Lecture Notes on

Unit 01 Differential Equations

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The contents of these pages constitute my authorized exam notes for Unit 01 of Differential Equations. This exam will take place at the Lake Worth campus under the supervision of Professor Tamara Johns on Wednesday, February 18, 2026 at 10:00 AM, in her physical office.

This is a retake opportunity granted after my initial Exam 01 Attempt 01 score of 25%, and these notes have been prepared to support a stronger performance on the retake. During the exam, I am permitted to reference these notes. The material included here is compiled directly from the following resource(s), which I consulted while preparing:

1. Lecture Recordings and Handouts from Prof. Johns
2. Differential Equations Textbook By Blanchard, Hall, and Devaney
3. Online notes by Professor Lebl
4. Houston Math's Youtube Channel

Contents

1 DEFINITIONS AND TERMINOLOGY, IVP, AND SLOPE FIELDS

1.1 Definitions and Terminology

1.2 Initial Value Problems

With initial value problems, we are given a differential equation, and then given a point in the form of y(x) = y to be plugged into the general solution (Once found) and then to be utilized to solve for the arbitrary constant(s).

Example Problems and Solutions From Houston Math

1. Answer the following parts corresponding to each differential equation below:

$$\frac{dy}{dx} = 2x \tag{1}$$
$$y' = 6x^2 + 4 \tag{2}$$
$$x^2 y' = -1 \tag{3}$$
$$y'' = xe^x \tag{4}$$

   (a) Find the **general solution**.
   (b) Find the **particular solution** given that $y(1) = 4$.
   (c) Find the **particular solution** given that $y(1) = 3$.

2. Answer the following parts corresponding to the **Second-Order Differential Equation**:

$$\frac{d^2y}{dt^2} = \cos(t) \tag{5}$$

   (a) Find the **particular solution** given that $y'(0) = 0$ and $y(0) = 1$.
   (b) Find the **particular solution** given that $y(1) = 3$.

1.3 (Lesson 2.1) - Solution Curves Without A Formula

2 SEPARABLE EQUATIONS

2.1 (2.2) - (General Overview) Separable Equations

If a **differential equation** can be written with all of the dependednt variable expressions on one side (usually y), and all of the independent variable expressions on the other side(usually x), then we say that the equation is **seprarable**.

The form we desire when identifying a **separable equation** is $\frac{dy}{g(y)} = f(x)\,d(x)$. If the form is not easy to spot on first glance, then this is the form that we wish to get our given differential equation into.

Something to note, is that we also want both sides to be integrable. These integrations or antiderivatives may differ by a constant at most, but what we really care about is that the right-side of the equation is integrable.

Small examples where the equations are already of the form $\boldsymbol{h(y)dy = g(x)dx}$

$$y^2\,dy = 4x\,dx \tag{6}$$

$$\frac{dy}{y} = te^t\,dt \tag{7}$$

$$\sec(t)\tan(t)\,dt = dx \tag{8}$$

$$\frac{x+1}{x-1}\,dx = \frac{dy}{y^2+1} \tag{9}$$

Here is an example where the equation is not in the form, but is separable (with a bit of work):

$$\frac{dy}{dx} - x = xy^2 \tag{10}$$

Here is an example of an equation that is **NOT** separable:

$$\frac{dy}{dx} - x = y^2 \tag{11}$$

The above **is not separable** because there is no way to create a product between x's and y's such that we attain the form $\boldsymbol{h(y)dy = g(x)dx}$ or $\boldsymbol{\frac{dy}{g(y)} = f(x)\,d(x)}$.

General Outline For Solving A Separable Equation:

1. Separate the variables, making sure each differential is on top
2. Integrate both sides **with respect to their particular variable**. For example:
   (a) In $\frac{dy}{dx}$, y is the dependent variable, as it is **dependent** on x.
   (b) In $\frac{dy}{dt}$, y is the dependent variable, as it is **dependent** on t.
   (c) In $\frac{dx}{dt}$, x is the dependent variable, as it is **dependent** on t.
3. Solve for the dependent variable, if reasonable.

Example Problems On Separable Equations From Houston Math

1. Solve the following separable differential equations:

$$y' = \frac{x}{y} \tag{12}$$

$$\frac{dy}{dx} - x = xy^2 \tag{13}$$

$$x^2 y' = -1 \tag{14}$$

$$y'' = xe^x \tag{15}$$

   (a) Find the **general solution**.
   (b) Find the **particular solution** given that $y(1) = 4$.
   (c) Find the **particular solution** given that $y(1) = 3$.

2. Answer the following parts corresponding to the **Second-Order Differential Equation**:

$$\frac{d^2y}{dt^2} = \cos(t) \tag{16}$$

   (a) Find the **particular solution** given that $y'(0) = 0$ and $y(0) = 1$.
   (b) Find the **particular solution** given that $y(1) = 3$.

2.2 Graphical Interpretations

2.3 Initial Value Problems (continued)

3 LINEAR EQUATIONS

3.1 (2.3) - (General Overview) Linear Equations

3.2 Integrating Factor Method

3.3 Interpret and Understand the Structure and Behavior of Solutions and Their Domain

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4.1 (2.4) - (General Overview) Exact Equations

4.2 Classifying and Solving Potential Functions

4.3 Solving Equations by Integration and Applying Initial Conditions

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5.2 Homogeneous Equations

5.3 Bernoulli Equations

5.4 Other substitutions and methods

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6.1 (3.1) - (General Overview) Modeling with 1st Order Differential Equations

6.2 Exponential Growth and Decay

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