CUBE CORNERS

(1,-1,1)   (1,1,1)   (-1,1,1)   (-1,-1,1)   (1,-1,1)

x=1 face   y=1 face   x=-1 face   y=-1 face

(1,-1,-1)   (1,1,-1)   (-1,1,-1)   (-1,-1,-1)   (1,-1,-1)

CYLINDER

+1

-1

0   π/2   π   3π/2   2π

INPUT PANO IMAGE

0

H

0   W

Given the cylinder $x^2 + y^2 = 1$ inscribed inside a $2 \times 2 \times 2$ cube centered at the origin, we can project the cylindrical panorama onto the coresponding four faces of the cube as follows. For every pixel in the output image we cast a ray from the cube's center at $(0,0,0)$ through the corresponding point on one of the cube's faces and determine where that ray intersects the cylinder. We then map this point on the cylinder to the corresponding point in the input image and (bilinearly) sample the input image at the point. The resulting pixel color is assigned to the current output pixel.

We parameterize the ray emanating from the cube's center $(0,0,0)$ through a point (u, v, w) on a face of the cube as

$$\mathbf{r}(t) = (0,0,0) + t \cdot (u, v, w). \tag{1}$$

This ray intersects the cylinder where

$$(ut)^2 + (vt)^2 = 1. \tag{2}$$

Solving we get $t = 1/\sqrt{u^2 + v^2}$, which we plug back into Equation 1 and get the cylindrical point

$$\mathbf{p} = (x, y, z) = \frac{(u, v, w)}{\sqrt{u^2 + v^2}}. \tag{3}$$

In cylindrical coordinates we have

$$\mathbf{p} = (\theta, z), \quad \theta = \text{atan2}\,(y, x) + \pi/4. \tag{4}$$

1

We added $\pi/4$ to θ so that the left edge of the image corresponds to the left edge of the cube's $x = 1$ face (instead of the middle of the face); Then we adjust for the appropriate image "wrap-around:"

$$\theta' = \begin{cases} \theta & \theta \geq 0 \\ \theta + 2\pi & \theta < 0 \end{cases} \tag{5}$$

Now the left edge of the input image corresponds to the left edge of the output image. The resulting pixel coordinate in the input image is

$$(r, c) = \left(\frac{H}{2}(z+1), \ \frac{W}{2\pi} \cdot \theta' \right). \tag{6}$$

```
for row = 0 .. outImage.H-1 {
    for col = 0 .. outImage.W-1 {
        s = 4.0*col/outImage.W
        face = floor(s)   // face = 0,1,2,3
        f = s - face      // f = frac(s)
        if face == 0 {    // (u,v,w) = point on cube
            u = 1
            v = 2*f - 1
        } else if face == 1 {
            u = 1 - 2*f
            v = 1
        } else if face == 2 {
            u = -1
            v = 1 - 2*f
        } else {
            u = 2*f - 1
            v = -1
        }
        w = 2.0*row/outImage.H - 1
        (x,y,z) = (u,v,w)/sqrt(u*u + v*v) // project onto cylinder
        theta = atan2(y,x) + pi/4;        // cyl. coords, -3pi/4 <= theta <= 5pi/4
        if theta < 0                      // map to [0,2*pi)
            theta += 2*pi
        r = inImage.H*(z + 1)/2.0         // map to input pixel
        c = imImage.W*theta/(2*pi)
        outImage(row,col) = inImage.sample(r,c)
    }
}
```