

数值代数第四次实验报告

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4.1 问题描述

1. 考虑两点边值问题

$$\begin{cases} \varepsilon \frac{d^2 y}{dx^2} + \frac{dy}{dx} = a, 0 < a < 1, \\ y(0) = 0, y(1) = 1. \end{cases}$$

容易知道它的精确解为

$$y = \frac{1-a}{1-e^{-\frac{1}{\varepsilon}}}(1-e^{-\frac{x}{\varepsilon}}) + ax$$

为了把微分方程离散化, 把 $[0, 1]$ 区间 n 等分, 令 $h = 1/n, x_i = ih, i = 1, \dots, n-1$, 得到差分方程

$$\varepsilon \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + \frac{y_{i+1} - y_i}{h} = a,$$

简化为

$$(\varepsilon + h)y_{i+1} - (2\varepsilon + h)y_i + \varepsilon y_{i-1} = ah^2$$

离散化后得到线性方程组 $Ay = b$, 其中

$$A = \begin{pmatrix} -(2\varepsilon + h) & \varepsilon + h & & & \\ \varepsilon & -(2\varepsilon + h) & \varepsilon + h & & \\ & \varepsilon & -(2\varepsilon + h) & \ddots & \\ & & \ddots & \ddots & \varepsilon + h \\ & & & \varepsilon & -(2\varepsilon + h) \end{pmatrix}$$

注意将线性方程组与上述差分方程进行对比得出正确的 b 向量 (尤其注意第一行和最后一行)。

对 $\varepsilon = 1, a = 1/2, n = 100$, 分别用 Jacobi 迭代法, G-S 迭代法和 SOR 迭代法求线性方程组的解, 要求 4 位有效数字, 然后比较迭代次数, 运行时间与精确解的误差。迭代法终止条件为 $\|x_{k+1} - x_k\| < 10^{-6}$ 。

对 $\varepsilon = 0.1, 0.01, 0.0001$, 考虑同样的问题。

4.2 程序运行结果

$e = 1$ $a = 0.5$ $n = 100$ 时三种方法的结果如下:

Jacobi法结果:

解为:

0.0128	0.0256	0.0382	0.0508	0.0633	0.0758	0.0881	0.1004	0.1126	0.1248
0.1369	0.1489	0.1608	0.1727	0.1845	0.1962	0.2079	0.2195	0.2310	0.2425
0.2539	0.2653	0.2766	0.2878	0.2990	0.3101	0.3211	0.3321	0.3430	0.3539
0.3647	0.3755	0.3862	0.3968	0.4074	0.4179	0.4284	0.4388	0.4492	0.4595
0.4698	0.4800	0.4902	0.5003	0.5104	0.5204	0.5304	0.5403	0.5502	0.5600
0.5698	0.5795	0.5892	0.5988	0.6084	0.6180	0.6275	0.6370	0.6464	0.6558
0.6651	0.6744	0.6837	0.6929	0.7020	0.7112	0.7203	0.7293	0.7383	0.7473
0.7562	0.7651	0.7740	0.7828	0.7916	0.8003	0.8090	0.8177	0.8263	0.8349
0.8435	0.8520	0.8605	0.8690	0.8774	0.8858	0.8942	0.9025	0.9108	0.9191
0.9273	0.9355	0.9437	0.9518	0.9599	0.9680	0.9760	0.9841	0.9920	

迭代次数为: 13172

运行时间为: 46350ms

与精确解的误差为: 0.00124715

G-S迭代法结果:

解为:

0.0128	0.0256	0.0382	0.0508	0.0633	0.0757	0.0881	0.1004	0.1126	0.1248
0.1368	0.1488	0.1608	0.1727	0.1845	0.1962	0.2079	0.2195	0.2310	0.2425
0.2539	0.2652	0.2765	0.2878	0.2989	0.3100	0.3211	0.3321	0.3430	0.3539
0.3647	0.3754	0.3861	0.3968	0.4074	0.4179	0.4284	0.4388	0.4492	0.4595
0.4698	0.4800	0.4902	0.5003	0.5103	0.5204	0.5303	0.5403	0.5501	0.5600
0.5697	0.5795	0.5892	0.5988	0.6084	0.6180	0.6275	0.6369	0.6464	0.6557
0.6651	0.6744	0.6836	0.6928	0.7020	0.7111	0.7202	0.7293	0.7383	0.7473
0.7562	0.7651	0.7740	0.7828	0.7916	0.8003	0.8090	0.8177	0.8263	0.8349
0.8435	0.8520	0.8605	0.8690	0.8774	0.8858	0.8942	0.9025	0.9108	0.9191
0.9273	0.9355	0.9437	0.9518	0.9599	0.9680	0.9760	0.9841	0.9920	

迭代次数为: 6574

运行时间为: 33245ms

与精确解的误差为: 0.00128824

SOR迭代法结果:

解为:

0.0129	0.0256	0.0383	0.0510	0.0635	0.0760	0.0884	0.1007	0.1130	0.1251
0.1372	0.1493	0.1613	0.1732	0.1850	0.1968	0.2085	0.2201	0.2317	0.2432
0.2546	0.2660	0.2773	0.2885	0.2997	0.3108	0.3219	0.3329	0.3438	0.3547
0.3656	0.3763	0.3870	0.3977	0.4083	0.4188	0.4293	0.4398	0.4501	0.4605
0.4708	0.4810	0.4911	0.5013	0.5113	0.5214	0.5313	0.5412	0.5511	0.5609
0.5707	0.5804	0.5901	0.5998	0.6093	0.6189	0.6284	0.6378	0.6473	0.6566
0.6659	0.6752	0.6845	0.6937	0.7028	0.7119	0.7210	0.7300	0.7390	0.7480
0.7569	0.7658	0.7746	0.7834	0.7922	0.8009	0.8096	0.8182	0.8268	0.8354
0.8440	0.8525	0.8609	0.8694	0.8778	0.8862	0.8945	0.9028	0.9111	0.9193
0.9275	0.9357	0.9438	0.9520	0.9600	0.9681	0.9761	0.9841	0.9921	

松弛因子为: 1.94

迭代次数为: 246

运行时间为: 1473ms

与精确解的误差为: 0.000298292

$\epsilon = 0.1$ $a = 0.5$ $n = 100$ 时三种方法的结果如下:

Jacobi法结果:

解为:

0.0504	0.0967	0.1393	0.1784	0.2144	0.2476	0.2782	0.3066	0.3327	0.3570
0.3795	0.4005	0.4199	0.4381	0.4551	0.4709	0.4858	0.4998	0.5130	0.5254
0.5372	0.5483	0.5589	0.5690	0.5786	0.5878	0.5966	0.6051	0.6133	0.6211
0.6288	0.6361	0.6433	0.6502	0.6570	0.6637	0.6701	0.6765	0.6827	0.6888
0.6948	0.7007	0.7066	0.7123	0.7180	0.7237	0.7292	0.7348	0.7402	0.7457
0.7511	0.7564	0.7617	0.7670	0.7723	0.7775	0.7828	0.7880	0.7932	0.7983
0.8035	0.8086	0.8137	0.8189	0.8240	0.8291	0.8341	0.8392	0.8443	0.8494
0.8544	0.8595	0.8645	0.8696	0.8746	0.8797	0.8847	0.8897	0.8948	0.8998
0.9048	0.9098	0.9148	0.9199	0.9249	0.9299	0.9349	0.9399	0.9449	0.9499
0.9549	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

迭代次数为: 5926

运行时间为: 29379ms

与精确解的误差为: 0.00906271

G-S迭代法结果:

解为:

0.0504	0.0967	0.1392	0.1784	0.2144	0.2476	0.2782	0.3065	0.3327	0.3570
0.3795	0.4004	0.4199	0.4381	0.4550	0.4709	0.4858	0.4998	0.5130	0.5254
0.5372	0.5483	0.5589	0.5690	0.5786	0.5878	0.5966	0.6051	0.6133	0.6211
0.6287	0.6361	0.6433	0.6502	0.6570	0.6636	0.6701	0.6765	0.6827	0.6888
0.6948	0.7007	0.7066	0.7123	0.7180	0.7237	0.7292	0.7348	0.7402	0.7457
0.7511	0.7564	0.7617	0.7670	0.7723	0.7775	0.7828	0.7880	0.7932	0.7983
0.8035	0.8086	0.8137	0.8189	0.8240	0.8291	0.8341	0.8392	0.8443	0.8494
0.8544	0.8595	0.8645	0.8696	0.8746	0.8797	0.8847	0.8897	0.8948	0.8998
0.9048	0.9098	0.9148	0.9199	0.9249	0.9299	0.9349	0.9399	0.9449	0.9499
0.9549	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

迭代次数为: 2981

运行时间为: 12774ms

与精确解的误差为: 0.00908668

SOR迭代法结果:

解为:

0.0505	0.0968	0.1394	0.1785	0.2146	0.2478	0.2784	0.3068	0.3330	0.3573
0.3798	0.4007	0.4202	0.4384	0.4553	0.4712	0.4861	0.5001	0.5133	0.5257
0.5375	0.5486	0.5592	0.5693	0.5789	0.5881	0.5969	0.6054	0.6135	0.6214
0.6290	0.6364	0.6435	0.6505	0.6572	0.6639	0.6703	0.6767	0.6829	0.6890
0.6950	0.7009	0.7067	0.7125	0.7182	0.7238	0.7294	0.7349	0.7404	0.7458
0.7512	0.7565	0.7618	0.7671	0.7724	0.7776	0.7829	0.7880	0.7932	0.7984
0.8035	0.8087	0.8138	0.8189	0.8240	0.8291	0.8342	0.8393	0.8443	0.8494
0.8545	0.8595	0.8646	0.8696	0.8746	0.8797	0.8847	0.8897	0.8948	0.8998
0.9048	0.9098	0.9149	0.9199	0.9249	0.9299	0.9349	0.9399	0.9449	0.9499
0.9550	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

松弛因子为: 1.895

迭代次数为: 201

运行时间为: 2542ms

与精确解的误差为: 0.00882342

$\epsilon = 0.01$ $\alpha = 0.5$ $n = 100$ 时三种方法的结果如下:

Jacobi法结果:

解为:

0.2550	0.3850	0.4525	0.4887	0.5094	0.5222	0.5311	0.5380	0.5440	0.5495
0.5548	0.5599	0.5649	0.5700	0.5750	0.5800	0.5850	0.5900	0.5950	0.6000
0.6050	0.6100	0.6150	0.6200	0.6250	0.6300	0.6350	0.6400	0.6450	0.6500
0.6550	0.6600	0.6650	0.6700	0.6750	0.6800	0.6850	0.6900	0.6950	0.7000
0.7050	0.7100	0.7150	0.7200	0.7250	0.7300	0.7350	0.7400	0.7450	0.7500
0.7550	0.7600	0.7650	0.7700	0.7750	0.7800	0.7850	0.7900	0.7950	0.8000
0.8050	0.8100	0.8150	0.8200	0.8250	0.8300	0.8350	0.8400	0.8450	0.8500
0.8550	0.8600	0.8650	0.8700	0.8750	0.8800	0.8850	0.8900	0.8950	0.9000
0.9050	0.9100	0.9150	0.9200	0.9250	0.9300	0.9350	0.9400	0.9450	0.9500
0.9550	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

迭代次数为: 569

运行时间为: 3473ms

与精确解的误差为: 0.0660671

G-S迭代法结果:

解为:

0.2550	0.3850	0.4525	0.4887	0.5094	0.5222	0.5311	0.5380	0.5440	0.5495
0.5548	0.5599	0.5649	0.5700	0.5750	0.5800	0.5850	0.5900	0.5950	0.6000
0.6050	0.6100	0.6150	0.6200	0.6250	0.6300	0.6350	0.6400	0.6450	0.6500
0.6550	0.6600	0.6650	0.6700	0.6750	0.6800	0.6850	0.6900	0.6950	0.7000
0.7050	0.7100	0.7150	0.7200	0.7250	0.7300	0.7350	0.7400	0.7450	0.7500
0.7550	0.7600	0.7650	0.7700	0.7750	0.7800	0.7850	0.7900	0.7950	0.8000
0.8050	0.8100	0.8150	0.8200	0.8250	0.8300	0.8350	0.8400	0.8450	0.8500
0.8550	0.8600	0.8650	0.8700	0.8750	0.8800	0.8850	0.8900	0.8950	0.9000
0.9050	0.9100	0.9150	0.9200	0.9250	0.9300	0.9350	0.9400	0.9450	0.9500
0.9550	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

迭代次数为: 333

运行时间为: 2298ms

与精确解的误差为: 0.0660678

SOR迭代法结果:

解为:

0.2550	0.3850	0.4525	0.4887	0.5094	0.5222	0.5311	0.5380	0.5440	0.5495
0.5548	0.5599	0.5649	0.5700	0.5750	0.5800	0.5850	0.5900	0.5950	0.6000
0.6050	0.6100	0.6150	0.6200	0.6250	0.6300	0.6350	0.6400	0.6450	0.6500
0.6550	0.6600	0.6650	0.6700	0.6750	0.6800	0.6850	0.6900	0.6950	0.7000
0.7050	0.7100	0.7150	0.7200	0.7250	0.7300	0.7350	0.7400	0.7450	0.7500
0.7550	0.7600	0.7650	0.7700	0.7750	0.7800	0.7850	0.7900	0.7950	0.8000
0.8050	0.8100	0.8150	0.8200	0.8250	0.8300	0.8350	0.8400	0.8450	0.8500
0.8550	0.8600	0.8650	0.8700	0.8750	0.8800	0.8850	0.8900	0.8950	0.9000
0.9050	0.9100	0.9150	0.9200	0.9250	0.9300	0.9350	0.9400	0.9450	0.9500
0.9550	0.9600	0.9650	0.9700	0.9750	0.9800	0.9850	0.9900	0.9950	

松弛因子为: 1.5

迭代次数为: 101

运行时间为: 977ms

与精确解的误差为: 0.0660603

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e = 0.0001 a = 0.5 n = 100时三种方法的结果如下：

Jacobi法结果：
解为：
0.5000 0.5100 0.5150 0.5200 0.5250 0.5300 0.5350 0.5400 0.5450 0.5500
0.5550 0.5600 0.5650 0.5700 0.5750 0.5800 0.5850 0.5900 0.5950 0.6000
0.6050 0.6100 0.6150 0.6200 0.6250 0.6300 0.6350 0.6400 0.6450 0.6500
0.6550 0.6600 0.6650 0.6700 0.6750 0.6800 0.6850 0.6900 0.6950 0.7000
0.7050 0.7100 0.7150 0.7200 0.7250 0.7300 0.7350 0.7400 0.7450 0.7500
0.7550 0.7600 0.7650 0.7700 0.7750 0.7800 0.7850 0.7900 0.7950 0.8000
0.8050 0.8100 0.8150 0.8200 0.8250 0.8300 0.8350 0.8400 0.8450 0.8500
0.8550 0.8600 0.8650 0.8700 0.8750 0.8800 0.8850 0.8900 0.8950 0.9000
0.9050 0.9100 0.9150 0.9200 0.9250 0.9300 0.9350 0.9400 0.9450 0.9500
0.9550 0.9600 0.9650 0.9700 0.9750 0.9800 0.9850 0.9900 0.9950
迭代次数为：118
运行时间为：656ms
与精确解的误差为：0.00495075

G-S迭代法结果：
解为：
0.5000 0.5100 0.5150 0.5200 0.5250 0.5300 0.5350 0.5400 0.5450 0.5500
0.5550 0.5600 0.5650 0.5700 0.5750 0.5800 0.5850 0.5900 0.5950 0.6000
0.6050 0.6100 0.6150 0.6200 0.6250 0.6300 0.6350 0.6400 0.6450 0.6500
0.6550 0.6600 0.6650 0.6700 0.6750 0.6800 0.6850 0.6900 0.6950 0.7000
0.7050 0.7100 0.7150 0.7200 0.7250 0.7300 0.7350 0.7400 0.7450 0.7500
0.7550 0.7600 0.7650 0.7700 0.7750 0.7800 0.7850 0.7900 0.7950 0.8000
0.8050 0.8100 0.8150 0.8200 0.8250 0.8300 0.8350 0.8400 0.8450 0.8500
0.8550 0.8600 0.8650 0.8700 0.8750 0.8800 0.8850 0.8900 0.8950 0.9000
0.9050 0.9100 0.9150 0.9200 0.9250 0.9300 0.9350 0.9400 0.9450 0.9500
0.9550 0.9600 0.9650 0.9700 0.9750 0.9800 0.9850 0.9900 0.9950
迭代次数为：109
运行时间为：401ms
与精确解的误差为：0.00495075

SOR迭代法结果：
解为：
0.5000 0.5100 0.5150 0.5200 0.5250 0.5300 0.5350 0.5400 0.5450 0.5500
0.5550 0.5600 0.5650 0.5700 0.5750 0.5800 0.5850 0.5900 0.5950 0.6000
0.6050 0.6100 0.6150 0.6200 0.6250 0.6300 0.6350 0.6400 0.6450 0.6500
0.6550 0.6600 0.6650 0.6700 0.6750 0.6800 0.6850 0.6900 0.6950 0.7000
0.7050 0.7100 0.7150 0.7200 0.7250 0.7300 0.7350 0.7400 0.7450 0.7500
0.7550 0.7600 0.7650 0.7700 0.7750 0.7800 0.7850 0.7900 0.7950 0.8000
0.8050 0.8100 0.8150 0.8200 0.8250 0.8300 0.8350 0.8400 0.8450 0.8500
0.8550 0.8600 0.8650 0.8700 0.8750 0.8800 0.8850 0.8900 0.8950 0.9000
0.9050 0.9100 0.9150 0.9200 0.9250 0.9300 0.9350 0.9400 0.9450 0.9500
0.9550 0.9600 0.9650 0.9700 0.9750 0.9800 0.9850 0.9900 0.9950
松弛因子为：1.01
迭代次数为：102
运行时间为：669ms
与精确解的误差为：0.00495082

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要在调试停止时自动关闭控制台，请启用“工具”->“选项”->“调试”->“调试停止时自动
按任意键关闭此窗口。 . .

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4.1 结果分析

根据运行结果得到如下表格（SOR 方法给出了最佳 ω 值）：

ε	迭代法	迭代次数	运行时间 (ms)	误差
1	Jacobi	13172	46350	0.001247
	G-S	6574	33245	0.001288
	SOR(1.94)	246	1473	0.000298
0.1	Jacobi	5926	29379	0.009063
	G-S	2981	12774	0.009087
	SOR(1.895)	201	2542	0.008823
0.01	Jacobi	569	3473	0.066067
	G-S	333	2298	0.066068
	SOR(1.5)	101	977	0.066060
0.0001	Jacobi	118	656	0.004951
	G-S	109	401	0.004951
	SOR(1.01)	102	669	0.004951

根据表格可以看出， ε 越小，三个方法的迭代次数越少；对于给定的 ε 值，迭代次数和运行时间排序为：Jacobi > G-S > SOR；表中也可以看出一般来说 SOR 的误差也比其他两个方法小。

4.2 问题描述

2. 考虑偏微分方程

$$-\Delta u + g(x, y)u = f(x, y), (x, y) \in [0, 1] \times [0, 1]$$

边界条件为 $u = 1$. 沿 x 方向和 y 方向均匀剖分 N 等份，令 $h = 1/N$ ，并设应用中心差分离散化后得到差分方程的代数方程组为

$$-u_{i-1,j} - u_{i,j-1} + (4 + h^2 g(ih, jh))u_{i,j} - u_{i+1,j} - u_{i,j+1} = h^2 f(ih, jh) \quad (1)$$

取 $g(x, y) = e^{xy}$, $f(x, y) = x + y$ ，分别用 **Jacobi 迭代法**，**G-S 迭代法**和 **SOR 迭代法**求解上述代数方程组，并列表比较 $N = 20, 40, 60$ 时收敛所需要的迭代次数和所用的 CPU 时间，迭代终止条件为 $\|x_{k+1} - x_k\| < 10^{-7}$. 要求仿照下面写的 Jacobi 迭代格式的推导过程推导处 G-S 迭代和 SOR 迭代的格式（在报告中写出推导过程），在用 SOR 迭代法求解的过程中，请对不同的 N 使用合适的松弛因子 ω ，并在程序输出中打印松弛因子的值。可以采用二分法 观察运行结果后选取合适的（代码中不需要体现选取过程，只需给出即可）。

注意本题中的三个迭代法的算法需要重新写，不能用之前的通用算法!!!

4.3G-S 和 SOR 推导过程

G-S:

考虑 G-S 迭代格式 $(D - L)x^{(k+1)} = Ux^{(k)} + b$, 考虑第 i 行的元素用 A 中的元素表示,得到 G-S 迭代格式:

$$A_{i1}x_1^{(k+1)} + A_{i2}x_2^{(k+1)} + \dots + A_{ii}x_i^{(k+1)} + A_{i(i+1)}x_{(i+1)}^{(k)} + \dots + A_{in}x_n^{(k)} = b_i$$

推广至矩阵可知代数方程 (1) 的 G-S 迭代格式为:

$$-u_{i-1,j}^{(k+1)} - u_{ij-1}^{(k+1)} + (4 + h^2 g(ih, jh))u_{ij}^{(k+1)} - u_{i+1,j}^{(k)} - u_{ij+1}^{(k)} = h^2 f(ih, jh)$$

SOR:

考虑 SOR 迭代格式 $(D - \omega L)x^{(k+1)} = ((1 - \omega)D + \omega U)x^{(k)} + \omega b$, 考虑第 i 行的元素用 A 中的元素表示, 得到 SOR 迭代格式:

$$\begin{aligned} &\omega A_{i1}x_1^{(k+1)} + \dots + \omega A_{i(i-1)}x_i^{(k+1)} + A_{ii}x_i^{(k+1)} + (\omega - 1)A_{ii}x_i^{(k)} + \omega A_{i(i+1)}x_{i+1}^{(k)} + \dots \\ &+ \omega A_{in}x_n^{(k)} = \omega b_i \end{aligned}$$

推广至矩阵可知代数方程 (1) 的 SOR 迭代格式为:

$$\begin{aligned} &-\omega u_{i-1,j}^{(k+1)} - \omega u_{ij-1}^{(k+1)} + (4 + h^2 g(ih, jh))u_{ij}^{(k+1)} + (\omega - 1)(4 + h^2 g(ih, jh))u_{ij}^{(k)} \\ &-\omega u_{i+1,j}^{(k)} - \omega u_{ij+1}^{(k)} = \omega h^2 f(ih, jh) \end{aligned}$$

4.3 程序运行结果

$$N = 20$$

Jacobi法结果:

解为:

```
0.9972 0.9956 0.9946 0.9940 0.9937 0.9936 0.9936 0.9938 0.9940 0.9943 0.9947
0.9956 0.9927 0.9908 0.9896 0.9889 0.9886 0.9886 0.9888 0.9892 0.9898 0.9905
0.9946 0.9908 0.9882 0.9865 0.9855 0.9849 0.9848 0.9850 0.9855 0.9862 0.9871
0.9940 0.9896 0.9865 0.9844 0.9830 0.9823 0.9821 0.9822 0.9827 0.9835 0.9845
0.9937 0.9889 0.9855 0.9830 0.9814 0.9805 0.9801 0.9802 0.9807 0.9815 0.9826
0.9936 0.9886 0.9849 0.9823 0.9805 0.9794 0.9790 0.9790 0.9794 0.9802 0.9812
0.9936 0.9886 0.9848 0.9821 0.9801 0.9790 0.9784 0.9783 0.9786 0.9794 0.9804
0.9938 0.9888 0.9850 0.9822 0.9802 0.9790 0.9783 0.9781 0.9784 0.9790 0.9800
0.9940 0.9892 0.9855 0.9827 0.9807 0.9794 0.9786 0.9784 0.9786 0.9791 0.9800
0.9943 0.9898 0.9862 0.9835 0.9815 0.9802 0.9794 0.9790 0.9791 0.9796 0.9803
0.9947 0.9905 0.9871 0.9845 0.9826 0.9812 0.9804 0.9800 0.9800 0.9803 0.9810
0.9952 0.9913 0.9881 0.9857 0.9838 0.9825 0.9817 0.9813 0.9812 0.9814 0.9819
0.9957 0.9921 0.9893 0.9870 0.9853 0.9841 0.9833 0.9828 0.9826 0.9828 0.9831
0.9962 0.9931 0.9905 0.9885 0.9870 0.9858 0.9850 0.9846 0.9843 0.9844 0.9846
0.9968 0.9941 0.9919 0.9902 0.9888 0.9878 0.9870 0.9866 0.9863 0.9862 0.9864
0.9974 0.9952 0.9934 0.9919 0.9908 0.9899 0.9892 0.9888 0.9885 0.9884 0.9884
0.9980 0.9963 0.9950 0.9938 0.9929 0.9922 0.9917 0.9912 0.9910 0.9908 0.9908
0.9987 0.9975 0.9966 0.9958 0.9952 0.9947 0.9942 0.9939 0.9937 0.9935 0.9934
0.9993 0.9987 0.9983 0.9979 0.9975 0.9973 0.9970 0.9968 0.9967 0.9966 0.9965
```

迭代次数为: 1123

运行时间为: 505ms

G-S迭代法结果:

解为:

```
0.9972 0.9956 0.9946 0.9940 0.9937 0.9936 0.9936 0.9938 0.9940 0.9943 0.9947
0.9956 0.9927 0.9908 0.9896 0.9889 0.9886 0.9886 0.9888 0.9892 0.9898 0.9905
0.9946 0.9908 0.9882 0.9865 0.9855 0.9849 0.9848 0.9850 0.9855 0.9862 0.9871
0.9940 0.9896 0.9865 0.9844 0.9830 0.9823 0.9821 0.9822 0.9827 0.9835 0.9845
0.9937 0.9889 0.9855 0.9830 0.9814 0.9805 0.9801 0.9802 0.9807 0.9815 0.9826
0.9936 0.9886 0.9849 0.9823 0.9805 0.9794 0.9790 0.9790 0.9794 0.9802 0.9812
0.9936 0.9886 0.9848 0.9821 0.9801 0.9790 0.9784 0.9783 0.9786 0.9794 0.9804
0.9938 0.9888 0.9850 0.9822 0.9802 0.9790 0.9783 0.9781 0.9784 0.9790 0.9800
0.9940 0.9892 0.9855 0.9827 0.9807 0.9794 0.9786 0.9784 0.9786 0.9791 0.9800
0.9943 0.9898 0.9862 0.9835 0.9815 0.9802 0.9794 0.9790 0.9791 0.9796 0.9803
0.9947 0.9905 0.9871 0.9845 0.9826 0.9812 0.9804 0.9800 0.9800 0.9803 0.9810
0.9952 0.9913 0.9881 0.9857 0.9838 0.9825 0.9817 0.9813 0.9812 0.9814 0.9819
0.9957 0.9921 0.9893 0.9870 0.9853 0.9841 0.9833 0.9828 0.9826 0.9828 0.9831
0.9962 0.9931 0.9905 0.9885 0.9870 0.9858 0.9850 0.9846 0.9843 0.9844 0.9846
0.9968 0.9941 0.9919 0.9902 0.9888 0.9878 0.9870 0.9866 0.9863 0.9862 0.9864
0.9974 0.9952 0.9934 0.9919 0.9908 0.9899 0.9892 0.9888 0.9885 0.9884 0.9884
0.9980 0.9963 0.9950 0.9938 0.9929 0.9922 0.9917 0.9912 0.9910 0.9908 0.9908
0.9987 0.9975 0.9966 0.9958 0.9952 0.9947 0.9942 0.9939 0.9937 0.9935 0.9934
0.9993 0.9987 0.9983 0.9979 0.9975 0.9973 0.9970 0.9968 0.9967 0.9966 0.9965
```

迭代次数为: 590

运行时间为: 346ms

SOR迭代法结果:

解为:

```
0.9972 0.9956 0.9946 0.9940 0.9937 0.9936 0.9936 0.9938 0.9940 0.9943 0.9947 0.9952
0.9956 0.9927 0.9908 0.9896 0.9889 0.9886 0.9886 0.9888 0.9892 0.9898 0.9905 0.9913
0.9946 0.9908 0.9882 0.9865 0.9855 0.9849 0.9848 0.9850 0.9855 0.9862 0.9871 0.9881
0.9940 0.9896 0.9865 0.9844 0.9830 0.9823 0.9821 0.9822 0.9827 0.9835 0.9845 0.9857
0.9937 0.9889 0.9855 0.9830 0.9814 0.9805 0.9801 0.9802 0.9807 0.9815 0.9826 0.9838
0.9936 0.9886 0.9849 0.9823 0.9805 0.9794 0.9790 0.9790 0.9794 0.9802 0.9812 0.9825
0.9936 0.9886 0.9848 0.9821 0.9801 0.9790 0.9784 0.9783 0.9786 0.9794 0.9804 0.9817
0.9938 0.9888 0.9850 0.9822 0.9802 0.9790 0.9783 0.9781 0.9784 0.9790 0.9800 0.9813
0.9940 0.9892 0.9855 0.9827 0.9807 0.9794 0.9786 0.9784 0.9786 0.9791 0.9800 0.9812
0.9943 0.9898 0.9862 0.9835 0.9815 0.9802 0.9794 0.9790 0.9791 0.9796 0.9803 0.9814
0.9947 0.9905 0.9871 0.9845 0.9826 0.9812 0.9804 0.9800 0.9800 0.9803 0.9810 0.9819
0.9952 0.9913 0.9881 0.9857 0.9838 0.9825 0.9817 0.9813 0.9812 0.9814 0.9819 0.9827
0.9957 0.9921 0.9893 0.9870 0.9853 0.9841 0.9833 0.9828 0.9826 0.9828 0.9831 0.9838
0.9962 0.9931 0.9905 0.9885 0.9870 0.9858 0.9850 0.9846 0.9843 0.9844 0.9846 0.9851
0.9968 0.9941 0.9919 0.9902 0.9888 0.9878 0.9870 0.9866 0.9863 0.9862 0.9864 0.9867
0.9974 0.9952 0.9934 0.9919 0.9908 0.9899 0.9892 0.9888 0.9885 0.9884 0.9884 0.9886
0.9980 0.9963 0.9950 0.9938 0.9929 0.9922 0.9917 0.9912 0.9910 0.9908 0.9908 0.9908
0.9987 0.9975 0.9966 0.9958 0.9952 0.9947 0.9942 0.9939 0.9937 0.9935 0.9934 0.9934
0.9993 0.9987 0.9983 0.9979 0.9975 0.9973 0.9970 0.9968 0.9967 0.9966 0.9965 0.9965
```

松弛因子为: 1.73

迭代次数为: 71

运行时间为: 18ms

Jacobi

0.9972	0.9947	0.9925	0.9904	0.9887	0.9871	0.9857	0.9845	0.9834	0.9825	0.9818	0.9812	0.9807
0.9810	0.9814	0.9819	0.9825	0.9831	0.9838	0.9846	0.9854	0.9863	0.9873	0.9884	0.9895	0.9907
0.9973	0.9949	0.9928	0.9908	0.9891	0.9876	0.9862	0.9850	0.9840	0.9831	0.9824	0.9818	0.9813
0.9814	0.9818	0.9823	0.9828	0.9834	0.9841	0.9848	0.9856	0.9865	0.9874	0.9884	0.9896	0.9907
0.9975	0.9952	0.9931	0.9912	0.9896	0.9881	0.9868	0.9856	0.9847	0.9838	0.9831	0.9825	0.9820
0.9819	0.9823	0.9827	0.9832	0.9837	0.9844	0.9851	0.9858	0.9867	0.9876	0.9886	0.9896	0.9908
0.9976	0.9954	0.9934	0.9917	0.9901	0.9887	0.9874	0.9863	0.9854	0.9845	0.9838	0.9833	0.9828
0.9825	0.9828	0.9832	0.9836	0.9842	0.9847	0.9854	0.9861	0.9869	0.9878	0.9887	0.9897	0.9909
0.9977	0.9957	0.9938	0.9921	0.9906	0.9893	0.9881	0.9870	0.9861	0.9853	0.9846	0.9841	0.9836
0.9831	0.9834	0.9837	0.9842	0.9846	0.9852	0.9858	0.9864	0.9872	0.9880	0.9889	0.9899	0.9910
0.9979	0.9959	0.9942	0.9926	0.9912	0.9899	0.9887	0.9877	0.9869	0.9861	0.9855	0.9849	0.9845
0.9838	0.9841	0.9844	0.9847	0.9852	0.9856	0.9862	0.9868	0.9875	0.9883	0.9891	0.9901	0.9911
0.9980	0.9962	0.9946	0.9931	0.9917	0.9905	0.9895	0.9885	0.9877	0.9870	0.9863	0.9858	0.9854
0.9846	0.9848	0.9851	0.9854	0.9858	0.9862	0.9867	0.9873	0.9879	0.9886	0.9894	0.9903	0.9913
0.9982	0.9965	0.9950	0.9936	0.9923	0.9912	0.9902	0.9893	0.9885	0.9879	0.9873	0.9868	0.9863
0.9854	0.9856	0.9858	0.9861	0.9864	0.9868	0.9873	0.9878	0.9883	0.9890	0.9897	0.9906	0.9915
0.9983	0.9968	0.9954	0.9941	0.9929	0.9919	0.9910	0.9902	0.9894	0.9888	0.9882	0.9878	0.9874
0.9863	0.9865	0.9867	0.9869	0.9872	0.9875	0.9879	0.9883	0.9889	0.9895	0.9901	0.9909	0.9918
0.9985	0.9971	0.9958	0.9946	0.9936	0.9926	0.9918	0.9910	0.9904	0.9898	0.9893	0.9888	0.9884
0.9873	0.9874	0.9876	0.9878	0.9880	0.9883	0.9886	0.9890	0.9895	0.9900	0.9906	0.9913	0.9921
0.9986	0.9974	0.9962	0.9952	0.9942	0.9934	0.9926	0.9919	0.9913	0.9908	0.9903	0.9899	0.9895
0.9884	0.9884	0.9886	0.9887	0.9889	0.9891	0.9894	0.9897	0.9901	0.9906	0.9911	0.9917	0.9925
0.9988	0.9977	0.9967	0.9957	0.9949	0.9942	0.9935	0.9929	0.9923	0.9918	0.9914	0.9910	0.9907
0.9895	0.9896	0.9896	0.9897	0.9899	0.9901	0.9903	0.9906	0.9909	0.9913	0.9917	0.9923	0.9929
0.9990	0.9980	0.9971	0.9963	0.9956	0.9949	0.9944	0.9938	0.9933	0.9929	0.9925	0.9922	0.9919
0.9907	0.9907	0.9908	0.9909	0.9910	0.9911	0.9913	0.9915	0.9918	0.9921	0.9925	0.9929	0.9935
0.9991	0.9983	0.9976	0.9969	0.9963	0.9957	0.9952	0.9948	0.9944	0.9940	0.9937	0.9934	0.9931
0.9920	0.9920	0.9920	0.9921	0.9922	0.9922	0.9924	0.9925	0.9927	0.9930	0.9933	0.9937	0.9942
0.9993	0.9986	0.9981	0.9975	0.9970	0.9966	0.9962	0.9958	0.9955	0.9952	0.9949	0.9946	0.9944
0.9934	0.9934	0.9934	0.9934	0.9935	0.9935	0.9936	0.9937	0.9939	0.9940	0.9943	0.9946	0.9949
0.9995	0.9990	0.9985	0.9981	0.9978	0.9974	0.9971	0.9968	0.9966	0.9963	0.9961	0.9959	0.9958
0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9950	0.9950	0.9951	0.9953	0.9954	0.9956	0.9959
0.9996	0.9993	0.9990	0.9987	0.9985	0.9983	0.9981	0.9979	0.9977	0.9975	0.9974	0.9973	0.9971
0.9965	0.9965	0.9965	0.9964	0.9964	0.9964	0.9965	0.9965	0.9966	0.9966	0.9967	0.9969	0.9970
0.9998	0.9997	0.9995	0.9994	0.9992	0.9991	0.9990	0.9989	0.9988	0.9988	0.9987	0.9986	0.9985
0.9982	0.9982	0.9982	0.9981	0.9981	0.9981	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9984

迭代次数为: 4290
运行时间为: 4343ms

G-S

0.9973	0.9949	0.9928	0.9908	0.9891	0.9876	0.9862	0.9850	0.9840	0.9831	0.9824	0.9818	0.9813
0.9814	0.9818	0.9823	0.9828	0.9834	0.9841	0.9848	0.9856	0.9865	0.9874	0.9884	0.9896	0.9907
0.9975	0.9952	0.9931	0.9912	0.9896	0.9881	0.9868	0.9856	0.9847	0.9838	0.9831	0.9825	0.9820
0.9819	0.9823	0.9827	0.9832	0.9837	0.9844	0.9851	0.9858	0.9867	0.9876	0.9886	0.9896	0.9908
0.9976	0.9954	0.9934	0.9917	0.9901	0.9887	0.9874	0.9863	0.9854	0.9845	0.9838	0.9833	0.9828
0.9825	0.9828	0.9832	0.9836	0.9842	0.9847	0.9854	0.9861	0.9869	0.9878	0.9887	0.9897	0.9909
0.9977	0.9957	0.9938	0.9921	0.9906	0.9893	0.9881	0.9870	0.9861	0.9853	0.9846	0.9841	0.9836
0.9831	0.9834	0.9837	0.9842	0.9846	0.9852	0.9858	0.9864	0.9872	0.9880	0.9889	0.9899	0.9910
0.9979	0.9959	0.9942	0.9926	0.9912	0.9899	0.9887	0.9877	0.9869	0.9861	0.9855	0.9849	0.9845
0.9838	0.9841	0.9844	0.9847	0.9852	0.9856	0.9862	0.9868	0.9875	0.9883	0.9891	0.9901	0.9911
0.9980	0.9962	0.9946	0.9931	0.9917	0.9905	0.9895	0.9885	0.9877	0.9870	0.9863	0.9858	0.9854
0.9846	0.9848	0.9851	0.9854	0.9858	0.9862	0.9867	0.9873	0.9879	0.9886	0.9894	0.9903	0.9913
0.9982	0.9965	0.9950	0.9936	0.9923	0.9912	0.9902	0.9893	0.9885	0.9879	0.9873	0.9868	0.9863
0.9854	0.9856	0.9858	0.9861	0.9864	0.9868	0.9873	0.9878	0.9883	0.9890	0.9897	0.9906	0.9915
0.9983	0.9968	0.9954	0.9941	0.9929	0.9919	0.9910	0.9902	0.9894	0.9888	0.9882	0.9878	0.9874
0.9863	0.9865	0.9867	0.9869	0.9872	0.9875	0.9879	0.9883	0.9889	0.9895	0.9901	0.9909	0.9918
0.9985	0.9971	0.9958	0.9946	0.9936	0.9926	0.9918	0.9910	0.9904	0.9898	0.9893	0.9888	0.9884
0.9873	0.9874	0.9876	0.9878	0.9880	0.9883	0.9886	0.9890	0.9895	0.9900	0.9906	0.9913	0.9921
0.9986	0.9974	0.9962	0.9952	0.9942	0.9934	0.9926	0.9919	0.9913	0.9908	0.9903	0.9899	0.9895
0.9884	0.9884	0.9886	0.9887	0.9889	0.9891	0.9894	0.9897	0.9901	0.9906	0.9911	0.9917	0.9925
0.9988	0.9977	0.9967	0.9957	0.9949	0.9942	0.9935	0.9929	0.9923	0.9918	0.9914	0.9910	0.9907
0.9895	0.9896	0.9896	0.9897	0.9899	0.9901	0.9903	0.9906	0.9909	0.9913	0.9917	0.9923	0.9929
0.9990	0.9980	0.9971	0.9963	0.9956	0.9949	0.9944	0.9938	0.9933	0.9929	0.9925	0.9922	0.9919
0.9907	0.9907	0.9908	0.9909	0.9910	0.9911	0.9913	0.9915	0.9918	0.9921	0.9925	0.9929	0.9935
0.9991	0.9983	0.9976	0.9969	0.9963	0.9957	0.9952	0.9948	0.9944	0.9940	0.9937	0.9934	0.9931
0.9920	0.9920	0.9920	0.9921	0.9922	0.9922	0.9924	0.9925	0.9927	0.9930	0.9933	0.9937	0.9942
0.9993	0.9986	0.9981	0.9975	0.9970	0.9966	0.9962	0.9958	0.9955	0.9952	0.9949	0.9946	0.9944
0.9934	0.9934	0.9934	0.9934	0.9935	0.9935	0.9936	0.9937	0.9939	0.9940	0.9943	0.9946	0.9949
0.9995	0.9990	0.9985	0.9981	0.9978	0.9974	0.9971	0.9968	0.9966	0.9963	0.9961	0.9959	0.9958
0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9950	0.9950	0.9951	0.9953	0.9954	0.9956	0.9959
0.9996	0.9993	0.9990	0.9987	0.9985	0.9983	0.9981	0.9979	0.9977	0.9975	0.9974	0.9973	0.9971
0.9965	0.9965	0.9965	0.9964	0.9964	0.9964	0.9965	0.9965	0.9966	0.9966	0.9967	0.9969	0.9970
0.9998	0.9997	0.9995	0.9994	0.9992	0.9991	0.9990	0.9989	0.9988	0.9988	0.9987	0.9986	0.9985
0.9982	0.9982	0.9982	0.9981	0.9981	0.9981	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9984

迭代次数为: 2252
运行时间为: 2479ms

SOR

```
0.9825 0.9828 0.9832 0.9836 0.9842 0.9847 0.9854 0.9861 0.9869 0.9878 0.9887 0.9897 0.9909
0.9977 0.9957 0.9938 0.9921 0.9906 0.9893 0.9881 0.9870 0.9861 0.9853 0.9846 0.9841 0.9836
0.9831 0.9834 0.9837 0.9842 0.9846 0.9852 0.9858 0.9864 0.9872 0.9880 0.9889 0.9899 0.9910
0.9979 0.9959 0.9942 0.9926 0.9912 0.9899 0.9887 0.9877 0.9869 0.9861 0.9855 0.9849 0.9845
0.9838 0.9841 0.9844 0.9847 0.9852 0.9856 0.9862 0.9868 0.9875 0.9883 0.9891 0.9901 0.9911
0.9980 0.9962 0.9946 0.9931 0.9917 0.9905 0.9895 0.9885 0.9877 0.9870 0.9863 0.9858 0.9854
0.9846 0.9848 0.9851 0.9854 0.9858 0.9862 0.9867 0.9873 0.9879 0.9886 0.9894 0.9903 0.9913
0.9982 0.9965 0.9950 0.9936 0.9923 0.9912 0.9902 0.9893 0.9885 0.9879 0.9873 0.9868 0.9863
0.9854 0.9856 0.9858 0.9861 0.9864 0.9868 0.9873 0.9878 0.9883 0.9890 0.9897 0.9906 0.9915
0.9983 0.9968 0.9954 0.9941 0.9929 0.9919 0.9910 0.9902 0.9894 0.9888 0.9882 0.9878 0.9874
0.9863 0.9865 0.9867 0.9869 0.9872 0.9875 0.9879 0.9883 0.9889 0.9895 0.9901 0.9909 0.9918
0.9985 0.9971 0.9958 0.9946 0.9936 0.9926 0.9918 0.9910 0.9904 0.9898 0.9893 0.9888 0.9884
0.9873 0.9874 0.9876 0.9878 0.9880 0.9883 0.9886 0.9890 0.9895 0.9900 0.9906 0.9913 0.9921
0.9986 0.9974 0.9962 0.9952 0.9942 0.9934 0.9926 0.9919 0.9913 0.9908 0.9903 0.9899 0.9895
0.9884 0.9884 0.9886 0.9887 0.9889 0.9891 0.9894 0.9897 0.9901 0.9906 0.9911 0.9917 0.9925
0.9988 0.9977 0.9967 0.9957 0.9949 0.9942 0.9935 0.9929 0.9923 0.9918 0.9914 0.9910 0.9907
0.9895 0.9896 0.9896 0.9897 0.9899 0.9901 0.9903 0.9906 0.9909 0.9913 0.9917 0.9923 0.9929
0.9990 0.9980 0.9971 0.9963 0.9956 0.9949 0.9944 0.9938 0.9933 0.9929 0.9925 0.9922 0.9919
0.9907 0.9907 0.9908 0.9909 0.9910 0.9911 0.9913 0.9915 0.9918 0.9921 0.9925 0.9929 0.9935
0.9991 0.9983 0.9976 0.9969 0.9963 0.9957 0.9952 0.9948 0.9944 0.9940 0.9937 0.9934 0.9931
0.9920 0.9920 0.9920 0.9921 0.9922 0.9922 0.9924 0.9925 0.9927 0.9930 0.9933 0.9937 0.9942
0.9993 0.9986 0.9981 0.9975 0.9970 0.9966 0.9962 0.9958 0.9955 0.9952 0.9949 0.9946 0.9944
0.9934 0.9934 0.9934 0.9934 0.9935 0.9935 0.9936 0.9937 0.9939 0.9940 0.9943 0.9946 0.9949
0.9995 0.9990 0.9985 0.9981 0.9978 0.9974 0.9971 0.9968 0.9966 0.9963 0.9961 0.9959 0.9958
0.9949 0.9949 0.9949 0.9949 0.9949 0.9949 0.9950 0.9951 0.9951 0.9953 0.9954 0.9956 0.9959
0.9996 0.9993 0.9990 0.9987 0.9985 0.9983 0.9981 0.9979 0.9977 0.9975 0.9974 0.9973 0.9971
0.9965 0.9965 0.9965 0.9964 0.9964 0.9964 0.9965 0.9965 0.9966 0.9966 0.9967 0.9969 0.9970
0.9998 0.9997 0.9995 0.9994 0.9992 0.9991 0.9990 0.9989 0.9988 0.9988 0.9987 0.9986 0.9985
0.9982 0.9982 0.9982 0.9981 0.9981 0.9981 0.9981 0.9982 0.9982 0.9982 0.9982 0.9983 0.9984
```

松弛因子为: 1.85
迭代次数为: 145
运行时间为: 124ms

N=60

Jacobi

```
0.9902 0.9904 0.9906 0.9909 0.9912 0.9915 0.9919 0.9923 0.9928 0.9933 0.9938 0.9945 0.9952
0.9992 0.9985 0.9978 0.9971 0.9965 0.9959 0.9954 0.9949 0.9944 0.9940 0.9936 0.9932 0.9928
0.9906 0.9905 0.9904 0.9903 0.9902 0.9901 0.9900 0.9900 0.9899 0.9899 0.9899 0.9899 0.9899
0.9908 0.9909 0.9912 0.9914 0.9917 0.9920 0.9923 0.9927 0.9931 0.9936 0.9941 0.9947 0.9953
0.9993 0.9986 0.9980 0.9974 0.9968 0.9963 0.9958 0.9954 0.9949 0.9945 0.9942 0.9938 0.9935
0.9915 0.9913 0.9912 0.9911 0.9910 0.9909 0.9909 0.9908 0.9908 0.9908 0.9907 0.9907 0.9907
0.9914 0.9916 0.9918 0.9920 0.9922 0.9925 0.9928 0.9931 0.9935 0.9939 0.9944 0.9949 0.9956
0.9994 0.9988 0.9982 0.9977 0.9972 0.9967 0.9963 0.9959 0.9955 0.9951 0.9948 0.9945 0.9942
0.9923 0.9922 0.9921 0.9920 0.9919 0.9918 0.9918 0.9917 0.9917 0.9916 0.9916 0.9916 0.9916
0.9921 0.9922 0.9924 0.9926 0.9928 0.9930 0.9933 0.9936 0.9939 0.9943 0.9947 0.9952 0.9958
0.9994 0.9989 0.9984 0.9980 0.9975 0.9971 0.9967 0.9964 0.9960 0.9957 0.9954 0.9951 0.9949
0.9932 0.9931 0.9930 0.9929 0.9928 0.9927 0.9927 0.9926 0.9926 0.9925 0.9925 0.9925 0.9925
0.9929 0.9930 0.9931 0.9932 0.9934 0.9936 0.9938 0.9941 0.9944 0.9947 0.9951 0.9956 0.9961
0.9995 0.9991 0.9986 0.9982 0.9979 0.9975 0.9972 0.9969 0.9966 0.9963 0.9960 0.9958 0.9956
0.9941 0.9940 0.9939 0.9938 0.9938 0.9937 0.9936 0.9936 0.9935 0.9935 0.9934 0.9934 0.9934
0.9937 0.9938 0.9939 0.9940 0.9941 0.9943 0.9945 0.9947 0.9949 0.9952 0.9956 0.9960 0.9964
0.9996 0.9992 0.9989 0.9985 0.9982 0.9979 0.9976 0.9974 0.9971 0.9969 0.9967 0.9965 0.9963
0.9950 0.9950 0.9949 0.9948 0.9947 0.9947 0.9946 0.9945 0.9945 0.9945 0.9944 0.9944 0.9944
0.9945 0.9946 0.9947 0.9948 0.9949 0.9950 0.9952 0.9953 0.9956 0.9958 0.9961 0.9964 0.9968
0.9997 0.9994 0.9991 0.9988 0.9986 0.9983 0.9981 0.9979 0.9977 0.9975 0.9973 0.9972 0.9970
0.9960 0.9959 0.9958 0.9958 0.9957 0.9957 0.9956 0.9956 0.9955 0.9955 0.9955 0.9954 0.9954
0.9955 0.9955 0.9956 0.9956 0.9957 0.9958 0.9959 0.9961 0.9962 0.9964 0.9967 0.9969 0.9972
0.9998 0.9995 0.9993 0.9991 0.9989 0.9987 0.9986 0.9984 0.9983 0.9981 0.9980 0.9979 0.9977
0.9970 0.9969 0.9968 0.9968 0.9967 0.9967 0.9967 0.9966 0.9966 0.9966 0.9965 0.9965 0.9965
0.9965 0.9965 0.9966 0.9966 0.9967 0.9967 0.9967 0.9968 0.9969 0.9970 0.9972 0.9973 0.9978
0.9998 0.9997 0.9995 0.9994 0.9993 0.9992 0.9990 0.9989 0.9988 0.9987 0.9986 0.9985 0.9985
0.9980 0.9979 0.9979 0.9978 0.9978 0.9978 0.9977 0.9977 0.9977 0.9977 0.9976 0.9976 0.9976
0.9976 0.9976 0.9976 0.9976 0.9977 0.9977 0.9978 0.9978 0.9979 0.9980 0.9981 0.9982 0.9984
0.9999 0.9998 0.9998 0.9997 0.9996 0.9996 0.9995 0.9995 0.9994 0.9994 0.9993 0.9993 0.9992
0.9990 0.9989 0.9989 0.9989 0.9989 0.9989 0.9989 0.9989 0.9988 0.9988 0.9988 0.9988 0.9988
0.9987 0.9987 0.9988 0.9988 0.9988 0.9988 0.9988 0.9988 0.9989 0.9989 0.9990 0.9990 0.9991
```

迭代次数为: 9377
运行时间为: 24656ms

G-S

0.9906	0.9905	0.9904	0.9903	0.9902	0.9901	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9899
0.9908	0.9909	0.9912	0.9914	0.9917	0.9920	0.9923	0.9927	0.9931	0.9936	0.9941	0.9947	0.9953
0.9993	0.9986	0.9980	0.9974	0.9968	0.9963	0.9958	0.9954	0.9949	0.9945	0.9942	0.9938	0.9935
0.9915	0.9913	0.9912	0.9911	0.9910	0.9909	0.9909	0.9908	0.9908	0.9908	0.9907	0.9907	0.9907
0.9914	0.9916	0.9918	0.9920	0.9922	0.9925	0.9928	0.9931	0.9935	0.9939	0.9944	0.9949	0.9956
0.9994	0.9988	0.9982	0.9977	0.9972	0.9967	0.9963	0.9959	0.9955	0.9951	0.9948	0.9945	0.9942
0.9923	0.9922	0.9921	0.9920	0.9919	0.9918	0.9918	0.9917	0.9917	0.9916	0.9916	0.9916	0.9916
0.9921	0.9922	0.9924	0.9926	0.9928	0.9930	0.9933	0.9936	0.9939	0.9943	0.9947	0.9952	0.9958
0.9994	0.9989	0.9984	0.9980	0.9975	0.9971	0.9967	0.9964	0.9960	0.9957	0.9954	0.9951	0.9949
0.9932	0.9931	0.9930	0.9929	0.9928	0.9927	0.9927	0.9926	0.9926	0.9925	0.9925	0.9925	0.9925
0.9929	0.9930	0.9931	0.9932	0.9934	0.9936	0.9938	0.9941	0.9944	0.9947	0.9951	0.9956	0.9961
0.9995	0.9991	0.9986	0.9982	0.9979	0.9975	0.9972	0.9969	0.9966	0.9963	0.9960	0.9958	0.9956
0.9941	0.9940	0.9939	0.9938	0.9938	0.9937	0.9936	0.9936	0.9935	0.9935	0.9934	0.9934	0.9934
0.9937	0.9938	0.9939	0.9940	0.9941	0.9943	0.9945	0.9947	0.9949	0.9952	0.9956	0.9960	0.9964
0.9996	0.9992	0.9989	0.9985	0.9982	0.9979	0.9976	0.9974	0.9971	0.9969	0.9967	0.9965	0.9963
0.9950	0.9950	0.9949	0.9948	0.9947	0.9947	0.9946	0.9945	0.9945	0.9945	0.9944	0.9944	0.9944
0.9945	0.9946	0.9947	0.9948	0.9949	0.9950	0.9952	0.9953	0.9956	0.9958	0.9961	0.9964	0.9968
0.9997	0.9994	0.9991	0.9988	0.9986	0.9983	0.9979	0.9977	0.9977	0.9975	0.9973	0.9972	0.9970
0.9960	0.9959	0.9958	0.9958	0.9957	0.9957	0.9956	0.9956	0.9955	0.9955	0.9955	0.9954	0.9954
0.9955	0.9955	0.9956	0.9956	0.9957	0.9958	0.9959	0.9961	0.9962	0.9964	0.9967	0.9969	0.9972
0.9998	0.9995	0.9993	0.9991	0.9989	0.9987	0.9986	0.9984	0.9983	0.9981	0.9980	0.9979	0.9977
0.9970	0.9969	0.9968	0.9968	0.9967	0.9967	0.9967	0.9966	0.9966	0.9966	0.9965	0.9965	0.9965
0.9965	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968	0.9969	0.9970	0.9972	0.9973	0.9975	0.9978
0.9998	0.9997	0.9995	0.9994	0.9993	0.9992	0.9990	0.9989	0.9988	0.9987	0.9987	0.9986	0.9985
0.9980	0.9979	0.9979	0.9978	0.9978	0.9978	0.9977	0.9977	0.9977	0.9977	0.9976	0.9976	0.9976
0.9976	0.9976	0.9976	0.9976	0.9977	0.9977	0.9978	0.9978	0.9979	0.9980	0.9981	0.9982	0.9984
0.9999	0.9998	0.9998	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994	0.9994	0.9993	0.9993	0.9992
0.9990	0.9989	0.9989	0.9989	0.9989	0.9989	0.9989	0.9988	0.9988	0.9988	0.9988	0.9988	0.9988
0.9987	0.9987	0.9988	0.9988	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9991

迭代次数为：4928
运行时间为：8468ms

SOR

0.9906	0.9905	0.9904	0.9903	0.9902	0.9901	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9899
0.9908	0.9909	0.9912	0.9914	0.9917	0.9920	0.9923	0.9927	0.9931	0.9936	0.9941	0.9947	0.9953
0.9993	0.9986	0.9980	0.9974	0.9968	0.9963	0.9958	0.9954	0.9949	0.9945	0.9942	0.9938	0.9935
0.9915	0.9913	0.9912	0.9911	0.9910	0.9909	0.9909	0.9908	0.9908	0.9908	0.9907	0.9907	0.9907
0.9914	0.9916	0.9918	0.9920	0.9922	0.9925	0.9928	0.9931	0.9935	0.9939	0.9944	0.9949	0.9956
0.9994	0.9988	0.9982	0.9977	0.9972	0.9967	0.9963	0.9959	0.9955	0.9951	0.9948	0.9945	0.9942
0.9923	0.9922	0.9921	0.9920	0.9919	0.9918	0.9918	0.9917	0.9917	0.9916	0.9916	0.9916	0.9916
0.9921	0.9922	0.9924	0.9926	0.9928	0.9930	0.9933	0.9936	0.9939	0.9943	0.9947	0.9952	0.9958
0.9994	0.9989	0.9984	0.9980	0.9975	0.9971	0.9967	0.9964	0.9960	0.9957	0.9954	0.9951	0.9949
0.9932	0.9931	0.9930	0.9929	0.9928	0.9927	0.9927	0.9926	0.9926	0.9925	0.9925	0.9925	0.9925
0.9929	0.9930	0.9931	0.9932	0.9934	0.9936	0.9938	0.9941	0.9944	0.9947	0.9951	0.9956	0.9961
0.9995	0.9991	0.9986	0.9982	0.9979	0.9975	0.9972	0.9969	0.9966	0.9963	0.9960	0.9958	0.9956
0.9941	0.9940	0.9939	0.9938	0.9938	0.9937	0.9936	0.9936	0.9935	0.9935	0.9934	0.9934	0.9934
0.9937	0.9938	0.9939	0.9940	0.9941	0.9943	0.9945	0.9947	0.9949	0.9952	0.9956	0.9960	0.9964
0.9996	0.9992	0.9989	0.9985	0.9982	0.9979	0.9976	0.9974	0.9971	0.9969	0.9967	0.9965	0.9963
0.9950	0.9950	0.9949	0.9948	0.9947	0.9947	0.9946	0.9945	0.9945	0.9945	0.9944	0.9944	0.9944
0.9945	0.9946	0.9947	0.9948	0.9949	0.9950	0.9952	0.9953	0.9956	0.9958	0.9961	0.9964	0.9968
0.9997	0.9994	0.9991	0.9988	0.9986	0.9983	0.9981	0.9979	0.9977	0.9975	0.9973	0.9972	0.9970
0.9960	0.9959	0.9958	0.9958	0.9957	0.9957	0.9956	0.9956	0.9955	0.9955	0.9955	0.9954	0.9954
0.9955	0.9955	0.9956	0.9956	0.9957	0.9958	0.9959	0.9961	0.9962	0.9964	0.9967	0.9969	0.9972
0.9998	0.9995	0.9993	0.9991	0.9989	0.9987	0.9986	0.9984	0.9983	0.9981	0.9980	0.9979	0.9977
0.9970	0.9969	0.9968	0.9968	0.9967	0.9967	0.9967	0.9966	0.9966	0.9966	0.9965	0.9965	0.9965
0.9965	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968	0.9969	0.9970	0.9972	0.9973	0.9975	0.9978
0.9998	0.9997	0.9995	0.9994	0.9993	0.9992	0.9990	0.9989	0.9988	0.9987	0.9987	0.9986	0.9985
0.9980	0.9979	0.9979	0.9978	0.9978	0.9978	0.9977	0.9977	0.9977	0.9977	0.9976	0.9976	0.9976
0.9976	0.9976	0.9976	0.9976	0.9977	0.9977	0.9978	0.9978	0.9979	0.9980	0.9981	0.9982	0.9984
0.9999	0.9998	0.9998	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994	0.9994	0.9993	0.9993	0.9992
0.9990	0.9989	0.9989	0.9989	0.9989	0.9989	0.9989	0.9988	0.9988	0.9988	0.9988	0.9988	0.9988
0.9987	0.9987	0.9988	0.9988	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9991

松弛因子为：1.9
迭代次数为：212
运行时间为：532ms

C:\Users\郑涛\Desktop\数值代数\数值代数实验\实验四\Homework4\Debug\Homework4.exe (进程 42716) 已退出。
要在调试停止时自动关闭控制台，请启用“工具”->“选项”->“调试”->“调试停止时自动关闭控制台”。
按任意键关闭此窗口。...

4.2 结果分析

根据程序运行结果可以得到如下表格：

N	迭代法	迭代次数	运行时间 (ms)
20	Jacobi	1123	505
	G-S	590	346
	SOR(1.73)	71	18
40	Jacobi	4290	4343
	G-S	2252	2479
	SOR(1.85)	145	124
60	Jacobi	9377	24656
	G-S	4928	8468
	SOR(1.9)	212	532

从表格中可以看出，不管是迭代次数还是运行时间，最佳的 SOR 迭代法都优于 G-S 迭代法优于 Jacobi 迭代法