The problem asks to find the function $f(x) \in \{0,1\}$ that minimizes the following cost function:

$$L(x) = \sum_{Y\in\{0,1\}} \int p(x,Y)\left[Y \log f(x) + (1-Y)\log(1-f(x))\right] dx$$

Step-by-Step Solution:

1. **Rewrite the Joint Probability**: Using Bayes' rule, we can express the joint probability $p(x,Y)$ as:

$$p(x,Y) = p(Y|x)p(x)$$

where: - $p(Y|x)$ is the conditional probability of Y given x, - $p(x)$ is the marginal probability of x.

Substituting into the cost function gives:

$$L(x) = \sum_{Y\in\{0,1\}} \int p(x)p(Y|x)\left[Y \log f(x) + (1-Y)\log(1-f(x))\right] dx$$

2. **Split the Sum Over Y**: The sum over $Y \in \{0,1\}$ results in two terms, for $Y=1$ and $Y=0$:

$$L(x) = \int p(x)\left[p(Y=1|x)\log f(x) + p(Y=0|x)\log(1-f(x))\right] dx$$

Since $p(Y=0|x) = 1 - p(Y=1|x)$, this simplifies to:

$$L(x) = \int p(x)\left[p(Y=1|x)\log f(x) + (1-p(Y=1|x))\log(1-f(x))\right] dx$$

3. **Optimize Over $f(x)$**: To minimize this cost function with respect to $f(x)$, we differentiate $L(x)$ with respect to $f(x)$:

$$\frac{dL(x)}{df(x)} = \int p(x)\left[\frac{p(Y=1|x)}{f(x)} - \frac{1-p(Y=1|x)}{1-f(x)}\right] dx$$

Setting this derivative to zero to find the optimal $f(x)$:

$$\frac{p(Y=1|x)}{f(x)} = \frac{1-p(Y=1|x)}{1-f(x)}$$

4. **Solve for $f(x)$**: Solving the above equation:

$$f(x)\cdot(1-p(Y=1|x)) = (1-f(x))\cdot p(Y=1|x)$$

Simplifying further:

$$f(x) = p(Y=1|x)$$

Conclusion: The function $f(x)$ that minimizes the cost function is $p(Y=1|x)$, which is the conditional probability of $Y=1$ given x.