# Assignment 3bis

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Find the solution of initial-value problem:

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = 2x - 5y + 3 & (1) \\ \frac{\mathrm{d}y}{\mathrm{d}t} = 5x - 6y + 1 & (2) \end{cases}$$

$$x(0) = 6 \qquad y(0) = 5$$

Let's start with equation $(2)$:

$$x = \frac{1}{5}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{6}{5}y - \frac{1}{5}$$

Go back to $(1)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{1}{5}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{6}{5}y - \frac{1}{5}\Big) = 2\Big(\frac{1}{5}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{6}{5}y - \frac{1}{5}\Big) - 5y + 3$$
$$\frac{1}{5}\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\mathrm{d}y}{\mathrm{d}t} + 6y - 1\Big) = \frac{2}{5}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{12}{5}y - \frac{2}{5} - 5y + 3$$
$$\frac{1}{5}\ddot{y} + \frac{6}{5}\dot{y} = \frac{2}{5}\dot{y} + \frac{12}{5}y - \frac{2}{5} - 5y + 3$$
$$\frac{1}{5}\ddot{y} + \frac{4}{5}\dot{y} + \frac{13}{5}y - \frac{13}{5} = 0$$
$$\ddot{y} + 4\dot{y} + 13y - 13 = 0$$

We received second-order DE:

$$\ddot{y} + 4\dot{y} + 13y - 13 = 0$$
$$y(t) = y_{gen}(t) + y_{par}(t)$$

Let's solve the homogeneous part:

$$\ddot{y} + 4\dot{y} + 13y = 0$$

The characteristic polynomial:

$$\lambda^2 + 4\lambda + 13 = 0$$
$$\lambda = -2 \pm 3i$$

Thus:

$$y_{gen} = \mathrm{e}^{-2t}\Big(C_1\cos(3t) + C_2\sin(3t)\Big)$$

Particular solution is linear:

$$y_{par} = \alpha t + \beta$$
$$\ddot{y} + 4\dot{y} + 13y = 13$$
$$0 + 4\alpha + 13\alpha t + 13\beta = 13$$
$$\alpha = 0 \qquad \beta = 1$$
$$y_{par} = 1$$

Thus:

$$y = \mathrm{e}^{-2t}\Big(C_1\cos(3t) + C_2\sin(3t)\Big) + 1$$

Go back:

$$x = \frac{1}{5}\Big(\frac{\mathrm{d}y}{\mathrm{d}t} + 6y - 1\Big)$$
$$5x = \frac{\mathrm{d}y}{\mathrm{d}t} + 6y - 1$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-2t}\Big((3C_2 - 2C_1)\cos(3t) - (3C_1 + 2C_2)\sin(3t)\Big)$$

Thus:

$$5x = \mathrm{e}^{-2t}\Big((3C_2 - 2C_1)\cos(3t) - (3C_1 + 2C_2)\sin(3t)\Big) +$$
$$+6\mathrm{e}^{-2t}\Big(C_1\cos(3t) + C_2\sin(3t)\Big) + 6 - 1$$
$$5x = \mathrm{e}^{-2t}\Big((3C_2 + 4C_1)\cos(3t) + (4C_2 - 3C_1)\sin(3t)\Big) + 5$$
$$x = \frac{1}{5}\mathrm{e}^{-2t}\Big((3C_2 + 4C_1)\cos(3t) + (4C_2 - 3C_1)\sin(3t)\Big) + 1$$

Initial value problem:

$$x(0) = \frac{1}{5}\cdot 1 \cdot \Big((3C_2 + 4C_1)\cos(0) + (4C_2 - 3C_1)\sin(0)\Big) + 1$$
$$x(0) = \frac{1}{5}\cdot(3C_2 + 4C_1) + 1$$
$$x(0) = 0.6C_2 + 0.8C_1 + 1$$

And:

$$y(0) = 1\cdot\Big(C_1\cos(0) + C_2\sin(0)\Big) + 1$$
$$y(0) = C_1 + 1 = 5$$
$$C_1 = 4$$
$$x(0) = 0.6C_2 + 0.8C_1 + 1 = 6$$
$$0.6C_2 + 4.2 = 6$$
$$6C_2 + 42 = 60$$
$$C_2 = 3$$

Thus, the solution to the initial-value problem is:

$$x = 5\mathrm{e}^{-2t}\cos(3t) + 1$$
$$y = \mathrm{e}^{-2t}\Big(4\cos(3t) + 3\sin(3t)\Big) + 1$$