HW06
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209517697 - דן חוזה

1. מודל Drude, מוליכות AC

1.1 סעיף א:

$$\vec{E} = E_0 e^{-j\omega t}$$
$$\frac{d}{dt} < \vec{P}(t) > + \frac{1}{\tau} < \vec{P}(t) > = f(t)$$
$$f(t) = -e \cdot \vec{E}$$
$$< \dot{\vec{P}}(t) > + \frac{1}{\tau} < \vec{P}(t) > = -e \cdot \vec{E}$$

1.2 סעיף ב:

ננחש פתרון מהצורה $P(t) = P(\omega)e^{-j\omega t}$

$$\frac{d}{dt}P(\omega)e^{-j\omega t} + \frac{1}{\tau}P(\omega)e^{-j\omega t} = -e \cdot \vec{E}$$
$$-j\omega P(\omega)e^{-j\omega t} + \frac{1}{\tau}P(\omega)e^{-j\omega t} = -e \cdot \vec{E} \qquad \cdot e^{j\omega t}$$
$$-j\omega P(\omega) + \frac{1}{\tau}P(\omega) = -eE_0$$
$$\left(-j\omega + \frac{1}{\tau}\right)P(\omega) = -eE_0$$
$$P(\omega) = \frac{\tau e E_0}{j\omega\tau - 1}$$
$$P(t) = \frac{\tau e E_0}{j\omega\tau - 1}e^{-j\omega t}$$

1.3 סעיף ג:

מהירות סחיפה: $\overrightarrow{V_{drift}} = \frac{< \vec{P}(t) >}{m_0}$.

$$\vec{J} = -e \cdot n \cdot \overrightarrow{V_{drift}} = \frac{\frac{n}{m_0}\tau e^2 E_0 e^{-j\omega t}}{1 - j\omega\tau}$$

1.4 סעיף ד:

חוק אום המיקרוסקופי: $\vec{J} = \sigma_{AC} \cdot \vec{E}$

$$\sigma_{AC} = \frac{\frac{n}{m_0}\tau e^2}{1 - j\omega\tau}$$
$$\omega = 0 \rightarrow \sigma_{DC} = \frac{n}{m_0}\tau e^2$$

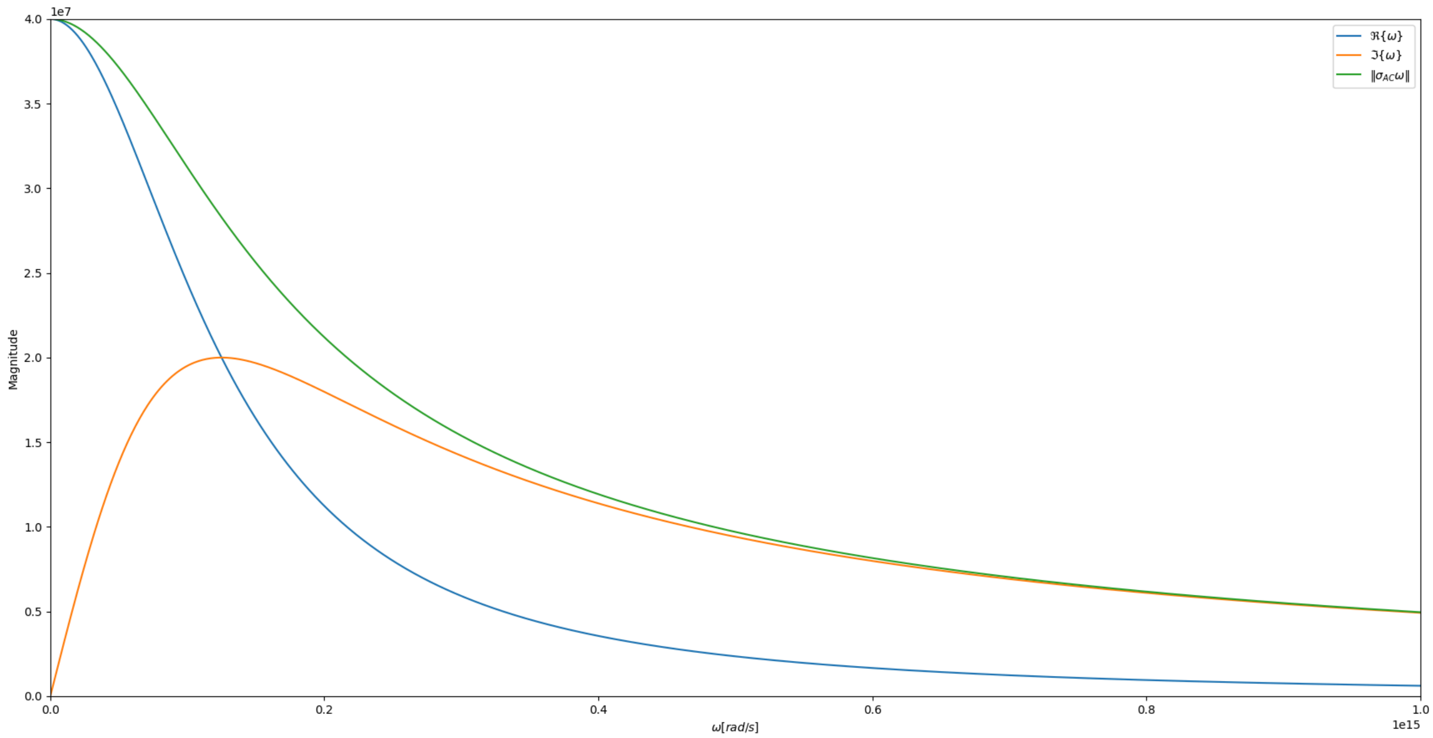
1.5 סעיף ה:

נתון:

$$\begin{aligned}\tau &= 8\cdot 10^{-15}\\ \sigma_{DC} &= 4\cdot 10^{7}\\ \sigma_{AC} &= \frac{\sigma_{DC}}{1-j\omega\tau} = \frac{\sigma_{DC}(1+j\omega\tau)}{1^2+\omega^2\tau^2}\end{aligned}$$

לכן:

$$\begin{aligned}\Re(\omega) &= \frac{\sigma_{DC}}{1+\omega^2\tau^2}\\ \Im(\omega) &= \frac{\sigma_{DC}\omega\tau}{1+\omega^2\tau^2}\\ \|\sigma_{AC}(\omega)\| &= \frac{\sigma_{DC}}{1+\omega^2\tau^2}\sqrt{1+\omega^2\tau^2}\end{aligned}$$



2. צפיפות מצבים

2.1. סעיף א:

נתון: $E=\frac{\hbar^2k^2}{2m}$ לכן:

$$k=\sqrt{\frac{2mE}{\hbar^2}}$$

במקרה 1D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi}{L}\\
V_{line}^{k} &= 2k\\
N(k) &= 2\times\frac{1}{2}\frac{V_{line}^{k}}{V_{single\ state}}=\frac{2kL}{\pi}\\
N(E) &= \frac{2L}{\pi}\sqrt{\frac{2mE}{\hbar^2}}\\
G(E) &= \frac{2}{\pi}\sqrt{\frac{2mE}{\hbar^2}}\\
g(E) &= \frac{dG(E)}{dE}=\frac{1}{\pi}\sqrt{\frac{2m}{E\hbar^2}}
\end{aligned}$$

במקרה 2D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi^2}{A}\\
V_{circle}^{k} &= \pi k^2\\
N(k) &= 2\times\frac{1}{4}\frac{V_{circle}^{k}}{V_{single\ state}}=\frac{k^2A}{2\pi}\\
N(E) &= \frac{A}{2\pi}\cdot\frac{2mE}{\hbar^2}\\
G(E) &= \frac{1}{2\pi}\cdot\frac{2mE}{\hbar^2}\\
g(E) &= \frac{dG(E)}{dE}=\frac{m}{\pi\hbar^2}
\end{aligned}$$

במקרה 3D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi^3}{V}\\
V_{sphere}^{k} &= \frac{4}{3}\pi k^3\\
N(k) &= 2\times\frac{1}{8}\frac{V_{sphere}^{k}}{V_{single\ state}}=\frac{Vk^3}{3\pi^2}\\
N(E) &= \frac{V}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}}=\frac{V}{3\pi^2\hbar^3}\cdot(2mE)^{\frac{3}{2}}
\end{aligned}$$

$$G(E) = \frac{1}{3\pi^2\hbar^3} \cdot (2mE)^{\frac{3}{2}}$$

$$g(E) = \frac{dG(E)}{dE} = \frac{m\sqrt{2m}}{\pi^2\hbar^3}\sqrt{E}$$

2.2. סעיף ב:

נתון: $E(k) = E_g + \frac{\hbar^2 k^2}{2m^*}$ לכן:

$$k = \sqrt{\frac{2m(E-E_g)}{\hbar^2}}$$

עבור 3D:

$$V_{single\ state} = \frac{\pi^3}{V}$$

$$V_{sphere}^k = \frac{4}{3}\pi k^3$$

$$N(k) = 2 \times \frac{1}{8}\frac{V_{sphere}^k}{V_{single\ state}} = \frac{Vk^3}{3\pi^2}$$

$$N(E) = \frac{V}{3\pi^2}\left(\frac{2m^*(E-E_g)}{\hbar^2}\right)^{\frac{3}{2}} = \frac{V}{3\pi^2\hbar^3} \cdot \left(2m^*(E-E_g)\right)^{\frac{3}{2}}$$

$$G(E) = \frac{1}{3\pi^2\hbar^3} \cdot \left(2m^*(E-E_g)\right)^{\frac{3}{2}}$$

$$g(E) = \frac{dG(E)}{dE} = \frac{m^*\sqrt{2m^*}}{\pi^2\hbar^3}\sqrt{E-E_g}$$

ותחום האנרגיות הוא: $E > E_g$.

2.3. סעיף ג:

$$E(\vec{k}) = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y}$$

$$1 = \frac{k_x^2}{\underbrace{\frac{2m_xE}{\hbar^2}}_{a^2}} + \frac{k_y^2}{\underbrace{\frac{2m_yE}{\hbar^2}}_{b^2}}$$

$$V^k = \pi \cdot a \cdot b = \pi \cdot \sqrt{\frac{2m_xE}{\hbar^2}} \cdot \sqrt{\frac{2m_xE}{\hbar^2}} = \frac{2\pi E}{\hbar^2}\sqrt{m_x m_y}$$

$$V_{single\ state} = \frac{\pi^2}{L^2}$$

$$N(E) = 2 \times \frac{1}{4}\frac{V^k}{V_{single\ state}} = \frac{L^2 E\sqrt{m_x m_y}}{\pi k^2}$$

$$
\begin{aligned}
G(E) &= \frac{N(E)}{L^2} = \frac{E\sqrt{m_x m_y}}{\pi\hbar^2} \\
g(E) &= \frac{dG(E)}{dE} = \frac{\sqrt{m_x m_y}}{\pi\hbar^2}
\end{aligned}
$$

Code for Plot .3

```python
from sympy import *
from sympy.abc import *
from numpy import *
import matplotlib.pyplot as plt

# Define the constants
tau = 8e-15
sigma = 4e7
omega = linspace(0,1e15,1000)

# Define the functions

RealOmega = sigma/(1+omega**2*tau**2)
ImagOmega = sigma*tau*omega/(1+omega**2*tau**2)
AbsOmega = sigma*sqrt(1+omega**2*tau**2)/(1+omega**2*tau**2)

# Plot the functions, label in latex
fig, ax = plt.subplots()
plt.plot(omega, RealOmega, label=r'$\Re\{\omega\}$')
plt.plot(omega, ImagOmega, label=r'$\Im\{\omega\}$')
plt.plot(omega, AbsOmega, label=r'$\Vert\sigma_{AC}\omega\Vert$')
plt.xlabel(r'$\omega [rad/s]$')
plt.ylabel('Magnitude')
plt.legend()
# Remove unnecessary white space
ax.autoscale()
ax.margins(0)  # Set margins to zero
plt.tight_layout()

plt.show()


```