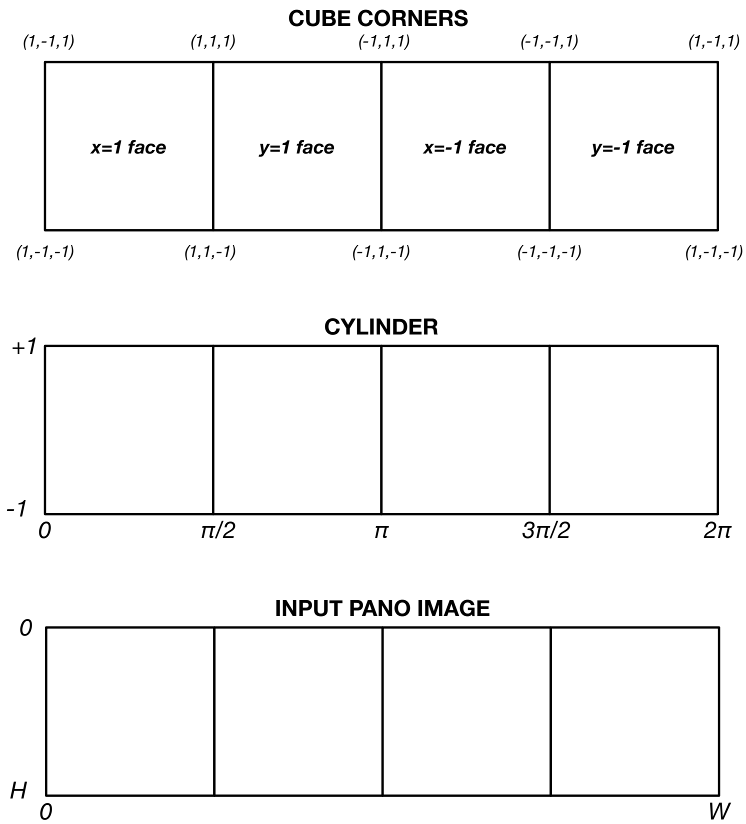
1 Mapping cylinder to 4 sides of cube



Given the cylinder $x^2 + y^2 = 1$ inscribed inside a $2 \times 2 \times 2$ cube centered at the origin, we can project the cylindrical panorama onto the corresponding four faces of the cube as follows. For every pixel in the output image we cast a ray from the cube's center at $(0, 0, 0)$ through the corresponding point on one of the cube's faces and determine where that ray intersects the cylinder. We then map this point on the cylinder to the corresponding point in the input image and (bilinearly) sample the input image at the point. The resulting pixel color is assigned to the current output pixel.

We parameterize the ray emanating from the cube's center $(0, 0, 0)$ through a point (u, v, w) on a face of the cube as

$$\mathbf{r}(t) = (0, 0, 0) + t \cdot (u, v, w). \tag{1}$$

This ray intersects the cylinder where

$$(ut)^2 + (vt)^2 = 1. \tag{2}$$

Solving we get $t = 1/\sqrt{u^2 + v^2}$, which we plug back into Equation 1 and get the cylindrical point

$$\mathbf{p} = (x, y, z) = \frac{(u, v, w)}{\sqrt{u^2 + v^2}}. \tag{3}$$

In cylindrical coordinates we have

$$\mathbf{p} = (\theta, z), \quad \theta = \operatorname{atan2}(y, x) + \pi/4. \tag{4}$$

We added $\pi/4$ to θ so that the left edge of the image corresponds to the left edge of the cube's $x = 1$ face (instead of the middle of the face); Then we adjust for the appropriate image "wrap-around:"

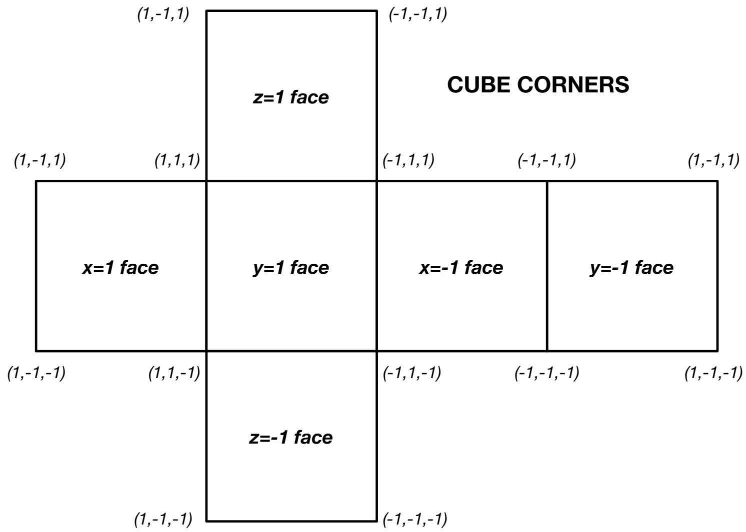
$$\theta' = \begin{cases} \theta & \theta \geq 0 \\ \theta + 2\pi & \theta < 0 \end{cases} \tag{5}$$

Now the left edge of the input image corresponds to the left edge of the output image. The resulting pixel coordinate in the input image is

$$(r, c) = \left(\frac{H}{2}(z+1),\ \frac{W}{2\pi} \cdot \theta' \right). \tag{6}$$

```
for row = 0 .. outImage.H-1 {
    for col = 0 .. outImage.W-1 {
       s = 4.0*col/outImage.W
       face = floor(s)  // face = 0,1,2,3
       f = s - face     // f = frac(s)
       if face == 0 {   // (u,v,w) = point on cube
           u = 1
           v = 2*f - 1
       } else if face == 1 {
           u = 1 - 2*f
           v = 1
       } else if face == 2 {
           u = -1
           v = 1 - 2*f
       } else {
           u = 2*f - 1
           v = -1
       }
       w = 2.0*row/outImage.H - 1
       (x,y,z) = (u,v,w)/sqrt(u*u + v*v) // project onto cylinder
       theta = atan2(y,x) + pi/4;        // cyl. coords, -3pi/4 <= theta <= 5pi/4
       if theta < 0                      // map to [0,2*pi)
           theta += 2*pi
       r = inImage.H*(z + 1)/2.0         // map to input pixel
       c = imImage.W*theta/(2*pi)
       outImage(row,col) = inImage.sample(r,c)
     }
}
```

2 Mapping sphere to 6 sides of cube



Given the sphere $x^2+y^2+z^2=1$ inscribed inside a $2\times 2\times 2$ cube centered at the origin, we can project the spherical panorama onto the corresponding four faces of the cube as follows. For every pixel in the output image we cast a ray from the cube's center at $(0,0,0)$ through the corresponding point on one of the cube's faces and determine where that ray intersects the sphere. We then map this point on the sphere to the corresponding point in the input image and (bilinearly) sample the input image at the point. The resulting pixel color is assigned to the current output pixel.

We parameterize the ray emanating from the cube's center $(0,0,0)$ through a point (u,v,w) on a face of the cube and find where it intersects the unit sphere. We use the mapping defined at

`http://mathproofs.blogspot.com/2005/07/mapping-cube-to-sphere.html`

yielding

$$x = u\sqrt{1-\frac{v^2}{2}-\frac{w^2}{2}+\frac{v^2w^2}{3}}, \tag{7}$$

$$y = v\sqrt{1-\frac{w^2}{2}-\frac{u^2}{2}+\frac{w^2u^2}{3}}, \tag{8}$$

$$z = w\sqrt{1-\frac{u^2}{2}-\frac{v^2}{2}+\frac{u^2v^2}{3}}. \tag{9}$$

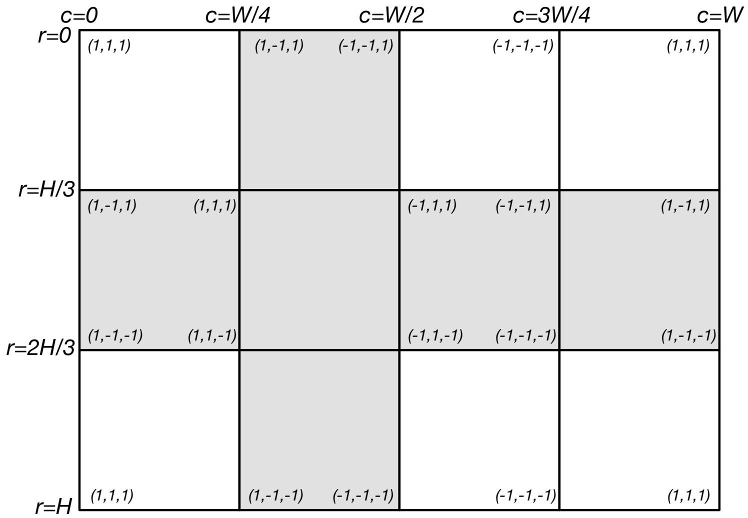
We convert (x,y,z) to the spherical coordinate (θ,ϕ) where the azimuthal angle is

$$\theta = \operatorname{atan2}(y,x) \tag{10}$$

and the elevation angle is

$$\phi = \operatorname{atan2}\left(\sqrt{x^2+y^2},\ z\right). \tag{11}$$

We divvy up the output image into $3 \times 4 = 12$ squares where only 6 of the squares actually map to the actual faces of the cube:



We pick (u, v, w) coordinates for the unused corners so that the image has a toroidal topology.

```
for row = 0 .. outImage.H-1 {
    for col = 0 .. outImage.W-1 {
       a = 4.0*col/outImage.W
       b = 3.0*row/outImage.H
       (i,j) = (floor(a),floor(b))            // map to one of 12 squares
       (s,t) = (a - i, b - j)                 // uvw interpolation values
       (u,v,w) = bilerp(corners[j][i]), s,t)  // get cube coords w/in square
       (x,y,z) = cubeToSphereCoords(u,v,w)
       (theta,phi) = cartesianToSpherical(x,y,z)
       r = theta*inImage.W/(2*pi)
       c = (phi + pi/2)*inImage.H/pi
       outImage(row,col) = inImage.sample(r,c)
     }
}
```