数值代数第七次实验报告

教师：邓建松

PB20061327 郑涛

7.1 问题描述

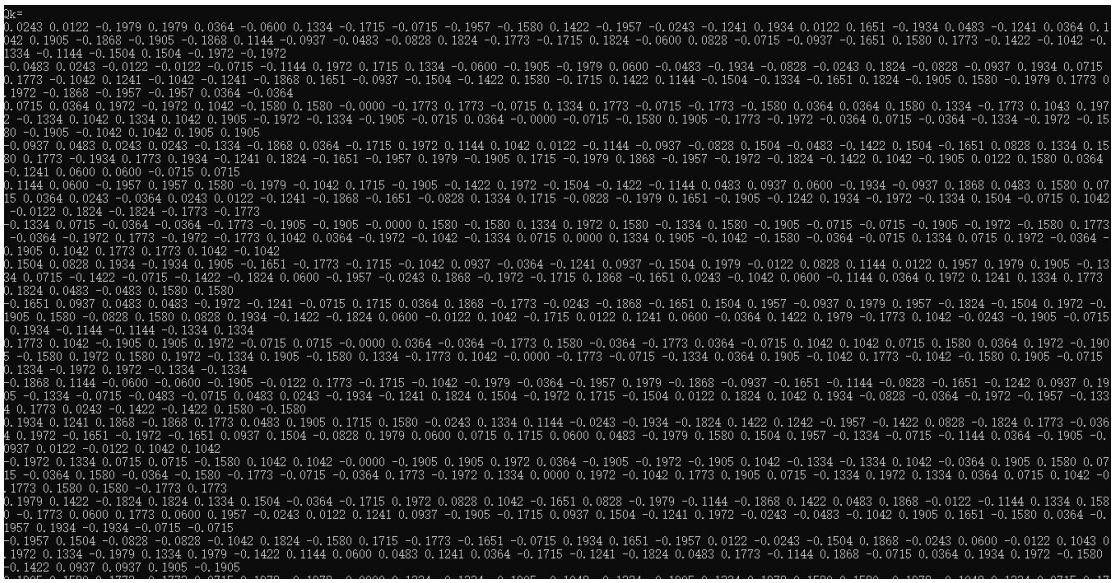
用 c++编制用过关 Jacobi 方法求实对称三对角阵的全部特征值和特征向量的通用子程序。

用该程序求下面矩阵（从 50 阶到 100 阶步长 10 阶）的全部特征值和特征向量：

$$\begin{pmatrix} 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & \dots & \dots & 0 & 0 \\ 0 & 0 & \dots & \dots & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

7.1 程序运行结果

比较多，就截了 Ak 和 Qk 的前面一部分



60--100 的升序特征值

60	70	80	90	100
2.00265	2.00196	2.0015	2.00119	2.00097
2.0106	2.00783	2.00601	2.00477	2.00387
2.02382	2.01759	2.01352	2.01072	2.0087
2.04229	2.03124	2.02402	2.01904	2.01546
2.06594	2.04875	2.03749	2.02972	2.02414
2.09473	2.07007	2.05391	2.04275	2.03473
2.12857	2.09517	2.07326	2.05812	2.04722
2.16737	2.124	2.0955	2.07579	2.0616
2.21103	2.1565	2.12061	2.09576	2.07786
2.25943	2.19261	2.14855	2.118	2.09597
2.31245	2.23226	2.17927	2.14249	2.11593
2.36994	2.27537	2.21273	2.16918	2.13771
2.43176	2.32186	2.24888	2.19806	2.16129
2.49774	2.37163	2.28767	2.22909	2.18665
2.5677	2.42458	2.32902	2.26222	2.21377
2.64145	2.48063	2.3729	2.29743	2.24261
2.71881	2.53964	2.41921	2.33467	2.27316
2.79957	2.60152	2.46791	2.37389	2.30537
2.88351	2.66613	2.51891	2.41505	2.33922
2.97041	2.73335	2.57214	2.45809	2.37468
3.06005	2.80306	2.62752	2.50298	2.41172
3.15217	2.8751	2.68496	2.54965	2.45029
3.24654	2.94935	2.74438	2.59804	2.49035
3.34292	3.02565	2.80568	2.64811	2.53188
3.44103	3.10387	2.86879	2.69979	2.57483
3.54062	3.18383	2.93359	2.75302	2.61916

3.64144	3.2654	3	2.80774	2.66482
3.7432	3.3484	3.06791	2.86387	2.71178
3.84565	3.43268	3.13723	2.92136	2.75998
3.9485	3.51806	3.20784	2.98013	2.80938
4.0515	3.6044	3.27964	3.04013	2.85994
4.15435	3.6915	3.35253	3.10126	2.91159
4.2568	3.77921	3.42639	3.16346	2.9643
4.35856	3.86735	3.50112	3.22667	3.01801
4.45938	3.95576	3.57659	3.29079	3.07267
4.55897	4.04424	3.6527	3.35576	3.12823
4.65708	4.13265	3.72934	3.4215	3.18463
4.75346	4.22079	3.80638	3.48792	3.24182
4.84783	4.3085	3.88371	3.55496	3.29975
4.93995	4.3956	3.96122	3.62252	3.35835
5.02959	4.48194	4.03878	3.69054	3.41757
5.11649	4.56732	4.11629	3.75893	3.47736
5.20043	4.6516	4.19362	3.8276	3.53765
5.28119	4.7346	4.27066	3.89648	3.59839
5.35855	4.81617	4.3473	3.96548	3.65951
5.4323	4.89613	4.42341	4.03452	3.72097
5.50226	4.97435	4.49888	4.10352	3.7827
5.56824	5.05065	4.57361	4.1724	3.84463
5.63006	5.1249	4.64747	4.24107	3.90672
5.68755	5.19694	4.72036	4.30946	3.9689
5.74057	5.26665	4.79216	4.37748	4.0311
5.78897	5.33387	4.86277	4.44504	4.09328
5.83263	5.39848	4.93209	4.51208	4.15537
5.87143	5.46036	5	4.5785	4.2173
5.90527	5.51937	5.06641	4.64424	4.27903
5.93406	5.57542	5.13121	4.70921	4.34049
5.95771	5.62837	5.19432	4.77333	4.40161
5.97618	5.67814	5.25562	4.83654	4.46235
5.9894	5.72463	5.31504	4.89874	4.52264
5.99735	5.76774	5.37248	4.95987	4.58243
	5.80739	5.42786	5.01987	4.64165
	5.8435	5.48109	5.07864	4.70025
	5.876	5.53209	5.13613	4.75818
	5.90483	5.58079	5.19226	4.81537
	5.92993	5.6271	5.24698	4.87177
	5.95125	5.67098	5.30021	4.92733
	5.96876	5.71233	5.35189	4.98199
	5.98241	5.75112	5.40196	5.0357
	5.99217	5.78727	5.45035	5.08841

	5.99804	5.82073	5.49702	5.14006
		5.85145	5.54191	5.19062
		5.87939	5.58495	5.24002
		5.9045	5.62611	5.28822
		5.92674	5.66533	5.33518
		5.94609	5.70257	5.38084
		5.96251	5.73778	5.42517
		5.97598	5.77091	5.46812
		5.98648	5.80194	5.50965
		5.99399	5.83082	5.54971
		5.9985	5.85751	5.58828
			5.882	5.62532
			5.90424	5.66078
			5.92421	5.69463
			5.94188	5.72684
			5.95725	5.75739
			5.97028	5.78623
			5.98096	5.81335
			5.98928	5.83871
			5.99523	5.86229
			5.99881	5.88407
				5.90403
				5.92214
				5.9384
				5.95278
				5.96527
				5.97586
				5.98454
				5.9913
				5.99613
				5.99903

7.1 结果分析

对于该矩阵阶数越高，最小特征值减小，最大特征值增大。

7.2 问题描述

用 c++编制用二分法求实对称三对角阵的指定特征值的程序。

用该程序求下面矩阵（100 阶）的最大最小特征值特征值并用反幂法求对应的特征向量：

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & \dots & \dots & 0 & 0 \\ 0 & 0 & \dots & \dots & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

7.2 程序运行结果

```
最小特征值:0.000967435  迭代次数:37  用时:5ms
特征向量:
0.0311036
0.0621772
0.0931905
0.124114
0.154917
0.18557
0.216044
0.246309
0.276335
0.306094
0.335557
0.364696
0.393481
0.421886
0.449883
0.477444
0.504544
0.531155
0.557253
0.582812
0.607806
0.632213
0.656008
0.679168
0.701672
0.723496
0.744621
0.765025
0.784689
0.803594
0.821722
0.839054
0.855575
0.871268
0.886119
0.900112
0.913234
0.925472
0.936816
0.947253
0.956773
0.965368
0.97303
0.979749
0.985521
0.99034
0.9942
0.997099
0.999033
1
1
0.999033
0.997099
0.9942
0.99034
0.985521
0.979749
0.97303
0.965368
0.956773
0.947253
0.936816
0.925472
0.913234
0.900112
0.886119
0.871268
0.855575
0.839054
0.821722
0.803594
0.784689
0.765025
0.744621
0.723496
0.701672
0.679168
0.656008
0.632213
0.607806
0.582812
0.557253
0.531155
0.504544
0.477444
0.449883
0.421886
0.393481
0.364696
0.335557
0.306094
0.276335
0.246309
0.216044
0.18557
0.154917
0.124114
0.0931905
0.0621772
0.0311036

最大特征值:3.99903  迭代次数:37  用时:5ms
特征向量:
-0.0311036
0.0621772
-0.0931905
0.124114
-0.154917
0.18557
-0.216044
0.246309
-0.276335
```

```
0.0621772
-0.0931905
0.124114
-0.154917
0.18557
-0.216044
0.246309
-0.276335
0.306094
-0.335557
0.364696
-0.393481
0.421886
-0.449883
0.477444
-0.504544
0.531155
-0.557253
0.582812
-0.607806
0.632213
-0.656008
0.679168
-0.701672
0.723496
-0.744621
0.765025
-0.784689
0.803594
-0.821722
0.839054
-0.855575
0.871268
-0.886119
0.900112
-0.913234
0.925472
-0.936816
0.947253
-0.956773
0.965368
-0.97303
0.979749
-0.985521
0.99034
-0.9942
0.997099
-0.999033
1
-1
0.999033
-0.997099
0.9942
-0.99034
0.985521
-0.979749
0.97303
-0.965368
0.956773
-0.947253
0.936816
-0.925472
0.913234
-0.900112
0.886119
-0.871268
0.855575
-0.839054
0.821722
-0.803594
0.784689
-0.765025
0.744621
-0.723496
0.701672
-0.679168
0.656008
-0.632213
0.607806
-0.582812
0.557253
-0.531155
0.504544
-0.477444
0.449883
-0.421886
0.393481
-0.364696
0.335557
-0.306094
0.276335
-0.246309
0.216044
-0.18557
0.154917
-0.124114
0.0931905
-0.0621772
0.0311036


C:\Users\郑涛\Desktop\数值代数\数值代数实验
要在调试停止时自动关闭控制台，请启用“工具”
按任意键关闭此窗口. . .
```

7.2 结果分析

最小特征值为 0.000967435，迭代 37 次，用时 5ms
最大特征值为 3.99903，迭代 37 次，用时 5ms