Continuous 1

Task 1

$$xy\mathrm{d}x + (x+1)\mathrm{d}y = 0$$
$$(x+1)\mathrm{d}y = -xy\mathrm{d}x$$
$$\frac{\mathrm{d}y}{y} = \frac{-x\mathrm{d}x}{x+1}$$
$$\int \frac{\mathrm{d}y}{y} = \int \frac{-x\mathrm{d}x}{x+1}$$
$$\ln y = -(x - \ln(x+1)) + C_0$$
$$\ln y = \ln(x+1) - x + C_0$$
$$y = \mathrm{e}^{\ln(x+1)-x+C_0}$$
$$y = C\frac{x+1}{\mathrm{e}^x}$$

Specific cases:

$$x = -1 \qquad -y\mathrm{d}x = 0 \qquad 0 = 0$$
$$y = 0 \qquad (x+1)\mathrm{d}y = 0 \qquad 0 = 0 \qquad C \text{ can be zero}$$

Answer:

$$y = C\frac{x+1}{\mathrm{e}^x}$$
$$x = -1$$

Task 2

$$\sqrt{y^2+1}\mathrm{d}x = xy\mathrm{d}y$$
$$\frac{\mathrm{d}x}{x} = \frac{y\mathrm{d}y}{\sqrt{y^2+1}}$$
$$u = y^2 + 1 \qquad \mathrm{d}u = 2y\mathrm{d}y$$
$$\ln x = \frac{1}{2}\int \frac{1}{\sqrt{u}}\mathrm{d}u$$
$$\ln x = \frac{1}{2}\cdot 2\sqrt{u} + C$$
$$\ln x = \sqrt{y^2+1} + C$$

Specific cases:

$$x = 0 \qquad \sqrt{y^2+1}\mathrm{d}x = 0 \qquad 0 = 0$$

Answer:

$$\ln x = \sqrt{y^2+1} + C$$
$$x = 0$$

Task bacteria

$$\dot{x} = ax$$
$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax$$
$$\frac{\mathrm{d}x}{x} = a\mathrm{d}t$$
$$\ln x = at + c$$
$$x = \mathrm{e}^{at}\mathrm{e}^{c}$$

Initial conditions:

$$x(0) = \mathrm{e}^{c}$$
$$x(0) = 500$$
$$c = \ln 500$$

and

$$x(3) = \mathrm{e}^{3a} \cdot 500$$
$$x(3) = 8000$$
$$\mathrm{e}^{3a} = \frac{8000}{500} = 16$$
$$a = \frac{1}{3}\ln 16$$
$$\mathrm{e}^{a} = \mathrm{e}^{1/3 \ln 16} = 16^{1/3}$$

Solution:

$$x(4) = 500\mathrm{e}^{4a} = 500(\mathrm{e}^{a})^{4} = 500(16^{1/3})^{4} = 500 \cdot 16^{\frac{4}{3}} \approx 20159$$

Task 3

$$(x + 2y)\mathrm{d}x - x\mathrm{d}y = 0$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x + 2y}{x}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 2\frac{y}{x}$$
$$t = \frac{1}{x}$$

Task 4

$$(x - y)\mathrm{d}x + (x + y)\mathrm{d}y = 0$$