

Continuous 1

Task 1

$$\begin{aligned}xydx + (x + 1)dy &= 0 \\(x + 1)dy &= -xydx \\\frac{dy}{y} &= \frac{-xdx}{x + 1} \\\int \frac{dy}{y} &= \int \frac{-xdx}{x + 1} \\\ln y &= -(x - \ln(x + 1)) + C_0 \\\ln y &= \ln(x + 1) - x + C_0 \\y &= e^{\ln(x+1) - x + C_0} \\y &= C \frac{x + 1}{e^x}\end{aligned}$$

Specific cases:

$$\begin{array}{llll}x = -1 & -ydx = 0 & 0 = 0 \\y = 0 & (x + 1)dy = 0 & 0 = 0 & C \text{ can be zero}\end{array}$$

Answer:

$$\begin{aligned}y &= C \frac{x + 1}{e^x} \\x &= -1\end{aligned}$$

Task 2

$$\begin{aligned}\sqrt{y^2 + 1}dx &= xydy \\\frac{dx}{x} &= \frac{ydy}{\sqrt{y^2 + 1}} \\u = y^2 + 1 & \quad du = 2ydy \\\ln x &= \frac{1}{2} \int \frac{1}{\sqrt{u}} du \\\ln x &= \frac{1}{2} \cdot 2\sqrt{u} + C \\\ln x &= \sqrt{y^2 + 1} + C\end{aligned}$$

Specific cases:

$$x = 0 \quad \sqrt{y^2 + 1}dx = 0 \quad 0 = 0$$

Answer:

$$\begin{aligned}\ln x &= \sqrt{y^2 + 1} + C \\x &= 0\end{aligned}$$

Task bacteria

$$\begin{aligned}\dot{x} &= ax \\ \frac{dx}{dt} &= ax \\ \frac{dx}{x} &= a dt \\ \ln x &= at + c \\ x &= e^{at} e^c\end{aligned}$$

Initial conditions:

$$\begin{aligned}x(0) &= e^c \\ x(0) &= 500 \\ c &= \ln 500\end{aligned}$$

and

$$\begin{aligned}x(3) &= e^{3a} \cdot 500 \\ x(3) &= 8000 \\ e^{3a} &= \frac{8000}{500} = 16 \\ a &= \frac{1}{3} \ln 16 \\ e^a &= e^{1/3 \ln 16} = 16^{1/3}\end{aligned}$$

Solution:

$$x(4) = 500e^{4a} = 500(e^a)^4 = 500(16^{1/3})^4 = 500 \cdot 16^{\frac{4}{3}} \approx 20159$$

Task 3

$$\begin{aligned}(x + 2y)dx - xdy &= 0 \\ \frac{dy}{dx} &= \frac{x + 2y}{x} \\ \frac{dy}{dx} &= 1 + 2\frac{y}{x} \\ t &= \frac{1}{x}\end{aligned}$$

Task 4

$$(x - y)dx + (x + y)dy = 0$$