Simulation of Central Limit Theorem

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Overview

By performing simulations in R we will show that average of n exponentialy distributed variables is approximately normal with mean 1/lambda and the standard deviation 1/(lambda*sqrt(n)). It will visually present the central limit theorem, that states that the distribution of average of a large number of independent, identically distributed variables will be approximately normal, regardless of the underlying distribution.

Simulations:

We wil perform 1000 simulation. The variable we simulate is average of 40 variables, exponentialy distributed with lambda 0.2.

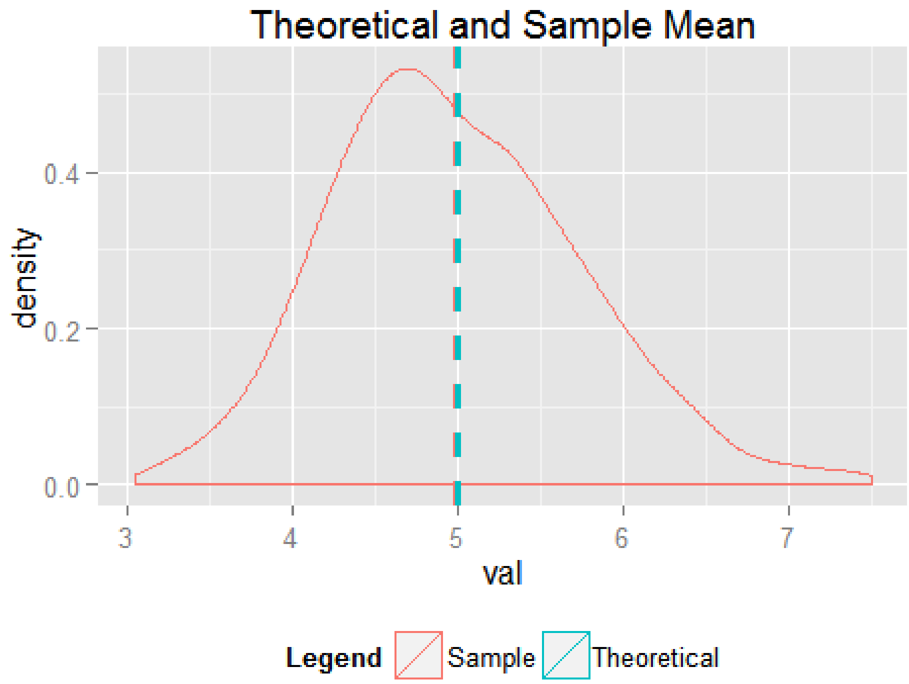
```
set.seed(7)
mns = NULL
lambda=0.2
n=40
for (i in 1 : 1000) mns = c(mns, mean(rexp(n,lambda)))
mns<-as.data.frame(merge('Sample',mns))
colnames(mns)<-c('Legend','val')
```

Sample Mean versus Theoretical Mean

Below we can compare sample mean (4.98) vs theoretical mean of exponentialy distributed variables (`r round(as.numeric( 1/lambda),2)`

```
# calculate mean
mean=as.data.frame(merge('Sample', as.numeric(mean(mns$val,na.rm=T))))
mean=rbind(mean,merge('Theoretical',as.numeric( 1/lambda)))
colnames(mean)<-c('Legend','mean_val')

#plot
library(ggplot2)
ggplot(data=mns,aes(x=val,colour=Legend)) +
geom_density() +
geom_vline(data=mean, aes(xintercept=mean_val, colour=Legend),linetype="dashed",
size=1.1) +
theme(legend.position="bottom") +
labs(title="Theoretical and Sample Mean")
```

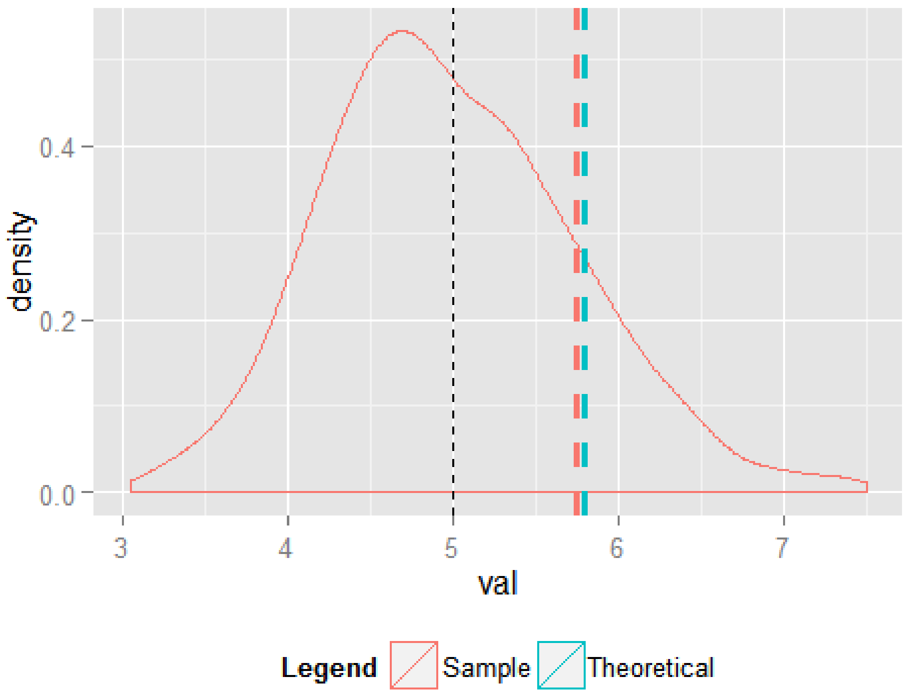


Sample Variance versus Theoretical Variance

Below we can compare sample standard deviation (0.76) vs theoretical standard deviation of exponentialy distributed variables (`r round(as.numeric( 1/(n*lambda)),2)`, Chart below shows mean + one standrd deviation.

```
# calculate mean
sdev=as.data.frame(merge('Sample', as.numeric(sd(mns$val,na.rm=T))))
sdev=rbind(sdev,merge('Theoretical',as.numeric( 1/(lambda*sqrt(n))) ))
sdev<-cbind(sdev,as.data.frame(sdev[,2]+mean[,2]))
colnames(sdev)<-c('Legend','sdev_val','sm_val')


#plot
ggplot(data=mns,aes(x=val,colour=Legend)) +
geom_density() +
geom_vline(data=mean[2, ], aes(xintercept=mean_val), colour='black',linetype="dashed",
size=0.25) +
geom_vline(data=sdev, aes(xintercept=sm_val, colour=Legend),linetype="dashed",
size=1.1) +
theme(legend.position="bottom")
```



Distribution

Comparing the shapes we can see that the distribution is approximately normal with mean 1/lambda and the standard deviation 1/(lambda*sqrt(n).

```
#normal distribution
set.seed(7)
rnormal = rnorm(1000,1/lambda,1/(sqrt(n)*lambda))
rnormal<-as.data.frame(merge('Normal',rnormal ))
colnames(rnormal)<-c('Legend','val')
dist<-rbind(mns,rnormal)

#plot
ggplot(data=dist,aes(x=val,colour=Legend)) +
geom_density() +
theme(legend.position="bottom") +
labs(title="Sample and Normal Distribution")
```