HW06
313545600 - דורון שפיגל
209517697 - דן חוזה

1. מודל Drude, מוליכות AC

1.1 סעיף א:

$$\vec{E} = E_0 e^{-j\omega t}$$
$$\frac{d}{dt} < \vec{P}(t) > + \frac{1}{\tau} < \vec{P}(t) > = f(t)$$
$$f(t) = -e \cdot \vec{E}$$
$$< \dot{\vec{P}}(t) > + \frac{1}{\tau} < \vec{P}(t) > = -e \cdot \vec{E}$$

1.2 סעיף ב:

ננחש פתרון מהצורה $P(t) = P(\omega)e^{-j\omega t}$

$$\frac{d}{dt}P(\omega)e^{-j\omega t} + \frac{1}{\tau}P(\omega)e^{-j\omega t} = -e \cdot \vec{E}$$
$$-j\omega P(\omega)e^{-j\omega t} + \frac{1}{\tau}P(\omega)e^{-j\omega t} = -e \cdot \vec{E} \qquad \cdot e^{j\omega t}$$
$$-j\omega P(\omega) + \frac{1}{\tau}P(\omega) = -eE_0$$
$$\left(-j\omega + \frac{1}{\tau}\right)P(\omega) = -eE_0$$
$$P(\omega) = \frac{\tau e E_0}{j\omega\tau - 1}$$
$$P(t) = \frac{\tau e E_0}{j\omega\tau - 1}e^{-j\omega t}$$

1.3 סעיף ג:

מהירות סחיפה: $\overrightarrow{V_{drift}} = \frac{< \vec{P}(t) >}{m_0}$.

$$\vec{J} = -e \cdot n \cdot \overrightarrow{V_{drift}} = \frac{\frac{n}{m_0}\tau e^2 E_0 e^{-j\omega t}}{1 - j\omega\tau}$$

1.4 סעיף ד:

חוק אום המיקרוסקופי: $\vec{J} = \sigma_{AC} \cdot \vec{E}$

$$\sigma_{AC} = \frac{\frac{n}{m_0}\tau e^2}{1 - j\omega\tau}$$
$$\omega = 0 \rightarrow \sigma_{DC} = \frac{n}{m_0}\tau e^2$$

1.5 סעיף ה:

נתון:

$$\begin{aligned}
\tau &= 8 \cdot 10^{-15} \\
\sigma_{DC} &= 4 \cdot 10^{7} \\
\sigma_{AC} &= \frac{\sigma_{DC}}{1 - j\omega\tau} = \frac{\sigma_{DC}(1 + j\omega\tau)}{1^2 + \omega^2\tau^2}
\end{aligned}$$

לכן:

$$\begin{aligned}
\Re(\omega) &= \frac{\sigma_{DC}}{1 + \omega^2\tau^2} \\
\Im(\omega) &= \frac{\sigma_{DC}\omega\tau}{1 + \omega^2\tau^2} \\
\|\sigma_{AC}(\omega)\| &= \frac{\sigma_{DC}}{1 + \omega^2\tau^2}\sqrt{1 + \omega^2\tau^2}
\end{aligned}$$

2. צפיפות מצבים

2.1. סעיף א:

נתון: $E = \frac{\hbar^2 k^2}{2m}$ לכן:

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

במקרה 1D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi}{L} \\
V_{line}^{k} &= 2k \\
N(k) &= 2 \times \frac{1}{2} \frac{V_{line}^{k}}{V_{single\ state}} = \frac{2kL}{\pi} \\
N(E) &= \frac{2L}{\pi} \sqrt{\frac{2mE}{\hbar^2}} \\
G(E) &= \frac{2}{\pi} \sqrt{\frac{2mE}{\hbar^2}} \\
g(E) &= \frac{dG(E)}{dE} = \frac{1}{\pi} \sqrt{\frac{2m}{E\hbar^2}}
\end{aligned}$$

במקרה 2D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi^2}{A} \\
V_{circle}^{k} &= \pi k^2 \\
N(k) &= 2 \times \frac{1}{4} \frac{V_{circle}^{k}}{V_{single\ state}} = \frac{k^2 A}{2\pi} \\
N(E) &= \frac{A}{2\pi} \cdot \frac{2mE}{\hbar^2} \\
G(E) &= \frac{1}{2\pi} \cdot \frac{2mE}{\hbar^2} \\
g(E) &= \frac{dG(E)}{dE} = \frac{m}{\pi\hbar^2}
\end{aligned}$$

במקרה 3D:

$$\begin{aligned}
V_{single\ state} &= \frac{\pi^3}{V} \\
V_{sphere}^{k} &= \frac{4}{3}\pi k^3 \\
N(k) &= 2 \times \frac{1}{8} \frac{V_{sphere}^{k}}{V_{single\ state}} = \frac{Vk^3}{3\pi^2} \\
N(E) &= \frac{V}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}} = \frac{V}{3\pi^2\hbar^3} \cdot (2mE)^{\frac{3}{2}}
\end{aligned}$$

$$
\begin{aligned}
G(E) &= \frac{1}{3\pi^2\hbar^3}\cdot(2mE)^{\frac{3}{2}} \\
g(E) &= \frac{dG(E)}{dE} = \frac{m\sqrt{2m}}{\pi^2\hbar^3}\sqrt{E}
\end{aligned}
$$

2.2. סעיף ב:

נתון: $E(k) = E_g + \frac{\hbar^2 k^2}{2m^*}$ לכן:

$$
k = \sqrt{\frac{2m(E-E_g)}{\hbar^2}}
$$

עבור 3D:

$$
\begin{aligned}
V_{single\ state} &= \frac{\pi^3}{V} \\
V^k_{sphere} &= \frac{4}{3}\pi k^3 \\
N(k) &= 2\times\frac{1}{8}\frac{V^k_{sphere}}{V_{single\ state}} = \frac{Vk^3}{3\pi^2} \\
N(E) &= \frac{V}{3\pi^2}\left(\frac{2m^*(E-E_g)}{\hbar^2}\right)^{\frac{3}{2}} = \frac{V}{3\pi^2\hbar^3}\cdot\left(2m^*(E-E_g)\right)^{\frac{3}{2}} \\
G(E) &= \frac{1}{3\pi^2\hbar^3}\cdot\left(2m^*(E-E_g)\right)^{\frac{3}{2}} \\
g(E) &= \frac{dG(E)}{dE} = \frac{m^*\sqrt{2m^*}}{\pi^2\hbar^3}\sqrt{E-E_g}
\end{aligned}
$$

ותחום האנרגיות הוא: $E > E_g$.

2.3. סעיף ג:

$$
\begin{aligned}
E(\vec{k}) &= \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} = \frac{m_y\hbar^2 k_x^2 + m_x\hbar^2 k_y^2}{2m_x m_y} \\
\frac{2m_x m_y E}{\hbar^2} &= m_y k_x^2 + m_x k_y^2 \\
\frac{2\sqrt{m_x m_y}E}{\hbar^2} &= \sqrt{\frac{m_y}{m_x}}k_x^2 + \sqrt{\frac{m_x}{m_y}}k_y^2
\end{aligned}
$$

$$
\begin{aligned}
V_{single\ state} &= \frac{\pi^2}{A} \\
V^k_{circle} &= \pi\left(k_x^2 + k_y^2\right) \\
N(k) &= 2\times\frac{1}{4}\frac{V^k_{circle}}{V_{single\ state}} = \frac{\left(k_x^2+k_y^2\right)A}{2\pi}
\end{aligned}
$$

$$
\begin{aligned}
G(k) &= \frac{\left(k_x^2 + k_y^2\right)}{2\pi} \\
G(E) &= E \cdot \frac{\sqrt{m_x m_y}}{\pi\hbar^2} \\
g(E) &= \frac{dG(E)}{dE} = \frac{m}{\pi\hbar^2}
\end{aligned}
$$