$\frac{\partial}{\partial\beta}\frac{1}{1-e^{-\beta\epsilon}} = \frac{1}{(1-e^{-\beta\epsilon})^2}$

שאלה 1

א) $E = n_1\epsilon_1 + n_2\epsilon_2 + n_3\epsilon_3$

$$Z = \sum_s e^{-\beta E(s)} = \sum_{n_1}\sum_{n_2}\sum_{n_3} e^{-\beta(n_1\epsilon_1+n_2\epsilon_2+n_3\epsilon_3)} =$$

ב)

$$= \sum_{n_1} e^{-\beta n_1\epsilon_1}\cdot\sum_{n_2} e^{-\beta n_2\epsilon_2}\cdot\sum_{n_3} e^{-\beta n_3\epsilon_3} = \frac{1}{1-e^{-\beta\epsilon_1}}\cdot\frac{1}{1-e^{-\beta\epsilon_2}}\cdot\frac{1}{1-e^{-\beta\epsilon_3}}$$

ג)

$$\langle E\rangle = -\frac{1}{Z}\cdot\frac{\partial Z}{\partial\beta} = \frac{\epsilon_1 e^{-\beta\epsilon_1}}{e^{-\beta\epsilon_1}-1} + \frac{\epsilon_2 e^{-\beta\epsilon_2}}{e^{-\beta\epsilon_2}-1} + \frac{\epsilon_3 e^{-\beta\epsilon_3}}{e^{-\beta\epsilon_3}-1} = \sum_{i=1}^{3}\frac{\epsilon_i}{e^{\beta\epsilon_i}-1}$$

ד)

$$C = \frac{\partial\langle E\rangle}{\partial T} = \sum_{i=1}^{3}\frac{\epsilon_i^2 e^{\frac{\epsilon_i}{k_BT}}}{k_BT^2\left(e^{\frac{\epsilon_i}{k_BT}}-1\right)^2}$$

ה)

$$\langle E\rangle \approx \frac{3\epsilon}{e^{\beta\epsilon}-1}$$

$T \gg \frac{\epsilon}{k_B}$ - טמפ' גבוהות

$T \ll \frac{\epsilon}{k_B}$ - טמפ' נמוכות

ו) $\lim_{T\to0} C = 0$ - מקיים את החוק השלישי ה תרמודינמיקה

$\lim_{T\to\infty} C = 3k_B$

שאלה 2

א)

$$Z = \int_{-\infty}^{\infty} g(V_x)\,dV_x = \int_{-\infty}^{\infty} e^{-\frac{mV_x^2}{2k_BT}}\,dV = \boxed{\sqrt{\frac{2\pi k_BT}{m}} = Z}$$

ב)

$$\langle V_x\rangle = \int_{-\infty}^{\infty} \overset{\text{פ' זוגית}}{V_x}\cdot \underset{\text{זוגי}}{g(V_x)}\,dV_x = 0$$

$$\langle |V_x|\rangle = 2\int_0^{\infty} V_x\cdot g(V_x)\,dV_x = 2\sqrt{\frac{m}{2\pi k_BT}}\int_0^{\infty} V_x e^{-\frac{mV_x^2}{2k_BT}}\,dV_x = \left(\frac{m}{2\pi k_BT}\right)^{\frac{1}{2}}\cdot 1\cdot\left(\frac{m}{2k_BT}\right)^{-1} =$$

$$\boxed{\langle |V_x|\rangle = \sqrt{\frac{2k_BT}{\pi m}}}$$

$$\langle V_x^2\rangle = \left(\frac{m}{2\pi k_BT}\right)^{\frac{1}{2}}\cdot\frac{\sqrt{\pi}}{2}\cdot\left(\frac{m}{2k_BT}\right)^{-\frac{3}{2}} = \frac{k_BT}{m}$$

ג)

$$f(V) = \left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} e^{-\frac{mV^2}{2k_BT}}\cdot V^2\sin\theta$$

$\langle v\rangle = \iiint f(v)\, v\, dv\, d\theta\, d\phi = \int f(v)\, dv \cdot 4\pi$

$= 4\pi \left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} \int_0^\infty v^3 e^{-\frac{mv^2}{2k_BT}} dv = 2\pi \left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} \cdot 1 \cdot \left(\frac{m}{2k_BT}\right)^{-2} = 2\sqrt{\frac{2k_BT}{m\pi}}$

$\langle v^2\rangle = \ldots = 4\pi \frac{1}{\pi}\left(\frac{m}{2k_BT}\right)^{\frac{3}{2}} \int_0^\infty v^4 e^{-\frac{mv^2}{2k_BT}} dv = \frac{2}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{\frac{3}{2}} \frac{3\sqrt{\pi}}{4}\left(\frac{m}{2k_BT}\right)^{-\frac{5}{2}}$

$\langle v^2\rangle = \frac{3k_BT}{m}$

ג) $\langle E\rangle = \langle \frac{mv^2}{2}\rangle = \frac{m\langle v^2\rangle}{2} = \frac{3}{2}k_BT$

$\Downarrow$

$\langle v^2\rangle = \frac{3k_BT}{m}$

1) ד+ק) מה???

ג) ה לא

$x^2+bx+c+\left(\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$

$x^2+bx+\left(\frac{b}{2}\right)^2$

$\left(x+\frac{b}{2}\right)^2$

א) כיוון [illegible] 2 דרגות חופש ב/ לכן יש 4 דרגות נוספות

ב) $E = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{k(x_1-x_a)^2}{2} + \frac{k(x_2-x_b)^2}{2}$

$Z = \iiiint_{-\infty}^{\infty} e^{-\beta E(p_1,p_2,x_1,x_2)} dp_1\, dp_2\, dx_1\, dx_2 =$

$= \int_{-\infty}^{\infty} e^{-\frac{\beta p_1^2}{2m_1}} dp_1 \int_{-\infty}^{\infty} e^{-\frac{\beta p_2^2}{2m_2}} dp_2 \int_{-\infty}^{\infty} e^{-\frac{\beta k(x_1-x_a)^2}{2}} dx_1 \cdot \int_{-\infty}^{\infty} e^{-\frac{\beta k(x_2-x_b)^2}{2}} =$

$= \sqrt{\frac{2m_1\pi}{\beta}} \cdot \sqrt{\frac{2m_2\pi}{\beta}} \cdot \sqrt{\frac{2\pi}{\beta k}} \cdot \sqrt{\frac{2\pi}{\beta k}} = \boxed{\frac{4\pi^2}{\beta^2}\frac{\sqrt{m_1m_2}}{k} = Z}$

ג) $\langle E\rangle = -\frac{1}{Z}\frac{dZ}{d\beta} = -\beta^2 \frac{-2}{\beta^3} = \frac{2}{\beta} = 2k_BT = 4\cdot\frac{1}{2}k_BT$

4 דרגות חופש, לכל דרגת חופש תרומה $\frac{1}{2}k_BT$

ד) $P(p_1,p_2,x_1,x_2) = \frac{e^{-\beta E(p_1,p_2,x_1,x_2)}}{Z} = \frac{\beta^2 k}{4\pi^2\sqrt{m_1m_2}} \exp\left[-\beta\left(\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{k(x_1-x_a)^2}{2}+\frac{k(x_2-x_b)^2}{2}\right)\right]$

$P(x_1,x_2) = \iint_{-\infty}^{\infty} P(p_1,p_2,x_1,x_2)\,dp_1\,dp_2 = \frac{\beta k}{2\pi}\exp\left[-\beta\left(\frac{k(x_1-x_a)^2}{2}+\frac{k(x_2-x_b)^2}{2}\right)\right]$

$P(x_1) = \sqrt{\frac{\beta k}{2\pi}}\exp\left(-\beta\frac{k(x_1-x_a)^2}{2}\right)$

ה) $P(x_2) = \sqrt{\frac{\beta k}{2\pi}}\exp\left(-\beta\frac{k(x_2-x_b)^2}{2}\right)$

ו) $E(|x_1-x_2|) = |E(x_1-x_2)| = |Ex_1 - Ex_2| = |x_a - x_b|$

ז) $E_{new} = E - qEx_1 + qEx_2$

אנרגיה שדה

$Z = \frac{2\pi}{\beta}\sqrt{m_1m_2}\int_{-\infty}^{\infty}\exp\left[-\beta\left(\frac{k(x_1-x_a)^2}{2}-qEx_1\right)\right]dx_1 \cdot \int_{-\infty}\exp\left[-\beta\left(\frac{k(x_2-x_b)^2}{2}\right)\right]dx_2$

$|E[q(x_1-x_2)]| = \ldots = q|x_a-x_b|$