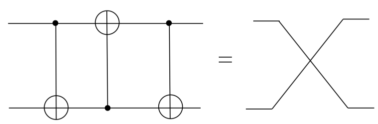
SWAP GATE

May 14, 2024

We can construct the swap gate using the identity:

$$\mathrm{SWAP} = \mathrm{CNOT}_{12} \cdot \mathrm{CNOT}_{21} \cdot \mathrm{CNOT}_{12} \tag{1}$$

the circuit diagram is:



We also know that:

$$\mathrm{CNOT}_{12} = P_0 \otimes I + P_1 \otimes X, \quad \mathrm{CNOT}_{21} = I \otimes P_0 + X \otimes P_1 \tag{2}$$

where, $P_0 = |0\rangle\langle 0|$ and $P_1 = |1\rangle\langle 1|$, therefore we can write the representations of the CNOTs and construct the SWAP gate out of that:

$$\tilde{M}_2[\mathrm{CNOT}_{12}] = \tilde{M}_1[P_0] \otimes \tilde{M}_1[I] + \tilde{M}_1[P_1] \otimes \tilde{M}_1[X] \tag{3}$$

$$\tilde{M}_2[\mathrm{CNOT}_{21}] = \tilde{M}_1[I] \otimes \tilde{M}_1[P_0] + \tilde{M}_1[X] \otimes \tilde{M}_1[P_1] \tag{4}$$

where, $\tilde{M}_1$ are the single qubit maps:

$$\tilde{M}[P_0] = \left(\begin{array}{c|c|c|c} P_0 & O & O & O \\ \hline O & P_0 & O & O \\ \hline O & O & P_0 & O \\ \hline O & O & O & P_0 \end{array}\right) \tag{5}$$

$$\tilde{M}[P_1] = \left(\begin{array}{c|c|c|c} P_1 & O & O & O \\ \hline O & P_1 & O & O \\ \hline O & O & P_1 & O \\ \hline O & O & O & P_1 \end{array}\right) \tag{6}$$

$$\tilde{M}[X] = \left(\begin{array}{c|c|c|c} X & O & O & O \\ \hline O & X & O & O \\ \hline O & O & X & O \\ \hline O & O & O & X \end{array}\right) \tag{7}$$

finally,

$$\tilde{M}_2[\mathrm{SWAP}] = \tilde{M}_2[\mathrm{CNOT}_{12}] \cdot \tilde{M}_2[\mathrm{CNOT}_{21}] \cdot \tilde{M}_2[\mathrm{CNOT}_{12}] \tag{8}$$