DEPARTMENT OF PHYSICAL AND MATHEMATICAL SCIENCES

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PORTFOLIO OPTIMIZATION OF STOCKS ON THE GHANA STOCK EXCHANGE

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THIS DISSERTATION IS SUBMITTED TO THE DEPARTMENT OF PHYSICAL AND MATHEMATICAL SCIENCES, THE UNIVERSITY OF GHANA IN PARTIAL FULFILMENT OF THE REQUIREMENT OF THE AWARD OF A BACHELOR DEGREE.

ABSTRACT

Portfolio optimization is the process of choosing the optimum portfolio (asset distribution) from a collection of all possible portfolios based on some criterion. ie (maximizing as well minimizing returns and risks respectfully). Building a portfolio with standardized optimization is still unheard of in Ghana, therefore this research demonstrates how the Markowitz mean-variance model may be used on the Ghana Stock Exchange and further reveals the most viable portfolio from a sample of select companies, much to the satisfaction of the investor. For the analysis, annual data on stock returns from 2015 through 2020 was used. According to the analysis, GSR had the highest expected annual returns of 65.14% and a 165.81% expected annual risk, while GOIL had the least annual expected return of 10.31% and annual risk of 30.05%. An inexperienced risk-averse investor may choose GOIL since it has the lowest expected annual risk with an expected return of 10.31%. Portfolios were created and instead of singling out GOIL for investment, the risk-averse investor may choose the optimal portfolio with the least risk, which is GOIL-TBL-ALU and expect an annual return of 24.61%, risk of 29.89 %, and a Sharpe ratio of 0.42189. However, it is more advisable to invest in the combination with the highest Sharpe ratio, that is ALU-CPC-GSR, in order to earn a 50.73% expected return with a 65.09% risk. The highest Sharpe ratio for ALU-CPC-GSR indicates the portfolio required to provide the best remuneration for the risk incurred by the investor, making it the best option for portfolio investment.

Also, the efficient frontier with the Capital Market Line was illustrated for investors who would be interested in investing in all the five assets. This implied that one invests 53.449% in ALU, 21.27% in CPC, 12.32% in GSR, 0.00% in GOIL and 12.92% in TBL, to achieve the market portfolio that yields the highest level of Sharpe ratio.

CHAPTER ONE

1.0 Background of Study

Of all the concerns of investors, deciding on the finest investment opportunity to enhance the value of their money is by far the most important. Portfolio optimization solves this concern. It refers to selecting the best-suited portfolio(asset distribution) based on some criterion from a collection of all potential portfolios ie (maximize and minimize returns and risks respectfully). Decisions on investment options are very challenging for most financial institutions and their shareholders who expect high returns at relatively low risk. As per George E. Redja, (2014) occurrence of a loss as a result of uncertainty is termed as risk. Investors are free to trade in various collection financial instruments, commonly referred to as portfolios. They comprise bonds, stocks, cash and cash equivalents, real estate certificates, etc. To reduce the shock experienced by a systematic risk when one loses by investing in a single financial instrument, Investors diversify. Diversification is the spreading of risk and returns within an asset class.

When dealing with Investment, evaluating risk and return at the same time is of utmost importance. (Brigham & Ehrhardt, 2008). As such, Diversification aims to capture all sectors' returns over time while reducing volatility (risk) at any given moment.

Risk-averse investors avoid taking risks and are thus prudent; risk-takers, on the other hand, are willing to take greater risks in the hopes of gaining a higher return; and

neither risk-averse nor risk-takers are characteristics of risk-indifferent investors. Although risk-takers are open to taking on greater danger, as rational people, we all demand a certain degree of return on investment for safety (Brigham & Houston, 2007).

Naturally, since investors are rationally averse, for riskier assets they would demand a higher expected return for said assets. Realized return is different from expected return, as such it isn't always a guarantee that an asset with a high expected return would yield similar results practically(reality). This is due to the uncertainty of return on assets that are deemed risky. As a result, an investment may not generate the promised(expected) return. The risk in holding deviates from an actual return from the expected return may be upward or downside.

Markowitz's conventional mean-variance optimization approach is commonly linked with portfolio optimization theory (Markowitz, 1952) where returns are calculated as the mean and risk as to the variance on those returns, Capital Asset Pricing (CAPM), Von Neumann Morgenstern, which takes into account the rationalizing of how portfolio choices are made on expected utility.

1.1 Problem of the study

Investors avoid using resources now in order to use them at a later date when the projected rates of return are higher (Kalyan & Subramanyam, 2018). However, the issue lies with what models to use to choose the appropriate companies that would maximize returns at relatively low risk.

Many investors focus solely on their profits, ignoring the importance of risk(Ketchen et al., 2018). In Ghana, this is rightly so because of the unavailability of data mostly pertaining to risk indicators (QR). Even the most standardized model which is seen as the foundation of modern portfolio theory, the classical Mean-Variance (MV) portfolio has its flaws. The primary disadvantage of mean-variance analysis is its susceptibility to covariance matrix and means

estimation error covariance matrix estimation of asset return (Logubayom, A.I, & Victor, T.A., 2019). Thus, there lie some discrepancies in using this model for choosing a portfolio (Merton, R.C.,1980) argued that it becomes a challenge when you try to estimate from a time series of realized stock return data the expected returns. Estimations derived from existing time series for variance and co-variance will be much more accurate than expected return estimations. Even if the market's expected return were known to remain constant over time, an accurate assessment would require a very extensive history of returns. And of course, predicting these changes becomes far more challenging if the projected return is predicted to fluctuate over time. There's also the issue of the model being deemed unrealistic. The model has been subjected to several critiques aimed at its unrealistic assumptions such as non-well-diversified equities in portfolio normal portfolio risk behavior and normal portfolio risk behavior (Rahnama H.,2016). As a result, investors try to maximize their utility (contentment) by reducing risk (variance) and maximizing expected return. The following is a summary of the Markowitz model: Calculate the portfolio's projected return rates for each stock. Calculate the risk of each stock in the portfolio by calculating the standard deviation (Variance). Calculate the correlation coefficient or covariance for all stocks as pairs (Fabozzi, 1999)

1.2 Objectives of the Study

The study’s objectives are as follows:

1. To determine the best combination of assets or portfolios are given a set of portfolios from the Ghana Stock Exchange.
2. To determine the risk and return associated with the optimal portfolio.

1.3 Significance of the Study

The study's conclusions will be resourceful to portfolio managers because it provides them with a basis for the construction of portfolios that for a given level of risks returns would be maximized. This will guide portfolio managers on which stocks to include in a portfolio and hence satisfy their clients. Also, since the data used is up to date, portfolio managers could use this as a source of information on updates and identify opportunities on the stock market (Thakur, n.d.). Further, investors in Ghana and beyond will benefit from this study as this study provides information and recommendation on which stocks to invest in and in what weights by reflecting the performance of stocks on the Ghana Stock Exchange. This will encourage investment and help create value for investors. Long-term capital for major sectors of the economy, such as businesses and the government, which contributes to development is a benefit of a stock exchange. Indexes of the Stock exchange are used as a measure of economic health; hence the government can use this information to ascertain the contribution of the Ghana Stock Exchange to economic growth and undertake any necessary approaches to improve upon it (Sarkissian, n.d.). Finally, researchers or academicians will find this study a useful tool of reference when conducting any similar research.

1.4 Organization of study

This project has five distinct (5) chapters. The introduction, which gives a brief background of the study opens chapter One (1). This is followed by Chapter two (2) which presents the general literature review of the basic theories introduced for Optimizing Portfolios. Chapter three (3) explains how data was retrieved and shows the methods used and all other relevant formulas used in this study. The Fourth (4th) Chapter gives the results to the various data, analyzed in Chapter three (3). The final chapter discusses the conclusions, findings, and the review of the models in the case of their limitations and suggested areas of further analysis.

CHAPTER THREE

3.0 Introduction

The goal of this research is to determine the best combination of assets or portfolios given a set of portfolios from the GSE and also determine the risk and return associated with each portfolio. To be able to achieve this, a quantitative survey approach is used. The primary components of this chapter will include the area of study, sample size and sampling technique, method of data collection, and method of data analysis. The main method of data analysis to be examined is the Markowitz mean-variance model. Additionally, a summary is provided.

3.1 Area of study

The Ghana stock exchange trades different financial instruments including stocks, commodities, and bonds. However, this study focuses on the listed stocks on the Ghana stock exchange.

3.4 Method of Data Analysis

The mean-variance analysis is a component of modern portfolio theory (MPT). This hypothesis is founded on the concept that investors make reasonable judgments when they have enough information. According to one of the theory's assumptions, investors enter the market to maximize their profits while minimizing unnecessary risk.We should be able to come up with a single risky asset portfolio with the lowest risk that is preferred over all other portfolios with comparable returns. Where the ray connecting risk-free investments to our risky portfolio becomes tangent to our collection of risky portfolios, our optimal portfolio is located. The

highest possible slope is at this point. Maths defines it (Optimization) as the best element selected from a set of feasible alternatives under particular constraints. Investors assess the

possible variance (volatility of an asset's returns) against the asset's expected returns when determining the degree of risk. The mean-variance analysis deals with the average variance in the expected return on investment.

3.5 Risk and Return

Sharpe Ratio

This ratio is used to calculate risk-adjusted returns and was developed by Noble Laurette William F. Sharpe. The Sharpe ratio helps determine whether a portfolio's excess returns are due to investment decisions or excess risk. The higher the returns mean a better investment option provided it does not come with excess additional risk.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

Where: R_P= return of the portfolio

R_f=risk-free rate

σ_ρ=standard deviation of the portfolio's excess return

Expected return of a portfolio

$$R_p = \sum_{\forall_j} w_j R_J$$

Where: R_p= The return on the portfolio

R_j= The weighted average of expected returns for individual assets in the portfolio

w_j = The weight which shows percentages invested in each asset.

= (0.3333*0.41342) +(0.3333*0.45714) +(0.3333*0.65143)

Risk of a portfolio

$$\sigma_p = \sqrt{w_x^2\sigma_x^2 + w_y^2\sigma_y^2 + w_z^2\sigma_z^2 + 2w_x w_y Cov(x,y) + 2w_y w_z Cov(y,z) + 2w_x w_z Cov(x,z)}$$

Where: σ_p = the risk(standard deviation) of the portfolio

w = the weight of assets in the portfolio

Cov = the covariance between the paired assets

=sqrt{[(0.3333^2*0.53063^2)+(0.3333^2*0.83504^2)+(0.3333^2*1.65814^2)]+(2*0.3333*0.3333*0.27755)+(2*0.3333*0.3333*-0.03144)+(2*0.3333*0.3333*-0.02638)}

3.5.1 Correlation (Multiple Correlation)

The correlation coefficient is a statistic for determining how closely two assets are correlated. However, we are dealing with more than two assets and multiple correlation will be used here. A multiple correlation coefficient (R) yields the maximum degree of linear relationship that can be obtained between two or more independent variables and a single dependent variable.

Multiple Correlation Coefficient: $R_{Z,XY} = \sqrt{\dfrac{r_{xz}^2 + r_{yz}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2}}$

Where: $r = Cor(x,y) = \dfrac{Cov(x,y)}{\sigma_x \sigma_y}$

$Cor(x,y)$ = correlation between paired assets

$Cov_{x,y}$ = covariance of returns for securities x and y

σ_x = standard deviation of returns for security x

σ_y = standard deviation of returns for security y

3.6 Efficient Frontier

The efficient frontier is a set of optimal portfolios that provides the best-expected return for a given level of risk or the least risk for a given level of expected return. The Markowitz efficient frontier is the section of the minimum-variance curve above and to the right of the global minimum variance portfolio. Rational risk-averse investors can choose multiple portfolio sets that fit their projected utility along the efficient frontier line. Outside of the efficient frontier line, all other combinations are inefficient. Portfolios that are below the efficient frontier are sub-optimal because they do not deliver enough return for the amount of risk they carry. Portfolios that concentrate on the right of the efficient frontier have a higher degree of risk for the given rate of return, making them sub-optimal.

Capital Market Line

$$R_p = R_f + \frac{R_p - R_f}{\sigma_p} * \sigma_m$$

Where: σ_m = market standard deviation

=0.12 + (0.72971*0)

=0.12.

CHAPTER FOUR

4.0 Introduction

This chapter aims to discuss and present data analysis and findings from the thirty- seven (37) companies listed on the Ghana stock exchange [as of 2020] further reduced to five (5) companies within a 6year period ranging from 2015-2020. The aim/purpose/objectives of the study are to:

1. To determine the best combination of assets or portfolios given a set of portfolios from the Ghana Stock Exchange.
2. To determine the risk and return associated with the optimal portfolio.

The method of data collection was done through secondary means. Thus, the Ghana Stock Exchange database was used to select companies' monthly returns as listed on the Stock Exchange. Data got from GSE's database(websites) were statistically analyzed by actuarial students. Excel sheets(solver) was the model used in the data analysis. Judgment (purposive) sampling, a form of non-probability sampling was used to sample the data. Thus, an initial list of 37 companies was narrowed down to a sample size of five (5) solely based on companies with relatively high returns.

4.1 Statistical Estimation for Each Asset

Table.1 displays a 6 year period of standard deviation(mean), and return of individual assets, as well as their skewness and kurtosis.

Table 1. Risk, Return, Skewness and Kurtosis of Stocks

STOCK	RETURNS	ST. DEV	SKEWNESS	KURTOSIS
Aluworks	41.34%	53.06%	2.072593338	7.234007188
Cocoa processing	45.71%	83.50%	2.663570211	10.19107565
Golden Star	65.14%	165.81%	8.354211573	69.86226727
Ghana Oil	10.31%	30.05%	0.468841457	0.48323701
Trust bank	22.18%	66.16%	4.016505593	25.8110273

If a return distribution has a negative skew, it means that the investment will have many little wins and a few significant losses. On the other hand, with a positively skewed distribution, investors might expect minor losses on a regular basis and few significant returns. As a result, negatively skewed investment return distribution is seen as a bad choice compared to a positively skewed investment return distribution. This is because with positively skewed investment return distribution the large profits may be sufficient to compensate for the frequent – but little – losses. Investing in assets with a negatively skewed return distribution, on the other hand, may appeal to investors. It could be because they prefer a lot of little victories and a few significant losses to a lot of small losses and a few big wins.

Positive skewness was seen in all of the companies' results. Trust Bank's positive skewness of 3.986, which is second highest behind GSR (with 8.354 skewness), indicates that it could have a good prospect. CPC and ALU come in second and third, with skewness of 2.664 and 2.073, respectively. With a skewness of 0.469, GOIL has the lowest skewness. The kurtosis coefficient high above a normal of 3, shows how more volatile the future return will be (that is, either extremely large or extremely small). Ghana Oil is the only company with kurtosis less than 3, hence it is platykurtic. The other four (4) companies had kurtosis of more than 3 hence they are leptokurtic. ALU, CPC, GSR, TBL had kurtosis of 7.234,10.191,69.862,25.410 respectively, implying a high possibility of their future returns being extremely little or extremely large.

Table 2. Optimum Portfolio Combinations

ASSET COMBINATION	PORTFOLIO RETURN	PORTFOLIO RISK	SHARPE RATIO
ALU-GSR-GOIL	38.93%	58.56%	0.459865205
ALU-CPC-GSR	50.73%	65.09%	0.595041246
CPC-GSR-GOIL	40.39%	62.67%	0.452985395
CPC-GOIL-TBL	26.07%	36.48%	0.385654401
GSR-GOIL-TBL	32.54%	59.80%	0.343553979
GSR-TBL-ALU	42.89%	61.10%	0.505552038
GOIL-ALU-CPC	32.45%	37.15%	0.550631114
GOIL-TBL-ALU	24.61%	29.89%	0.421892255
TBL-ALU-CPC	36.41%	42.66%	0.57222839
TBL-CPC-GSR	44.35%	106.87%	0.302668952

We explored a three-asset portfolio from the five (5) companies chosen. As a result, there were 10 alternative combinations from which to choose the best portfolio. To achieve this, each combination was given equal weights, and the Sharpe ratio of each combination was calculated. The Sharpe ratio measures how well the return on assets adjusts for the risk incurred, with a higher ratio indicating a more efficient portfolio. The best combination would have the highest Sharpe ratio since it reflects the best excess return over the risk of the various portfolio combinations. Table 2, presents the returns, risk, and Sharpe ratios for the ten (10) asset combinations. The table suggests that the combination of ALU-CPC-GSR yields the Sharpe ratio with the highest value. The Sharpe ratio for this portfolio was 0.59504, 65.09% risk. With the highest Sharpe ratio, the portfolio of ALU-CPC-GSR is projected to give the most efficient combination of risks to returns an investor could incur, making it the easiest choice a rationale investor would go in for. Also, the combination of ALU-CPC-GSR had the highest expected returns of 50.73% with a risk of 65.09 %. It is followed closely by the combination of TBL-CPC-GSR having returns of 44.35% and the highest risk of 106.87%. Despite the comparatively high expected return, TBL-CPC-GSR had the least Sharpe ratio of 0.30267 due to the high level of risk, implying that the portfolio poorly compensates for the high level of

risk incurred. Further, the combination of GOIL-TBL-ALU, which has a Sharpe ratio of 0.42189 has the lowest risk of 29.89% and the lowest return of 24.61%.

Table 3. Correlation of Three Asset Combinations

THREE ASSET	COMBINATION
ALU-GSR-GOIL	0.121281307
ALU-CPC-GSR	0.032946027
CPC-GSR-GOIL	0.055713623
CPC-GOIL-TBL	0.038555666
GSR-GOIL-TBL	0.054648424
GSR-TBL-ALU	0.066710152
GOIL-ALU-CPC	0.2284737
GOIL-TBL-ALU	0.269763555
TBL-ALU-CPC	0.228443421
TBL-CPC-GSR	0.05105608

Table 3, presenting the correlation coefficients of the three-asset combined portfolios shows that the compounded portfolios have a low overall positive correlation, with the highest correlation coefficient 0.26976 occurring at the combination of GOIL-TBL-ALU. For investors who want to diversify and avoid unsystematic risk, a compounded portfolio with a low overall correlation is essential (Sharpe, 1994). The correlation coefficient (0.26976) for GOIL-TBL-ALU indicates that when return values are increasing (or decreasing) in one of either company, it will result in the return values of the other two companies also increasing (or decreasing). ALU-CPC-GSR, our most efficient portfolio has the weakest positive correlation (0.004566), implying that the increasing (or decreasing) returns of one will only result in a lesser increase (or decrease) in the others.

Table 4. Optimal Portfolio

	OPTIMAL	PORTFOLIO	
Equal Weights	Max Returns	Min Risk	Max Sharpe Ratio
33.33%	0.00%	69.98%	62.47%
33.33%	0.00%	21.17%	23.72%
33.33%	100.00%	8.84%	13.81%
100%	100.00%	100.00%	100.00%
50.73%	65.14%	44.37%	45.67%
65.09%	165.81%	46.42%	47.34%
59.50%	32.05%	69.73%	71.11%

Table 4 shows that attempting to maximize returns resulted in weights of 100% allocated to GSR and no weights allocated to ALU and CPC with 65.14% expected return and a very high risk of 165.81%. However, in an attempt to reduce risk, the expected returns and risk were 44.37% and 46.42 %, respectively, with weights of 69.98% assigned to ALU, 21.17% to CPC, and 8.84% to GSR. The weights, 62.47% for ALU, 23.72% for CPC and 13.81% were obtained when the Sharpe ratio was maximized, with expected returns and risk of 45.67%, respectively. The weights in the columns indicate what proportion of assets an investor should put in each company at the optimal level. A risk-tolerant investor who is likely to invest in the efficient portfolio ALU-CPC-GSR and seeks to achieve a maximum return of 65.14% by investing in an efficient portfolio annually despite the high risk of 165.83%, should invest all of his assets in GSR but should expect a Sharpe ratio of 32.05%. A risk-averse investor interested in a least risk of 46.42% from investing in the efficient portfolio must put 69.98% of their assets in ALU,21.17% in CPC and 8.84% in GSR so that he/she could earn a 44.37% expected return with a high Sharpe ratio of 69.73% annually. In addition, an investor who wants to achieve a maximum Sharpe ratio of 71.11% in the efficient portfolio should invest 62.47% of his assets

in ALU.23.72% in CPC and 13.81% in GSR, with a risk of 47.34% and 45.67% returns annually.

EFFICIENT FRONTIER

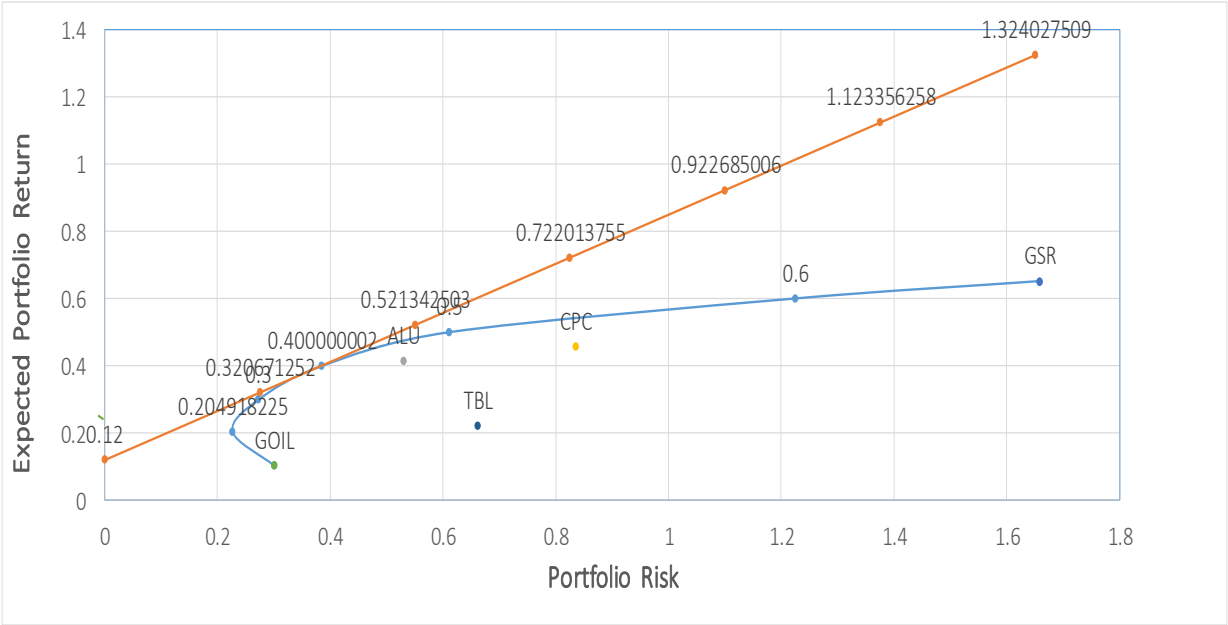

Figure 1. Efficient Frontier and Capital Market Line

Figure 1 represents the efficient frontier and the capital market line. A combination of risky assets is depicted by the efficient frontier. Excel solver was used to generate the values for drawing the efficient frontier by minimizing risk. A line that is drawn tangentially to the efficient frontier from the rate of return that is risk-free is the Capital Market Line. Once we introduce a risk-free asset(rate), Capital Market Line becomes the new efficient frontier. Each point along the CML expresses an efficient allocation between risk-free and risky assets. The minimum-variance portfolio occurs at GOIL. The most efficient portfolio(market portfolio) is formed at the point of intercept of the Capital Market Line and the Efficient Frontier. The Sharpe ratio of the portfolio is represented by the slope of the CML. The CML denotes the expected return of a portfolio at varying levels of risk.

The Sharpe ratio was maximized using Excel Solver and the results implied that when one invests 53.449% in ALU, 21.27% in CPC, 12.32% in GSR, 0.00% in GOIL and 12.92% in TBL, he/she can achieve the market portfolio that yields the highest level of Sharpe ratio. The standard deviation of the CML was calculated by creating dummy variables based on the length of the efficient frontier on the x-axis on the efficient frontier, that is from 0 to 1.65. The expected return was also derived with the Capital Market Line Formula.

CHAPTER FIVE

5.0 Conclusions and Recommendations

The research aimed to take one through the procedure of finding the optimal portfolio among portfolios of stocks listed on the Ghana Stock Exchange. In conclusion, the standard deviation(risk), expected return, kurtosis, and skewness of selected companies have been successfully estimated. Based on analysis using the Markowitz mean-variance model, of the five selected individual stocks,10 three-asset combination portfolios were created to give the investor various options to choose from, based on his/her risk tolerance. By modifying the weights of the different stocks, returns maximized, Sharpe ratios maximized and the risk of the optimal portfolio might be decreased, according to a subsequent analysis using Excel Solver. These findings add to the current information on the GSE as they show how an average investor may construct portfolios optimal in nature to aid in the choices of investment. Also, the efficient frontier of all the five assets was drawn with the intercept of the Capital Market Line and the old efficient frontier indicating the market portfolio.

The research for the three-asset combinations explains that GSR with 65.14% has the highest expected annual return with 165.81% as its expected annual risk, while GOIL has the least annual risk of 30.05% with an expected annual return of 10.31%. An uninformed risk-averse investor may quickly decide to invest only in GOIL because it has the lowest expected annual risk of 30.05% with an expected annual return of 10.31%. The investor is, however, required to diversify and not just invest in one stock. The purpose of diversification is to reduce volatility and improve overall performance. Out of the ten three-asset combinations created, the

combination of ALU-CPC-GSR, considered the most efficient, yielded an expected return of 50.73%, risk of 65.09%, and the highest Sharpe ratio of 0.59504.

The portfolio with the highest Sharpe ratio is the most efficient because it yields a return that best adjusts the risk incurred by the investor. If an individual invests in this portfolio and allocates the same amount of his or her assets to the three stocks, 0.33333 to each, he will still attain 65.09% expected annual risk with a 50.73% expected annual return, emphasizing the significance of diversification. Instead of singling out GOIL for investment, the risk-averse investor can expect an annual risk and return of 29.89% and 24.61% respectively, with a 0.42189 Sharpe ratio if he is well up-to-date and goes for the least risk-driven portfolio, thus GOIL-TBL-ALU.

*By altering weights assigned to the different stocks that make up the portfolio by lowering the weight assigned to the riskier stock, there is a reduction in risk as per the study. Through excel solver the efficient portfolio was optimized (per Table 4), which highlighted the fact that if the investor allocates 100% of his assets to GSR , the expected annual return may be increased to 65.14%, producing a risk of 165.81%. As such, an investor who is known to be risk-averse when investing in the efficient portfolio must assign 100% of his assets to GSR and to enjoy a 65.14% expected annual return with a very high risk of 165.81% which offers far better prospects to the investor compared to ignorantly investing in only GOIL or even in the least risk optimal portfolio (GOIL-TBL-ALU).

Investors are therefore recommended to invest in the combination of ALU-CPC-GSR since it is the most efficient portfolio. Risk-tolerant investors should invest all of their assets in GSR for maximum returns. Risk-averse investors should invest 69.98% of their assets in ALU,21.17% in CPC and 8.84% in GSR. A risk-neutral investor should devote 62.47% of his

or her assets to ALU, 23.72% to CPC, and 13.81 to GSR or invest in any of the study's optimal portfolios.

Lastly, because Excel may create more than ten (10) ideal portfolios, investors should create portfolios with a higher number of stocks to aid in making informed investing choices.

However, investors who wish to invest in all five stocks are recommended to invest 53.449% in ALU, 21.27% in CPC, 12.32% in GSR, 0.00% in GOIL and 12.92% in TBL, in order to achieve the market portfolio that yields the highest level of Sharpe ratio.

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APPENDIX

CPC Cocoa Processing Company

ALU Aluworks limited

GOIL Ghana Oil Company Limited

TBL Trust Bank Limited

GSR Golden Star Resources

GSE Ghana Stock Exchange

CML Capital Market Line