$$\frac{\partial}{\partial\beta}\frac{1}{1-e^{-\beta\varepsilon}} = \frac{1}{(1-e^{-\beta\varepsilon})^2}$$

שאלה 1

(א

$$E = m_1\varepsilon_1 + m_2\varepsilon_2 + m_3\varepsilon_3$$

$$Z = \sum_s e^{-\beta E(s)} = \sum_{m_1}\sum_{m_2}\sum_{m_3} e^{-\beta(m_1\varepsilon_1+m_2\varepsilon_2+m_3\varepsilon_3)} =$$

(ב

$$= \sum_{m_1} e^{-\beta m_1\varepsilon_1}\cdot\sum_{m_2} e^{-\beta m_2\varepsilon_2}\cdot\sum_{m_3} e^{-\beta m_3\varepsilon_3} = \frac{1}{1-e^{-\beta\varepsilon_1}}\cdot\frac{1}{1-e^{-\beta\varepsilon_2}}\cdot\frac{1}{1-e^{-\beta\varepsilon_3}}$$

(ג

$$\langle E\rangle = -\frac{1}{Z}\cdot\frac{\partial Z}{\partial\beta} = \frac{\varepsilon_1 e^{-\beta\varepsilon_1}}{e^{-\beta\varepsilon_1}-1}+\frac{\varepsilon_2 e^{-\beta\varepsilon_2}}{e^{-\beta\varepsilon_2}-1}+\frac{\varepsilon_3 e^{-\beta\varepsilon_3}}{e^{-\beta\varepsilon_3}-1} = \sum_{i=1}^{3}\frac{\varepsilon_i}{e^{\beta\varepsilon_i}-1}$$

(ד

$$C = \frac{\partial\langle E\rangle}{\partial T} = \sum_{i=1}^{3}\frac{\varepsilon_i^2 e^{\frac{\varepsilon_i}{k_BT}}}{k_BT^2\left(e^{\frac{\varepsilon_i}{k_BT}}-1\right)^2}$$

(ה

$$\langle E\rangle \approx \frac{3\varepsilon}{e^{\beta\varepsilon}-1}$$

$T \gg \frac{\varepsilon}{k_B}$ - טמפ' גבוהות

$T \ll \frac{\varepsilon}{k_B}$ - טמפ' נמוכות

(ו

$\lim_{T\to 0} C = 0$ - מקיים את החוק ג' של תרמודינמיקה

$$\lim_{T\to\infty} C = 3k_B$$

שאלה 2

(א

$$Z = \int_{-\infty}^{\infty} g(V_x)\,dV_x = \int_{-\infty}^{\infty} e^{-\frac{mV_x^2}{2k_BT}}\,dV = \boxed{\sqrt{\frac{2\pi k_BT}{m}} = Z}$$

(ב

$$\langle V_x\rangle = \int_{-\infty}^{\infty} \overset{\text{אי זוגי}}{V_x}\cdot \underset{\text{זוגי}}{g(V_x)}\,dV_x = 0$$

$$\langle |V_x|\rangle = 2\int_0^{\infty} V_x\cdot g(V_x)\,dV_x = 2\sqrt{\frac{m}{2\pi k_BT}}\int_0^{\infty} V_x e^{-\frac{mV_x^2}{2k_BT}}\,dV_x = \left(\frac{m}{2\pi k_BT}\right)^{\frac{1}{2}}\cdot 1\cdot\left(\frac{m}{2k_BT}\right)^{-1} =$$

$$\boxed{\langle |V_x|\rangle = \sqrt{\frac{2k_BT}{\pi m}}}$$

$$\langle V_x^2\rangle = \left(\frac{m}{2\pi k_BT}\right)^{\frac{1}{2}}\cdot\frac{\sqrt{\pi}}{2}\cdot\left(\frac{m}{2k_BT}\right)^{-\frac{3}{2}} = \frac{k_BT}{m}$$

(ג

$$f(v) = \left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2k_BT}}\, v^2\sin\theta$$

$\langle v\rangle = \iiint f(v)\, v\, dv\, d\theta\, d\phi = \int f(v)\, v\, dv \cdot 4\pi$

$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} \int_0^{\infty} v^3 e^{-\frac{mv^2}{2k_BT}} dv = 2\pi \left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} \cdot 1 \cdot \left(\frac{m}{2k_BT}\right)^{-2} = 2\sqrt{\frac{2k_BT}{m\pi}}$

$\langle v^2\rangle = \ldots = 4\pi \frac{1}{\pi}\left(\frac{m}{2k_BT}\right)^{\frac{3}{2}} \int_0^{\infty} v^4 e^{-\frac{mv^2}{2k_BT}} dv = 2\frac{1}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{\frac{3}{2}} \frac{3\sqrt{\pi}}{4}\left(\frac{m}{2k_BT}\right)^{-\frac{5}{2}}$

$\langle v^2\rangle = \frac{3k_BT}{m}$

ה) $\langle E\rangle = \langle \frac{mv^2}{2}\rangle = \frac{m\langle v^2\rangle}{2} = \frac{3}{2}k_BT$

$\Downarrow$

$\langle v^2\rangle = \frac{3k_BT}{m}$

1) + 5) מה???

3 חלק

א) על כל אחד מ־4 דרגות החופש יש אנרגיה $\frac{1}{2}k_BT$ לפי משפט ה־4 דרגות נוספות

ב) $E = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{k(x_1-x_a)^2}{2} + \frac{k(x_2-x_b)^2}{2}$

$x^2+bx+c + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

$x^2+bx+\left(\frac{b}{2}\right)^2$

$\left(x+\frac{b}{2}\right)^2$

$Z = \iiiint_{-\infty}^{\infty} e^{-\beta E(p_1,p_2,x_1,x_2)} dp_1\, dp_2\, dx_1\, dx_2 =$

$= \int_{-\infty}^{\infty} e^{-\frac{\beta p_1^2}{2m_1}} dp_1 \int_{-\infty}^{\infty} e^{-\frac{\beta p_2^2}{2m_2}} dp_2 \int_{-\infty}^{\infty} e^{-\frac{\beta k(x_1-x_a)^2}{2}} dx_1 \cdot \int_{-\infty}^{\infty} e^{-\frac{\beta k(x_2-x_b)^2}{2}} =$

$= \sqrt{\frac{2m_1\pi}{\beta}} \cdot \sqrt{\frac{2m_2\pi}{\beta}} \cdot \sqrt{\frac{2\pi}{\beta k}} \cdot \sqrt{\frac{2\pi}{\beta k}} = \boxed{\frac{4\pi^2}{\beta^2}\frac{\sqrt{m_1 m_2}}{k} = Z}$

ג) $\langle E\rangle = -\frac{1}{Z}\frac{dZ}{d\beta} = -\beta^2 \cdot \frac{-2}{\beta^3} = \frac{2}{\beta} = 2k_BT = 4\cdot\frac{1}{2}k_BT$

כי דרגות חופש של דרגות חופש נותנות $\frac{1}{2}k_BT$

ד) $P(p_1,p_2,x_1,x_2) = \frac{e^{-\beta E(p_1,p_2,x_1,x_2)}}{Z} = \frac{\beta^2 k}{4\pi^2\sqrt{m_1m_2}} \exp\left[-\beta\left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{k(x_1-x_a)^2}{2} + \frac{k(x_2-x_b)^2}{2}\right)\right]$

$P(x_1,x_2) = \iint_{-\infty}^{\infty} P(p_1,p_2,x_1,x_2)\, dp_1\, dp_2 = \frac{\beta k}{2\pi} \exp\left[-\beta\left(\frac{k(x_1-x_a)^2}{2} + \frac{k(x_2-x_b)^2}{2}\right)\right]$

ה) $P(x_1) = \sqrt{\frac{\beta k}{2\pi}} \exp\left(-\beta \frac{k(x_1-x_a)^2}{2}\right)$

$P(x_2) = \sqrt{\frac{\beta k}{2\pi}} \exp\left(-\beta \frac{k(x_2-x_b)^2}{2}\right)$

ו) $E(|x_1-x_2|) = |E(x_1-x_2)| = |Ex_1 - Ex_2| = |x_a - x_b|$

ז) $E_{new} = E - qEx_1 + qEx_2$

אנרגיה / שדה

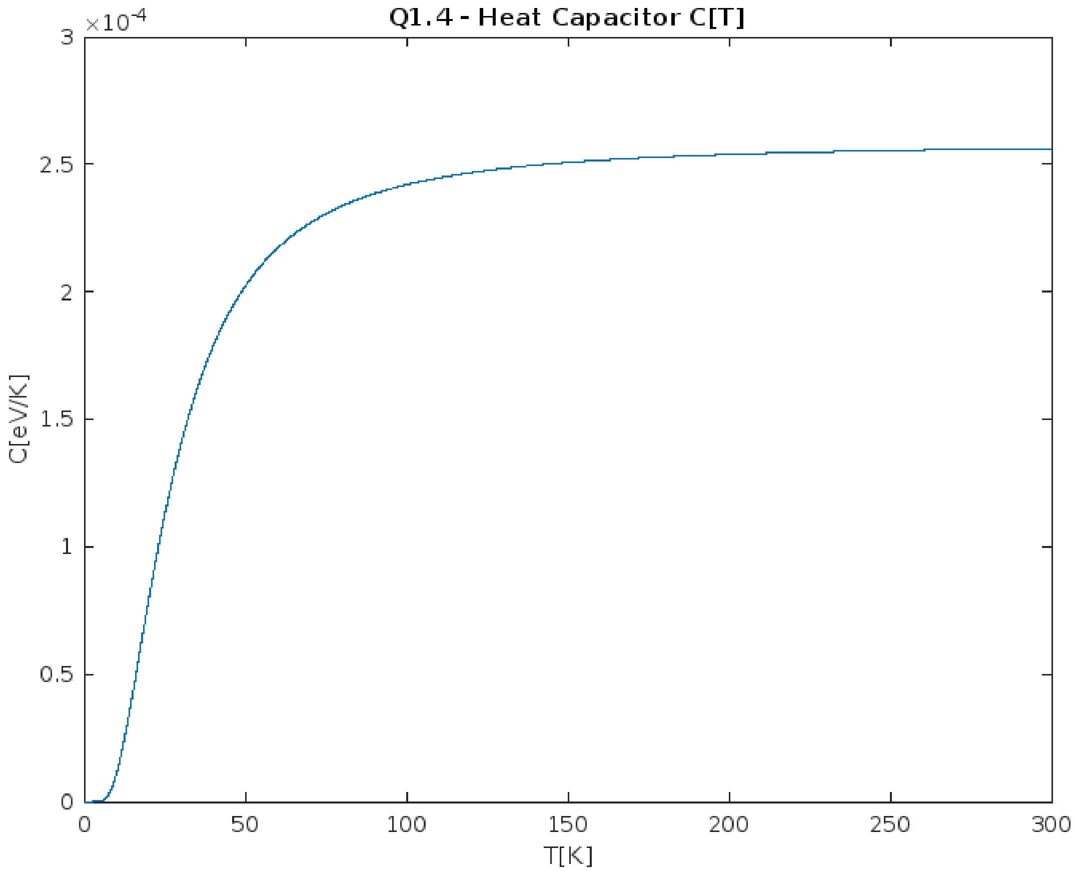
$Z = \frac{2\pi}{\beta}\sqrt{m_1m_2} \int_{-\infty}^{\infty} \exp\left[-\beta\left(\frac{k(x_1-x_a)^2}{2} - qEx_1\right)\right] dx_1 \cdot \int_{-\infty} \exp\left[-\beta\left(\frac{k(x_2-x_b)^2}{2}\right)\right] dx_2$

$|E[q(x_1-x_2)]| = \ldots = q|x_a - x_b|$

```
k = 8.6e-5;
e1 = 5e-3;
e2 = 7e-3;
e3 = 10e-3;

T = linspace(0, 300, 10000);
E1 = e1 .* e1 .*exp(-e1./(k.*T))./ ((1-exp(- e1./(k.*T))).^2);
E2 = e2 .* e2 .*exp(-e2./(k.*T))./ ((1-exp(- e2./(k.*T))).^2);
E3 = e3 .* e3 .*exp(-e3./(k.*T))./ ((1-exp(- e3./(k.*T))).^2);
f = (1./(k .* (T.^2))) .* (E1 + E2 + E3);

figure(1);
plot(T, f);
title("Q1.4 - Heat Capacitor C[T]")
xlabel('T[K]');
ylabel('C[eV/K]')
```



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