               0.3211
                                                                          0.3321
 0.2539
           0.2652
                     0.2765
                                          0.2989
                                                     0.3100
                                                                                              0.3539
                                                    0.4179
0.5204
0.6180
                     0.3861
 0.3647
           0.3754
                                0.3968
                                          0.4074
                                                               0.4284
                                                                          0.4388
                                                                                    0.4492
                                                                                              0.4595
                                                               0.5303
           0.4800
                     0.4902
                                0.5003
                                          0.5103
                                                                          0.5403
                                                                                    0.5501
 0.4698
                                                                                              0.5600
                     0. 5892
0. 6836
0. 7740
                                          0.6084
0.7020
0.7916
0.8774
                                                               0.6275
0.7202
0.8090
                                                                         0. 6369
0. 7293
0. 8177
           0.5795
0.6744
                                0.5988
0.6928
 0.5697
                                                                                    0.6464
                                                                                              0.6557
                                                     0.7111
0.8003
                                                                                              0.7473
0.8349
                                                                                    0.7383
 0.6651
                                0.7828
                                                                                    0.8263
 0.7562
           0.7651
           0.8520
                     0.8605
                                0.8690
                                                     0.8858
                                                               0.8942
                                                                          0.9025
                                                                                    0.9108
 0.8435
                                                                                               0.9191
0.9273 0.9355 0.9437 0.99
迭代次数为:6574
运行时间为:33245ms
与精确解的误差为:0.00128824
                                0.9518
                                          0.9599
                                                     0.9680
                                                               0.9760
                                                                          0.9841
                                                                                    0.9920
SOR迭代法结果:
解为:
 0.0129
                                         0.0635
          0.0256
                     0.0383
                               0.0510
                                                    0.0760
                                                              0.0884
                                                                         0.1007
                                                                                   0.1130
                                                                                             0.1251
                                                                                   0. 2317
0. 3438
                               0. 1732
0. 2885
0. 3977
0. 5013
                                                                                             0. 2432
0. 3547
                                                              0.2085
0.3219
                                                                         0.2201
0.3329
 0.1372
0.2546
          0.1493
0.2660
                     0.1613
                                          0.1850
0.2997
                                                    0.1968
                     0. 2773
0. 3870
                                                    0.3108
 0.3656
           0.3763
                                          0.4083
                                                              0.4293
                                                    0.4188
                                                                         0.4398
                                                                                   0.4501
                                                                                             0.4605
                                                              0.5313
0.6284
                     0.4911
                                                                                   0.5511
 0.4708
           0.4810
                                          0.5113
                                                    0.5214
                                                                         0.5412
                                                                                              0.5609
           0.5804
                     0.5901
                                0.5998
                                          0.6093
                                                                         0.6378
                                                                                             0.6566
 0.5707
                                                    0.6189
                                                                                   0.6473
                                          0. 7028
0. 7922
0. 8778
          0.6752
0.7658
                               0.6937
                                                                                             0.7480
 0.6659
                     0.6845
                                                    0.7119
                                                              0.7210
                                                                         0.7300
                                                                                   0.7390
                     0.7746
0.8609
                                                              0.8096
 0.7569
                                0.7834
                                                    0.8009
                                                                         0.8182
                                                                                   0.8268
                                                                                              0.8354
           0.8525
                                                    0.8862
                                                                                   0.9111
                                                              0.8945
                                                                         0.9028
 0.8440
                                0.8694
                                                                                              0.9193
0.9275 0.9357
松弛因子为: 1.94
迭代次数为: 246
                                0.9520
                                          0.9600
                     0.9438
                                                    0.9681
                                                              0.9761
                                                                         0.9841
                                                                                   0.9921
 5行时间为: 1473ms
5精确解的误差为: 0.000298292
```

```
e = 0.1 a = 0.5 n = 100时三种方法的结果如下:
Jacobi法结果:
解为:
 0.0504
                   0.1393 0.1784 0.2144
                                                0.2476
         0.0967
                                                          0.2782
                                                                    0.3066
                                                                              0.3327
                                                                                        0.3570
                                                                              0.5130
                                                                                       0.5254
          0.4005
                   0. 4199
0. 5589
                             0.4381
                                       0.4551
                                                 0. 4709
0. 5878
                                                           0.4858
 0.3795
                                                                    0.4998
          0.5483
 0.5372
                             0.5690
                                       0.5786
                                                           0.5966
                                                                    0.6051
                                                                              0.6133
                                                                                       0.6211
                             0.6502
0.7123
0.7670
                                                          0.6701
0.7292
0.7828
                                                 0.6637
0.7237
0.7775
0.8291
                                       0.6570
0.7180
                                                                    0.6765
0.7348
 0.6288
          0.6361
                    0.6433
                                                                              0.6827
                                                                                        0.6888
 0.6948
          0.7007
                                                                              0.7402
                    0.7066
                                                                                       0.7457
                                       0.7723
                                                                                        0.7983
 0.7511
          0.7564
                    0.7617
                                                                    0.7880
                                                                              0.7932
 0.8035
          0.8086
                    0.8137
                              0.8189
                                       0.8240
                                                           0.8341
                                                                    0.8392
                                                                              0.8443
                                                                                        0.8494
          0.8595
                                                                                        0.8998
 0.8544
                                       0.8746
                                                 0.8797
                                                           0.8847
                                                                    0.8897
                                                                              0.8948
                    0.8645
                             0.8696
                                       0.9249
0.9750
                                                           0.9349
                                                 0.9299
                                                                    0.9399
 0.9048
          0.9098
                    0.9148
                             0.9199
                                                                              0.9449
                                                                                        0.9499
                                                 0.9800
   9549
          0.9600
                    0.9650
                             0.9700
                                                           0.9850
                                                                    0.9900
                                                                              0.9950
5.9549 0.9600 0.9650 0.9
迭代次数为: 5926
运行时间为: 29379ms
与精确解的误差为: 0.00906271
 -S迭代法结果:
解为:
0.0504
                                                          0.2782
0.4858
                                                                              0.3327
0.5130
          0.0967
                    0.1392
                             0.1784
                                       0.2144
                                                 0.2476
                                                                    0.3065
                             0.4381
                                       0.4550
                                                 0. 4709
0. 5878
                                                                                       0.5254
 0.3795
                   0.4199
                                                                    0.4998
          0.4004
 0.5372
          0.5483
                    0.5589
                                       0.5786
                                                           0.5966
                                                                                       0.6211
                             0.5690
                                                                    0.6051
                                                                              0.6133
 0.6287
                             0.6502
                                                           0.6701
0.7292
          0.6361
                    0.6433
                                       0.6570
                                                 0.6636
                                                                    0.6765
                                                                              0.6827
                                                                                        0.6888
                             0.7123
                                       0.7180
                                                 0.7237
 0.6948
          0.7007
                    0.7066
                                                                    0.7348
                                                                              0.7402
                                                                                        0.7457
                                       0. 7723
0. 8240
0. 8746
                                                 0.7775
0.8291
                   0.7617
0.8137
                             0.7670
                                                           0.7828
                                                                    0.7880
0.8392
 0.7511
          0.7564
                                                                              0.7932
                                                                                        0.7983
                                                           0.8341
                                                                              0.8443
 0.8035
          0.8086
                              0.8189
                                                                                        0.8494
 0.8544
          0.8595
                                                 0.8797
0.9299
                                                           0.8847
                                                                    0.8897
                                                                                       0.8998
                    0.8645
                                                                              0.8948
                             0.8696
                             0.9199
                                       0.9249
                                                           0.9349
 0.9048
         0.9098
                    0.9148
                                                                    0.9399
                                                                              0.9449
                                                                                        0.9499
        0.9600
   9549
                    0.9650
                             0.9700
                                       0.9750
                                                 0.9800
                                                           0.9850
                                                                    0.9900
                                                                              0.9950
0.9349 0.9000 0.9030 0.9
迭代次数为: 2981
运行时间为: 12774ms
与精确解的误差为: 0.00908668
SOR迭代法结果:
解为:
0.0505
                              0.1785
0.4384
                                                 0.2478
0.4712
                    0.1394
                                        0.2146
                                                           0.2784
                                                                     0.3068
                                                                               0.3330
          0.0968
                                                                                         0.3573
                    0.4202
                                        0.4553
                                                           0.4861
                                                                     0.5001
                                                                               0.5133
                                                                                         0.5257
 0.3798
          0.4007
                              0.5693
 0.5375
          0.5486
                    0.5592
                                        0.5789
                                                 0.5881
                                                           0.5969
                                                                     0.6054
                                                                               0.6135
                                                                                         0.6214
                                       0.6572
0.7182
                                                           0.6703
 0.6290
          0.6364
                    0.6435
                              0.6505
                                                 0.6639
                                                                     0.6767
                                                                               0.6829
                                                                                         0.6890
                                                                     0. 7349
0. 7880
                                                 0.7238
 0.6950
          0.7009
                                                           0.7294
                                                                               0.7404
                              0.7125
                                                                                         0.7458
                    0.7067
          0.7565
                    0.7618
                              0.7671
                                                           0.7829
                                                                                         0.7984
 0.7512
                                        0.7724
                                                 0.7776
                                                                               0.7932
                                       0.8240
0.8746
0.9249
                    0.8138
                                                                     0.8393
                                                                               0.8443
 0.8035
          0.8087
                              0.8189
                                                 0.8291
                                                           0.8342
                                                                                         0.8494
                                                 0.8797
0.9299
                                                           0.8847
                                                                     0.8897
 0.8545
          0.8595
                    0.8646
                              0.8696
                                                                               0.8948
                                                                                         0.8998
                                                                     0.9399
          0.9098
                    0.9149
                              0.9199
                                                           0.9349
                                                                               0.9449
 0.9048
                                                                                         0.9499
   9550
                    0.9650
                              0.9700
                                       0.9750
                                                 0.9800
                                                                     0.9900
                                                                               0.9950
          0.9600
                                                           0.9850
公动因子为: 1.895
生代次数为: 201
至行时间为: 2542ms
与精确解的误差为: 0.00882342
```

```
e = 0.01 a = 0.5 n = 100时三种方法的结果如下:
[acobi法结果:
解为:
0.2550
                                                                           0.5440
         0.3850
                   0.4525
                            0.4887
                                      0.5094
                                              0.5222
                                                         0.5311
                                                                  0.5380
                                                                                     0.5495
                                               0.5800
 0.5548
          0.5599
                   0.5649
                            0.5700
                                      0.5750
                                                         0.5850
                                                                  0.5900
                                                                           0.5950
                                                                                     0.6000
                                      0.6250
0.6750
 0.6050
          0.6100
                   0.6150
                             0.6200
                                               0.6300
                                                         0.6350
                                                                  0.6400
                                                                           0.6450
                                                                                     0.6500
 0.6550
                   0.6650
                             0.6700
                                                         0.6850
                                                                  0.6900
                                                                           0.6950
          0.6600
                                               0.6800
                                                                                     0.7000
                                      0.7250
0.7750
          0.7100
                   0.7150
                             0.7200
                                                         0.7350
                                                                  0.7400
 0.7050
                                               0.7300
                                                                           0.7450
                                                                                     0.7500
                             0.7700
                                                                           0.7950
 0.7550
          0.7600
                   0.7650
                                               0.7800
                                                         0.7850
                                                                  0.7900
                                                                                     0.8000
                                      0.8250
 0.8050
                   0.8150
                                                         0.8350
                                                                  0.8400
                                                                           0.8450
         0.8100
                             0.8200
                                               0.8300
                                                                                     0.8500
                   0.8650
                                                         0.8850
                                                                  0.8900
                                                                           0.8950
 0.8550
         0.8600
                                      0.8750
                            0.8700
                                               0.8800
                                                                                     0.9000
                                     0.9250
0.9750
                   0.9150
 0.9050
         0.9100
                                                        0.9350
                            0.9200
                                               0.9300
                                                                 0.9400
                                                                           0.9450
                                                                                     0.9500
0.9550 0.9600 0.9650 0.9
迭代次数为:569
运行时间为:3473ms
与精确解的误差为:0.0660671
                            0.9700
                                               0.9800
                                                        0.9850
                                                                 0.9900
                                                                           0.9950
G-S迭代法结果:
解为:
0.2550
0.5548
         0.3850
                   0.4525
                            0.4887
                                      0.5094
                                               0.5222
                                                         0.5311
                                                                  0.5380
                                                                           0.5440
                                                                                     0.5495
                                      0.5750
                                                                  0.5900
          0.5599
                             0.5700
                   0.5649
                                               0.5800
                                                         0.5850
                                                                           0.5950
                                                                                     0.6000
                             0.6200
                                      0.6250
                                                         0.6350
 0.6050
          0.6100
                   0.6150
                                               0.6300
                                                                  0.6400
                                                                           0.6450
                                                                                     0.6500
                                                        0.6850
0.7350
 0.6550
          0.6600
                   0.6650
                             0.6700
                                      0.6750
                                               0.6800
                                                                  0.6900
                                                                           0.6950
                                                                                     0.7000
 0.7050
          0.7100
                   0.7150
                             0.7200
                                      0.7250
                                               0.7300
                                                                  0.7400
                                                                           0.7450
                                                                                     0.7500
                                      0.7750
                                                                           0.7950
                                                                                     0.8000
 0.7550
                   0.7650
                             0.7700
                                               0.7800
                                                         0.7850
          0.7600
                                                                  0.7900
                                      0.8250
0.8750
                                                                           0.8450
 0.8050
          0.8100
                   0.8150
                             0.8200
                                               0.8300
                                                         0.8350
                                                                  0.8400
                                                                                     0.8500
                                               0.8800
                                                        0.8850
0.9350
 0.8550
          0.8600
                   0.8650
                             0.8700
                                                                  0.8900
                                                                           0.8950
                                                                                     0.9000
   9050
                                     0.9250
0.9750
         0.9100
                   0.9150
                             0.9200
                                               0.9300
                                                                  0.9400
                                                                           0.9450
                                                                                     0.9500
0.9550 0.9600 0.9650 0.9
迭代次数为:333
运行时间为:2298ms
与精确解的误差为:0.0660678
                            0.9700
                                               0.9800
                                                         0.9850
                                                                  0.9900
                                                                           0.9950
SOR迭代法结果:
解为:
0.2550 0.385
0.5548 0.559
                                               0.5222
0.5800
                                                                 0.5380
0.5900
                   0.4525
                                      0.5094
          0.3850
                            0.4887
                                                        0.5311
                                                                           0.5440
                                                                                    0.5495
          0.5599
                   0.5649
                            0.5700
                                      0.5750
                                                        0.5850
                                                                           0.5950
                                                                                    0.6000
          0.6100
                   0.6150
                            0.6200
                                      0.6250
                                               0.6300
                                                        0.6350
 0.6050
                                                                 0.6400
                                                                           0.6450
                                                                                    0.6500
                                     0. 6750
0. 7250
                            0.6700
 0.6550
          0.6600
                   0.6650
                                               0.6800
                                                        0.6850
                                                                 0.6900
                                                                           0.6950
                                                                                    0.7000
                   0.7150
                            0.7200
                                                        0.7350
                                                                           0.7450
                                                                                    0.7500
 0.7050
          0.7100
                                               0.7300
                                                                 0.7400
                   0.7650
          0.7600
 0.7550
                            0.7700
                                      0.7750
                                               0.7800
                                                        0.7850
                                                                 0.7900
                                                                           0.7950
                                                                                    0.8000
                                     0. 8250
0. 8750
0. 9250
0. 9750
          0.8100
                            0.8200
0.8700
                                               0.8300
0.8800
 0.8050
                   0.8150
                                                        0.8350
                                                                 0.8400
                                                                           0.8450
                                                                                    0.8500
          0.8600
                   0.8650
 0.8550
                                                        0.8850
                                                                 0.8900
                                                                           0.8950
                                                                                    0.9000
                                                                 0.9400
                   0.9150
                            0.9200
                                                        0.9350
 0.9050
         0.9100
                                               0.9300
                                                                           0.9450
                                                                                    0.9500
                            0.9700
   9550 0.9600
                   0.9650
                                               0.9800
                                                        0.9850
                                                                 0.9900
                                                                          0.9950
松弛因子为: 1.5
迭代次数为: 101
运行时间为: 977ms
与精确解的误差为: 0.0660603
```

```
e = 0.0001 a = 0.5 n = 100时三种方法的结果如下:
Jacobi法结果.
解为:
 0.5000
                    0.5150 0.5200 0.5250 0.5650 0.5700 0.5750
                                                   0.5300
                                                                      0.5400
0.5900
                                                                                           0.5500
          0.5100
                                                             0.5350
                                                                                 0.5450
          0.5600
 0.5550
                                                   0.5800
                                                             0.5850
                                                                                 0.5950
                                                                                           0.6000
                                         0.6250
0.6750
                              0.6200
0.6700
                                                             0.6350
                    0.6150
 0.6050
          0.6100
                                                   0.6300
                                                                       0.6400
                                                                                 0.6450
                                                                                           0.6500
                                                                                           0.7000
 0.6550
                    0.6650
                                                             0.6850
                                                                                 0.6950
          0.6600
                                                   0.6800
                                                                       0.6900
                              0.7200
0.7700
0.8200
                                         0. 7250
0. 7750
0. 8250
                    0.7150
                                                             0.7350
0.7850
                                                                       0.7400
                                                                                           0.7500
 0.7050
          0.7100
                                                   0.7300
                                                                                 0.7450
 0.7550
          0.7600
                    0.7650
                                                                                 0.7950
                                                   0.7800
                                                                       0.7900
                                                                                           0.8000
                    0.8150
                                                             0.8350
 0.8050
          0.8100
                                                   0.8300
                                                                                 0.8450
                                                                       0.8400
                                                                                           0.8500
 0.8550
                               0.8700
                                         0.8750
          0.8600
                    0.8650
                                                   0.8800
                                                             0.8850
                                                                       0.8900
                                                                                 0.8950
                                                                                           0.9000
   9050
          0.9100
                    0.9150
                              0.9200
                                         0.9250
                                                             0.9350
                                                   0.9300
                                                                       0.9400
                                                                                 0.9450
                                                                                           0.9500
0.9550 0.9600 0.9650 0.97
迭代次数为: 118
运行时间为: 656ms
与精确解的误差为: 0.00495075
                              0.9700
                                        0.9750
                                                   0.9800
                                                             0.9850
                                                                      0.9900
                                                                                 0.9950
G-S迭代法结果:
  为:
0.5000
                                                             0.5350
                                                                                0.5450
          0.5100
                    0.5150
                              0.5200
                                         0.5250
                                                   0.5300
                                                                      0.5400
                                                                                           0.5500
                                                             0.5850
0.6350
 0.5550
          0.5600
                    0.5650
                               0.5700
                                         0.5750
                                                   0.5800
                                                                       0.5900
                                                                                 0.5950
                                                                                           0.6000
                                         0.6250
 0.6050
          0.6100
                    0.6150
                               0.6200
                                                   0.6300
                                                                       0.6400
                                                                                 0.6450
                                                                                           0.6500
                                                             0.6850
                    0.6650
                               0.6700
                                                                       0.6900
 0.6550
          0.6600
                                         0.6750
                                                   0.6800
                                                                                 0.6950
                                                                                           0.7000
          0.7100
0.7600
0.8100
                    0.7150
0.7650
                                                                       0.7400
                                                             0.7350
                                                                                 0.7450
 0.7050
                               0.7200
                                         0.7250
                                                   0.7300
                                                                                           0.7500
   7550
8050
                               0.7700
0.8200
                                         0.7750
0.8250
                                                   0.7800
0.8300
                                                             0.7850
0.8350
                                                                       0.7900
0.8400
                                                                                 0.7950
0.8450
                                                                                           0.8000
                    0.8150
                                                                                           0.8500
          0.8600
0.9100
                                        0.8750
0.9250
                               0.8700
 0.8550
                    0.8650
                                                   0.8800
                                                             0.8850
                                                                       0.8900
                                                                                 0.8950
                                                                                           0.9000
                               0.9200
                                                             0.9350
 0.9050
                    0.9150
                                                   0.9300
                                                                       0.9400
                                                                                 0.9450
                                                                                           0.9500
0.9550 0.9600 0.9650 0.9
迭代次数为: 109
运行时间为: 401ms
与精确解的误差为: 0.00495075
                               0.9700
                                         0.9750
                                                             0.9850
                                                                                 0.9950
                                                   0.9800
                                                                       0.9900
SOR迭代法结果:
解为:
0.5000
                   0.5150
0.5650
                              0.5200
0.5700
                                       0.5250
0.5750
                                                  0.5300
0.5800
                                                           0.5350
0.5850
          0.5100
                                                                      0.5400
                                                                               0.5450
                                                                                         0.5500
                                                                      0.5900
 0.5550
          0.5600
                                                                               0.5950
                                                                                         0.6000
                                                           0.6350
0.6850
          0.6100
                    0.6150
                              0.6200
                                        0.6250
                                                  0.6300
                                                                      0.6400
                                                                               0.6450
 0.6050
                                                                                         0.6500
                              0.6700
                                        0.6750
                                                                               0.6950
 0.6550
          0.6600
                    0.6650
                                                  0.6800
                                                                      0.6900
                                                                                         0.7000
                              0. 7200
0. 7700
0. 8200
                                       0. 7250
0. 7750
0. 8250
                                                           0. 7350
0. 7850
                                                                               0. 7450
0. 7950
0. 8450
                                                  0.7300
0.7800
                    0.7150
0.7650
 0.7050
          0.7100
                                                                      0.7400
                                                                                         0.7500
 0.7550
          0.7600
                                                                      0.7900
                                                                                         0.8000
                    0.8150
                                                            0.8350
 0.8050
                                                  0.8300
                                                                                         0.8500
          0.8100
                                                                      0.8400
 0.8550
                    0.8650
                              0.8700
                                        0.8750
                                                           0.8850
0.9350
                                                                               0.8950
          0.8600
                                                  0.8800
                                                                      0.8900
                                                                                         0.9000
 0.9050
          0.9100
                    0.9150
                              0.9200
                                        0.9250
                                                  0.9300
                                                                      0.9400
                                                                               0.9450
                                                                                         0.9500
                                       0.9750
                                                  0.9800
                                                          0.9850
   9550
                    0.9650
                              0.9700
                                                                     0.9900
                                                                               0.9950
          0.9600
松弛因子为: 1.01
迭代次数为: 102
运行时间为: 669ms
与精确解的误差为: 0.00495082
 :\Users\郑涛\Desktop\数值代数\数值代数实验\实验四\Homework4\x64\Debug\Homework4.ε
要在调试停止时自动关闭控制台,请启用"工具"->"选项"->"调试"->"调试停止时自动
安任意键关闭此窗口...
```

4.1 结果分析

根据运行结果得到如下表格(SOR方法给出了最佳ω值):

3	迭代法	迭代次数	运行时间 (ms)	误差	
	Jacobi	13172	46350	0. 001247	
1	G-S	6574	33245	0.001288	
	SOR (1.94)	246	1473	0. 000298	
	Jacobi	5926	29379	0.009063	
0.1	G-S	2981	12774	0.009087	
	SOR(1.895	201	2542	0.008823	
	Jacobi	569	3473	0.066067	
0.01	G-S	333	2298	0.066068	
	SOR (1.5)	101	977	0.066060	
0.0001	Jacobi	118	656	0. 004951	
	G-S	109	401	0. 004951	
	SOR (1.01)	102	669	0.004951	

根据表格可以看出, ε 越小,三个方法的迭代次数越少;对于给定的 ε 值,迭代次数和运行时间排序为: Jacobi > G-S > SOR;表中也可以看出一般来说SOR的误差也比其他两个方法小。

4.2 问题描述

2. 考虑偏微分方程

$$-\Delta u + g(x,y)u = f(x,y), (x,y) \in [0,1] \times [0,1]$$

边界条件为 u=1. 沿 x 方向和 y 方向均匀剖分 N 等份,令 h=1/N,并设应用中心差分离散化后得到差分方程的代数方程组为

$$-u_{i-1,j} - u_{i,j-1} + (4 + h^2g(ih, jh))u_{i,j} - u_{i+1,j} - u_{i,j+1} = h^2f(ih, jh)$$
(1)

取 $g(x,y)=e^{xy}$, f(x,y)=x+y, 分别用Jacobi 迭代法,G-S 迭代法和 SOR 迭代法求解上述代数方程组,并列表比较 N=20,40,60 时收敛所需要的迭代次数和所用的 CPU 时间,迭代终止条件为 $\||x_{k+1}-x_k\||<10^{-7}$. 要求仿照下面写的 Jacobi 迭代格式的推导过程推导处 G-S 迭代和 SOR 迭代的格式(在报告中写出推导过程),在用 SOR 迭代法求解的过程中,请对不同的 N 使用合适的松弛因子 ω ,并在程序输出中打印松弛因子的值。可以采用<u>二分法</u>观察运行结果后选取合适的(代码中不需要体现选取过程,只需给出即可)。

注意本题中的三个迭代法的算法需要重新写,不能用之前的通用算法!!!

4.3G-S 和 SOR 推导过程

G-S:

考虑 G-S 迭代格式 $(D-L) x^{(k+1)} = Ux^{(k)} + b$,考虑第 i 行的元素用 A 中的元素表示,得到 G-S 迭代格式:

$$A_{i1}x_1^{(k+1)} + A_{i2}x_2^{(k+1)} + \dots + A_{ii}x_i^{(k+1)} + A_{i(i+1)}x_{(i+1)}^{(k)} + \dots + A_{in}x_n^{(k)} = b_i$$

推广至矩阵可知代数方程(1)的 G-S 迭代格式为:
$$-u_{i-1,j}^{(k+1)} - u_{i,j-1}^{(k+1)} + (4 + h^2g(ih,jh))u_{i,j}^{(k+1)} - u_{i+1,j}^{(k)} - u_{i,j+1}^{(k)} = h^2f(ih,jh)$$

SOR:

考虑 SOR 迭代格式 $(D-\omega L)$ $x^{(k+1)}=((1-\omega)D+\omega U)$ $x^{(k)}+\omega b$,考虑第 i 行的元素用 A 中的元素表示,得到 SOR 迭代格式:

$$\omega A_{i1} x_1^{(k+1)} + \dots + \omega A_{i(i-1)} x_i^{(k+1)} + A_{ii} x_i^{(k+1)} + (\omega - 1) A_{ii} x_i^{(k)} + \omega A_{i(i+1)} x_{i+1}^{(k)} + \dots$$
$$+ \omega A_{in} x_n^{(k)} = \omega b_i$$

推广至矩阵可知代数方程(1)的 SOR 迭代格式为:

$$-\omega u_{i-1,j}^{(k+1)} - \omega u_{i,j-1}^{(k+1)} + (4 + h^2 g(ih, jh)) u_{i,j}^{(k+1)} + (\omega - 1)(4 + h^2 g(ih, jh)) u_{i,j}^{(k)}$$
$$-\omega u_{i+1,j}^{(k)} - \omega u_{i,j+1}^{(k)} = \omega h^2 f(ih, jh)$$

4.3 程序运行结果

```
Jacobi法结果:
 解为:
0.9972
                      0.9946
0.9908
                                            0.9937
0.9889
                                                                  0.9936
0.9886
            0.9956
                                 0.9940
                                                       0.9936
                                                                             0.9938
                                                                                        0.9940
                                                                                                   0.9943
                                                                                                              0.9947
 0.9956
           0.9927
                                 0.9896
                                                       0.9886
                                                                             0.9888
                                                                                        0.9892
                                                                                                   0.9898
                                                                                                              0.9905
           0.9908
 0.9946
                      0.9882
                                 0.9865
                                            0.9855
                                                       0.9849
                                                                  0.9848
                                                                             0.9850
                                                                                        0.9855
                                                                                                   0.9862
                                                                                                              0.9871
                                            0.9830
                                                                  0.9821
 0.9940
           0.9896
0.9889
                      0.9865
                                                       0.9823
                                                                             0.9822
                                                                                        0.9827
                                                                                                   0.9835
                                                                                                              0.9845
                                 0.9844
                                 0. 9830
0. 9823
0. 9821
0. 9822
                                                                  0.9801
0.9790
0.9784
                                                                                        0.9807
0.9794
0.9786
    9937
                      0.9855
                                            0.9814
                                                       0.9805
                                                                             0.9802
                                                                                                   0.9815
                                                                                                              0.9826
                                                                                                              0.9812
 0.9936
            0.9886
                      0.9849
                                            0.9805
                                                       0.9794
                                                                             0.9790
                                                                                                   0.9802
                                                                             0.9783
                                                       0.9790
                                                                                                   0.9794
                                                                                                              0.9804
 0.9936
            0.9886
                      0.9848
                                            0.9801
                                                       0.9790
0.9794
0.9802
                                                                  0. 9783
0. 9786
                                                                             0.9781
0.9784
                                                                                        0.9784
0.9786
0.9791
                                                                                                   0.9790
0.9791
 0.9938
            0.9888
                      0.9850
                                            0.9802
                                                                                                              0.9800
                                 0. 9827
0. 9835
                                            0.9807
 0.9940
            0.9892
                      0.9855
                                                                                                              0.9800
            0.9898
                                                                                                              0.9803
                      0.9862
                                                                  0.9794
                                                                             0.9790
                                                                                                   0.9796
 0.9943
                                            0.9815
                      0.9871
0.9881
                                                       0. 9812
0. 9825
                                                                             0.9800
 0.9947
            0.9905
                                 0.9845
                                            0.9826
                                                                  0.9804
                                                                                        0.9800
                                                                                                   0.9803
                                                                                                              0.9810
 0. 9952
0. 9957
            0.9913
0.9921
                                 0.9857
                                                                             0.9813
                                                                  0.9817
                                                                                                              0.9819
                                            0.9838
                                                                                        0.9812
                                                                                                   0.9814
                      0.9893
                                            0.9853
                                                                  0.9833
                                                                                                              0.9831
                                 0.9870
                                                       0.9841
                                                                             0.9828
                                                                                        0.9826
                                                                                                   0.9828
                                                                             0.9846
                                                                                                   0.9844
            0.9931
                      0.9905
                                 0.9885
                                                                  0.9850
                                                                                        0.9843
 0.9962
                                            0.9870
                                                       0.9858
                                                                                                              0.9846
 0.9968
                                                                  0. 9870
0. 9892
           0.9941
                      0.9919
                                 0.9902
                                                                                        0.9863
                                            0.9888
                                                       0.9878
                                                                             0.9866
                                                                                                   0.9862
                                                                                                              0.9864
                                                                                                              0.9884
            0.9952
                      0.9934
 0.9974
                                 0.9919
                                            0.9908
                                                       0.9899
                                                                             0.9888
                                                                                        0.9885
                                                                                                   0.9884
                                                                  0.9917
0.9942
0.9970
                                            0. 9929
0. 9952
                                 0.9938
0.9958
                                                                             0.9912
0.9939
                                                                                                   0. 9908
0. 9935
                                                                                                              0.9908
0.9934
 0.9980
            0.9963
                      0.9950
                                                       0.9922
                                                                                        0.9910
           0.9975
                      0.9966
                                                                                           9937
   9987
                                                       0.9947
 0.
    9993
            0.9987
                      0.9983
                                 0.9979
                                            0.9975
                                                       0.9973
                                                                             0.9968
                                                                                        0.9967
                                                                                                   0.9966
                                                                                                              0.9965
迭代次数为: 1123
运行时间为: 505ms
G-S迭代法结果:
解为:
0.9972 0.995
            0.9956
                      0.9946
                                 0.9940
                                            0.9937
                                                       0.9936
                                                                  0.9936
                                                                             0.9938
                                                                                       0.9940
                                                                                                  0.9943
                                                                                                             0.9947
                                 0.9896
                                                                                       0.9892
 0.9956
                      0.9908
                                            0.9889
                                                       0.9886
                                                                  0.9886
                                                                                                   0.9898
           0.9927
                                                                             0.9888
                                                                                                             0.9905
                      0.9882
                                            0.9855
                                                       0.9849
                                                                             0.9850
                                                                                       0.9855
                                                                                                  0.9862
           0.9908
                                 0.9865
                                                                  0.9848
                                                                                                             0.9871
 0.9946
 0.9940
            0.9896
                      0.9865
                                 0.9844
                                            0.9830
                                                       0.9823
                                                                  0.9821
                                                                             0.9822
                                                                                       0.9827
                                                                                                   0.9835
                                                                                                             0.9845
                                 0. 9830
0. 9823
0. 9821
 0.9937
                      0.9855
                                                                  0.9801
                                                                                       0.9807
                                                                                                             0.9826
            0.9889
                                            0.9814
                                                       0.9805
                                                                             0.9802
                                                                                                  0.9815
                                            0.9805
0.9801
                                                                 0.9790
0.9784
0.9783
0.9786
                                                                                       0.9794
                      0.9849
           0.9886
                                                       0.9794
                                                                             0.9790
                                                                                                  0.9802
 0.9936
                                                                                                             0.9812
                                                       0.9790
0.9790
 0.9936
            0.9886
                      0.9848
                                                                             0.9783
                                                                                        0.9786
                                                                                                   0.9794
                                                                                                             0.9804
           0.9888
                                                                             0.9781
0.9784
                      0.9850
                                 0.9822
                                            0.9802
                                                                                       0.9784
                                                                                                   0.9790
 0.9938
                                                                                                             0.9800
                                 0. 9827
0. 9835
                      0.9855
                                            0.9807
                                                                                       0.9786
                                                                                                             0.9800
                                                       0.9794
                                                                                                  0.9791
 0.9940
            0.9892
                                                                  0.9794
0.9804
                                                                             0.9790
0.9800
            0.9898
                                            0.9815
                                                       0.9802
 0.9943
                      0.9862
                                                                                        0.9791
                                                                                                   0.9796
                                                                                                              0.9803
                                            0. 9826
0. 9838
0. 9853
                      0.9871
0.9881
                                 0. 9845
0. 9857
                                                       0.9812
0.9825
 0.9947
0.9952
           0.9905
                                                                                       0.9800
                                                                                                   0.9803
                                                                                                             0.9810
                                                                             0.9813
0.9828
                                                                  0. 9817
                                                                                       0.9812
                                                                                                   0.9814
            0.9913
                                                                                                             0.9819
                                    9870
                                                                                                   0.9828
   9957
            0.9921
                      0.9893
                                                       0.9841
                                                                  0.9833
                                                                                        0.9826
                                                                                                              0.9831
 O.
                                 Û.
                                                                  0.9850
 0.9962
            0.9931
                      0.9905
                                 0.9885
                                            0.9870
                                                       0.9858
                                                                             0.9846
                                                                                        0.9843
                                                                                                   0.9844
                                                                                                             0.9846
                                            0.9888
                      0.9919
                                 0.9902
                                                                             0.9866
                                                                                                  0.9862
                                                                                                             0.9864
            0.9941
                                                       0.9878
                                                                                       0.9863
   9968
                                                                  0.9870
                                                                  0.9892
0.9917
                                                                                                              0.9884
 0.
   9974
            0.9952
                      0.9934
                                 0.
                                    9919
                                            0.9908
                                                       0.9899
                                                                             0.9888
                                                                                        0.
                                                                                          9885
                                                                                                   0.9884
 0.9980
            0.9963
                      0.9950
                                 0.9938
                                            0.9929
                                                       0.9922
                                                                             0.9912
                                                                                        0.9910
                                                                                                   0.9908
                                                                                                             0.9908
                                 0.9958
                                            0.9952
                                                                                                  0.9935
                                                                             0.9939
                      0.9966
                                                                  0.9942
                                                                                       0.9937
                                                                                                             0.9934
           0.9975
                                                       0.9947
 0.9987
0.9993 0.9987
迭代次数为:590
运行时间为:346ms
                      0.9983
                                 0.9979
                                            0.9975
                                                       0.9973
                                                                  0.9970
                                                                             0.9968
                                                                                        0.9967
                                                                                                  0.9966
                                                                                                             0.9965
       弋法结果:
解为:
0.9972
                                                                                0.9940
0.9892
0.9855
0.9827
0.9807
0.9784
0.9786
0.9784
                    0. 9946
0. 9908
0. 9882
0. 9865
0. 9855
                                                  0.9936
0.9886
          0.9956
                              0.9940
                                        0. 9937
0. 9889
0. 9855
                                                                       0.9938
                                                                                          0.9943
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                                                                       0.9888
                              0. 9896
0. 9865
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0. 9862
                                                                                                     0.9905
          0.9927
                                                            0.9886
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0.9956
                                                                       0.9850
0.9946
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   9943
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0. 9827
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0.9825
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   9947
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0. 9826
0. 9843
0. 9863
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0. 9870
0. 9885
                                                            0.9817
0.9833
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0.9828
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          0.9913
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   9952
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   9957
           0.9921
                                         0.9853
                                                   0.9841
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                    0. 9905
0. 9919
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                                                                                           0. 9844
0. 9862
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0. 9888
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   9962
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                    0.9966
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   9987
          0.9975
                                                   0.9947
                                                            0.
                                                               9942
                                                                       0.9939
 0.9993 0.9
公驰因子为:
迭代次数为:
运行时间为:
           0.9987
                    0.9983
                              0.9979
                                         0.9975
                                                   0.9973
                                                            0.9970
                                                                       0.9968
                                                                                 0.9967
                                                                                           0.9966
                                                                                                     0.9965
                                                                                                               0.9965
              1.73
71
18ms
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Jacobi

0.9972 0.9810	0.9947 0.9814	0.9925 0.9819	0.9904 0.9825	0.9887 0.9831	0.9871 0.9838	0.9857 0.9846	0.9845 0.9854	0.9834 0.9863	0.9825	0.9818 0.9884	0.9812 0.9895	0.9807 0.9907
0.9010	0.9014	0.9928	0.9825	0.9891	0.9876	0.9840	0.9850	0.9840	0.9873 0.9831	0.9824	0.9818	0.9907
0.9814	0.9949	0.9823	0.9828	0.9834	0.9841	0.9848	0.9856	0.9840	0.9874	0.9824	0.9896	0.9913
0.9014	0.9952	0.9023	0.9020	0.9896	0.9841	0.9868	0.9856	0.9847	0.9838	0.9831	0.9825	0.9820
0.9819	0.9823	0.9931	0.9832	0.9837	0.9844					0.9886	0.9825	0.9820
		0.9827				0.9851 0.9874	0.9858	0.9867 0.9854	0.9876	0.9838	0.9833	0.9908
0.9976	0.9954	0.9934	0.9917 0.9836	0.9901 0.9842	0.9887		0.9863		0.9845		0.9833	0.9828
0.9825	0.9828				0.9847	0.9854	0.9861	0.9869	0.9878	0.9887	0.9841	0.9836
0.9977	0.9957	0.9938	0.9921	0.9906	0.9893	0.9881	0.9870	0.9861	0.9853	0.9846		
0.9831	0.9834	0.9837	0.9842	0.9846	0.9852	0.9858	0.9864	0.9872	0.9880	0.9889	0.9899	0.9910
0.9979	0.9959	0.9942	0.9926	0.9912	0.9899	0.9887	0.9877	0.9869	0.9861	0.9855	0.9849	0.9845
0.9838	0.9841	0.9844	0.9847	0.9852	0.9856	0.9862	0.9868	0.9875	0.9883	0.9891	0.9901	0.9911
0.9980	0.9962	0.9946	0.9931	0.9917	0.9905	0.9895	0.9885	0.9877	0.9870	0.9863	0.9858	0.9854
0.9846	0.9848	0.9851	0.9854	0.9858	0.9862	0.9867	0.9873	0.9879	0.9886	0.9894	0.9903	0.9913
0.9982	0.9965	0.9950	0.9936	0.9923	0.9912	0.9902	0.9893	0.9885	0.9879	0.9873	0.9868	0.9863
0.9854	0.9856	0.9858	0.9861	0.9864	0.9868	0.9873	0.9878	0.9883	0.9890	0.9897	0.9906	0.9915
0.9983	0.9968	0.9954	0.9941	0.9929	0.9919	0.9910	0.9902	0.9894	0.9888	0.9882	0.9878	0.9874
0.9863	0.9865	0.9867	0.9869	0.9872	0.9875	0.9879	0.9883	0.9889	0.9895	0.9901	0.9909	0.9918
0.9985	0.9971	0.9958	0.9946	0.9936	0.9926	0.9918	0.9910	0.9904	0.9898	0.9893	0.9888	0.9884
0.9873	0.9874	0.9876	0.9878	0.9880	0.9883	0.9886	0.9890	0.9895	0.9900	0.9906	0.9913	0.9921
0.9986	0.9974	0.9962	0.9952	0.9942	0.9934	0.9926	0.9919	0.9913	0.9908	0.9903	0.9899	0.9895
0.9884	0.9884	0.9886	0. 9887	0.9889	0.9891	0.9894	0.9897	0.9901	0.9906	0.9911	0.9917	0.9925
0.9988	0.9977	0.9967	0.9957	0.9949	0.9942	0.9935	0.9929	0.9923	0.9918	0.9914	0.9910	0.9907
0.9895	0.9896	0.9896	0.9897	0.9899	0.9901	0.9903	0.9906	0.9909	0.9913	0.9917	0.9923	0.9929
0.9990	0.9980	0.9971	0.9963	0.9956	0.9949	0.9944	0. 9938	0.9933	0.9929	0.9925	0.9922	0.9919
0.9907	0.9907	0.9908	0.9909	0.9910	0.9911	0.9913	0.9915	0.9918	0.9921	0.9925	0.9929	0.9935
0.9991	0.9983	0.9976	0.9969	0.9963	0.9957	0.9952	0.9948	0.9944	0.9940	0.9937	0.9934	0.9931
0.9920	0.9920	0.9920	0.9921	0.9922	0.9922	0.9924	0.9925	0.9927	0.9930	0.9933	0.9937	0.9942
0.9993	0.9986	0.9981	0.9975	0.9970	0.9966	0.9962	0.9958	0.9955	0.9952	0.9949	0.9946	0.9944
0.9934	0.9934	0.9934	0.9934	0.9935	0.9935	0.9936	0.9937	0.9939	0.9940	0.9943	0.9946	0.9949
0.9995	0.9990	0.9985	0.9981	0. 9978	0.9974	0.9971	0.9968	0.9966	0.9963	0.9961	0.9959	0.9958
0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9950	0.9950	0.9951	0.9953	0.9954	0.9956	0.9959
0.9996	0.9993	0.9990	0. 9987	0. 9985	0.9983	0.9981	0.9979	0. 9977	0. 9975	0.9974	0. 9973	0.9971
0.9965	0.9965	0.9965	0.9964	0.9964	0.9964	0.9965	0.9965	0.9966	0.9966	0.9967	0.9969	0.9970
0.9998	0.9997	0.9995	0.9994	0.9992	0.9991	0.9990	0.9989	0.9988	0.9988	0.9987	0.9986	0.9985
0.9982	_0. 9982	0.9982	0.9981	0.9981	0.9981	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9984
迭代次数	为: 4290											
运行时间	为: 4343	ms										

G-S

```
0.9973 0.9949 0.9928 0.9928 0.9981 0.9876 0.9876 0.9882 0.9850 0.9840 0.9831 0.9824 0.9818 0.9917 0.9915 0.9915 0.9923 0.9823 0.9824 0.9841 0.9848 0.9856 0.9856 0.9856 0.9857 0.9837 0.9931 0.9915 0.9916 0.9881 0.9881 0.9856 0.9856 0.9856 0.9857 0.9838 0.9931 0.9925 0.9826 0.9819 0.9923 0.9827 0.9832 0.9837 0.9844 0.9858 0.9856 0.9856 0.9857 0.9838 0.9831 0.9825 0.9826 0.9948 0.9944 0.9948 0.9948 0.9934 0.9917 0.9901 0.9987 0.9838 0.9858 0.9867 0.9856 0.9866 0.9866 0.9966 0.9968 0.9968 0.9925 0.9825 0.9825 0.9825 0.9826 0.9842 0.9846 0.9852 0.9838 0.9831 0.9826 0.9833 0.9928 0.9937 0.9942 0.9946 0.9952 0.9838 0.9858 0.9854 0.9856 0.9858 0.9866 0.9831 0.9836 0.9837 0.9846 0.9852 0.9858 0.9858 0.9864 0.9852 0.9838 0.9931 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910 0.9910
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GN Micros	oft Visual S	itudio 调试法	空制台									
0.9825	0.9828	0.9832	0.9836	0.9842	0.9847	0.9854	0.9861	0.9869	0.9878	0.9887	0.9897	0.9909
0.9977	0.9957	0.9938	0.9921	0.9906	0.9893	0.9881	0.9870	0.9861	0.9853	0.9846	0.9841	0.9836
0.9831	0.9834	0.9837	0.9842	0.9846	0.9852	0.9858	0.9864	0.9872	0.9880	0.9889	0.9899	0.9910
0.9979	0.9959	0.9942	0.9926	0.9912	0.9899	0.9887	0.9877	0.9869	0.9861	0.9855	0.9849	0.9845
0.9838	0.9841	0.9844	0.9847	0.9852	0.9856	0.9862	0.9868	0.9875	0.9883	0.9891	0.9901	0.9911
0.9980	0.9962	0.9946	0.9931	0.9917	0.9905	0.9895	0.9885	0.9877	0.9870	0.9863	0.9858	0.9854
0.9846	0.9848	0.9851	0.9854	0.9858	0.9862	0.9867	0.9873	0.9879	0.9886	0.9894	0.9903	0.9913
0.9982	0.9965	0.9950	0.9936	0.9923	0.9912	0.9902	0.9893	0.9885	0.9879	0.9873	0.9868	0.9863
0.9854	0.9856	0.9858	0.9861	0.9864	0.9868	0.9873	0. 9878	0.9883	0.9890	0.9897	0.9906	0.9915
0.9983	0.9968	0.9954	0.9941	0.9929	0.9919	0.9910	0.9902	0.9894	0.9888	0.9882	0.9878	0.9874
0.9863	0. 9865	0. 9867	0.9869	0. 9872	0. 9875	0. 9879	0.9883	0. 9889	0.9895	0.9901	0.9909	0.9918
0.9985	0.9971	0. 9958	0.9946	0.9936	0.9926	0.9918	0.9910	0.9904	0.9898	0.9893	0.9888	0.9884
0.9873	0.9874	0.9876	0.9878	0.9880	0.9883	0.9886	0.9890	0.9895	0.9900	0.9906	0.9913	0.9921
0.9986	0.9974	0.9962	0.9952	0.9942	0.9934	0.9926	0.9919	0.9913	0.9908	0.9903	0.9899	0.9895
0.9884	0.9884	0.9886	0. 9887	0.9889	0.9891	0.9894	0. 9897	0.9901	0.9906	0.9911	0.9917	0.9925
0.9988	0.9977	0.9967	0.9957	0.9949	0.9942	0.9935	0.9929	0.9923	0.9918	0.9914	0.9910	0.9907
0.9895	0.9896	0.9896	0.9897	0.9899	0.9901	0.9903	0.9906	0.9909	0.9913	0.9917	0.9923	0.9929
0.9990	0.9980	0.9971	0.9963	0.9956	0.9949	0.9944	0.9938	0.9933	0.9929	0.9925	0.9922	0.9919
0.9907	0.9907	0.9908	0.9909	0.9910	0.9911	0.9913	0.9915	0.9918	0.9921	0.9925	0.9929	0.9935
0.9991	0.9983	0.9976	0.9969	0.9963	0.9957	0.9952	0.9948	0.9944	0.9940	0.9937	0.9934	0.9931
0.9920	0.9920	0.9920	0.9921	0.9922	0.9922	0.9924	0.9925	0.9927	0.9930	0.9933	0.9937	0.9942
0.9993	0.9986	0.9981	0.9975	0.9970	0.9966	0.9962	0.9958	0.9955	0.9952	0.9949	0.9946	0.9944
0.9934	0.9934	0.9934	0.9934	0.9935	0.9935	0.9936	0.9937	0.9939	0.9940	0.9943	0.9946	0.9949
0.9995	0.9990	0.9985	0.9981	0.9978	0.9974	0.9971	0.9968	0.9966	0.9963	0.9961	0.9959	0.9958
0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9950	0.9950	0.9951	0.9953	0.9954	0.9956	0.9959
0.9996	0.9993	0.9990	0.9987	0.9985	0.9983	0.9981	0.9979	0.9977	0.9975	0.9974	0.9973	0.9971
0. 9965 0. 9998	0.9965	0.9965	0.9964	0.9964	0.9964	0.9965	0.9965	0.9966	0.9966	0.9967	0.9969	0.9970
0.9998 0.9982	0.9997 0.9982	0. 9995 0. 9982	0.9994 0.9981	0.9992 0.9981	0.9991 0.9981	0.9990 0.9981	0. 9989 0. 9982	0. 9988 0. 9982	0. 9988 0. 9982	0. 9987 0. 9982	0.9986 0.9983	0.9985 0.9984
#W5WEDZ	0.9982 为: 1.85		0.9981	0.9981	0.9981	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9984
松弛因子 迭代次数	グ: 1.00 为: 145											
医行跃器	力: 143 为: 124π	-										
	/): IZHI	15										

N = 60

Jacobi

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0.9985
0.9905
0.9909
0.9986
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迭代次数为: 4928
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0.9987 0.9987
0.9987 0.9987
松弛因子为: 1.9
迭代次数为: 212
运行时间为: 532ms
                                    0.9988
                                                                                            0.9988
                                                                                                              0.9988
                                                                                                                                0.9989
                                                       0.9988
                                                                          0.9988
C:\Users\郑涛\Desktop\数值代数\数值代数实验\实验四\Homework4\x64\Debug\Homework4.exe(进程 42716)已退出
要在调试停止时自动关闭控制台,请启用"工具"->"选项"->"调试"->"调试停止时自动关闭控制台"。
按任意键关闭此窗口. . .
```

4.2 结果分析

根据程序运行结果可以得到如下表格:

N	迭代法	迭代次数	运行时间 (ms)		
	Jacobi	1123	505		
20	G-S	590	346		
	SOR (1.73)	71	18		
40	Jacobi	4290	4343		
	G-S	2252	2479		
	SOR (1.85)	145	124		
60	Jacobi	9377	24656		
	G-S	4928	8468		
	SOR (1.9)	212	532		

从表格中可以看出,不管是迭代次数还是运行时间,最佳的 SOR 迭代法都优于 G-S 迭代法优于 Jacobi 迭代法