

Supervised Machine Learning

Part One: Regression



Agenda

- Overview of Supervised Learning
- Regression Models - Linear Regression
 - OLS
 - LASSO
 - Ridge
 - ElasticNet
- Model Evaluation
 - Pipelines & Feature Selection (GridSearchCV)
- Model Selection

Supervised Learning

Definition

Supervised learning is the machine learning task of inferring a function from labeled training data. The training data consist of a set of training examples. In supervised learning, each example is a pair consisting of an input object (typically a vector) and a desired output value (also called the supervisory signal). A supervised learning algorithm analyzes the training data and produces an inferred function, which can be used for mapping new examples.

https://en.wikipedia.org/wiki/Supervised_learning

Important Points

1. Labeled training data
2. Desired output
3. Produces an inferred function
4. Used for novel examples

Approaches

1. Classification

2. Regression

Regression Models

Regression Models

- Supervised learning algorithms that estimate the relationship among variables.
- Focus is on the relationship between a dependent variable (target) and 1(+) independent variables (predictor)
- Does the dependent variable change when the independent variable(s) change?
- Common algorithms
 - Generalized linear models

Generalized Linear Models

Linear Models

Linear Regression fits a linear model to the data by adjusting a set of coefficients w to minimize the residual sum of squares between observed responses & prediction.

(1) Linear model $y = X\beta + \epsilon$

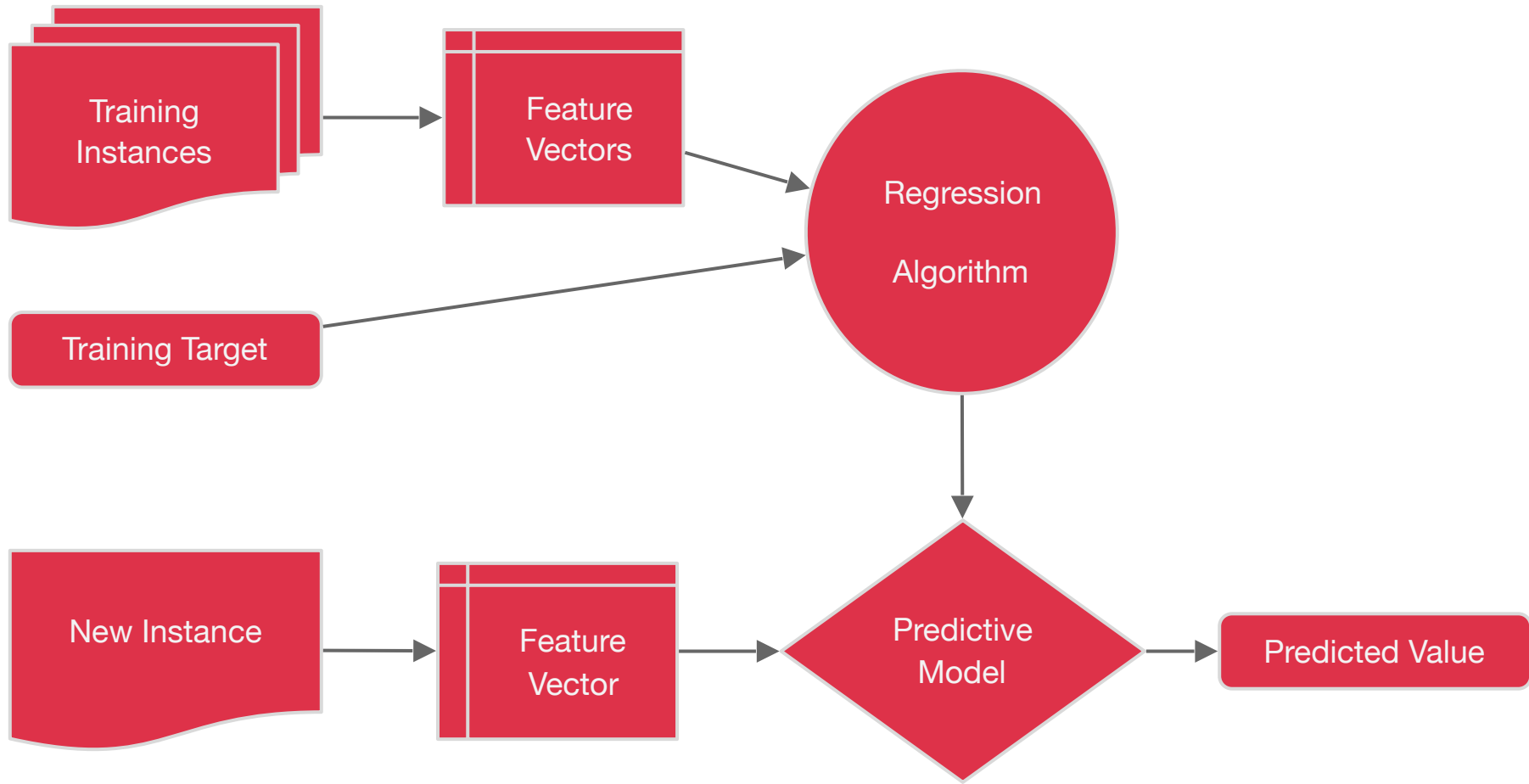
(2) Objective function $\min_w \sum (Xw - y)^2$

(3) Predictive model $\hat{y}(w, x) = w_0 + w_1x_1 + \dots + w_px_p$

Notation:

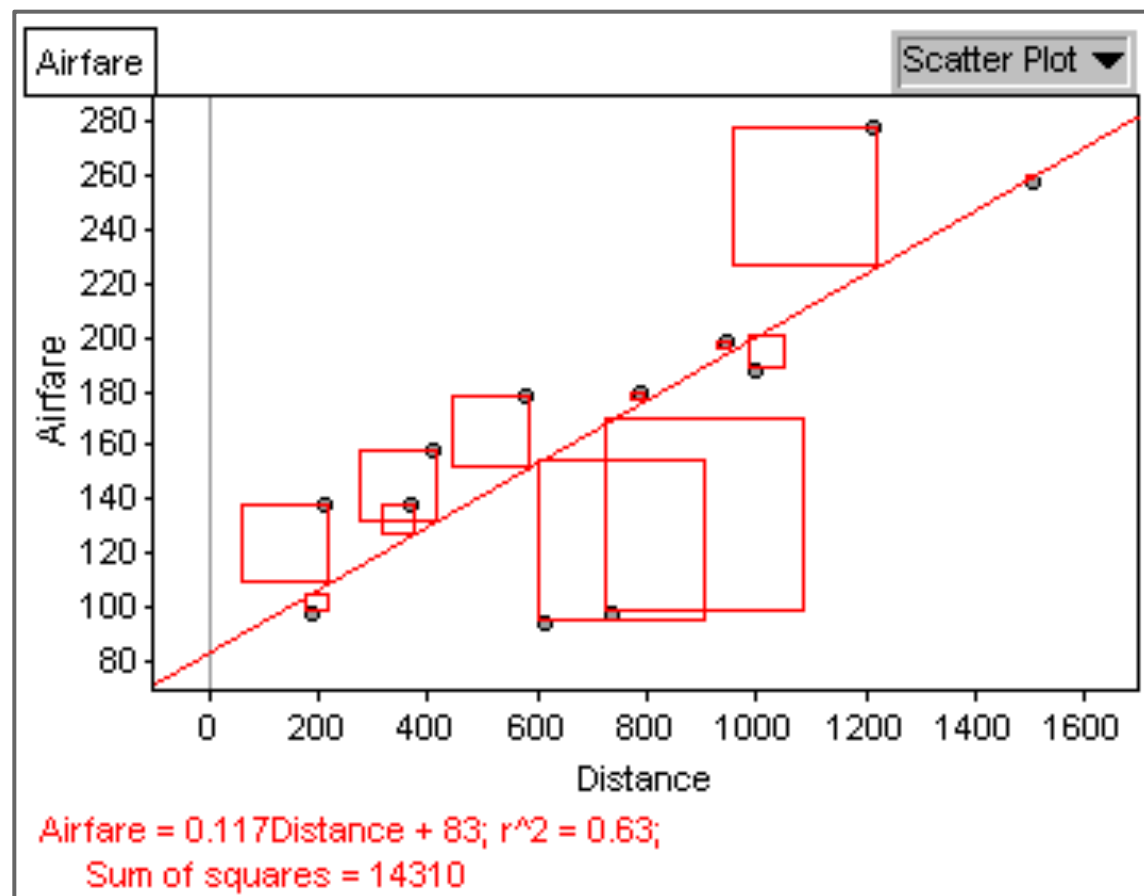
- y is the observed value
- x is the input variables
- β is the set of coefficients
- ϵ is noise or randomness in observation
- w is the set of weights
- w_0 is the ability to adjust the plane in space
- \hat{y} is the predicted value

Regression Pipeline



Ordinary Least Squares

- Method for estimating unknown parameters in a linear regression model
- Keep adjusting parameters until minimum squared residuals (e.g. minimize some cost function).
- Relies on the independence of the model terms
- *multicollinearity*: two or more predictor variables in a multiple regression model are highly correlated, one can be linearly predicted from the others
- If this happens, the estimate becomes sensitive to error.



Simple Regression with OLS

```
from sklearn import linear_model
from sklearn.metrics import mean_squared_error, r2_score

regr = linear_model.LinearRegression()
regr.fit(X_train, y_train)
LinearRegression(copy_X=True, fit_intercept=True, normalize=False)

print regr.coef_
[ -6.02985639e+01  -3.02367158e+11   3.02367158e+11   6.04734316e+11
   4.17860883e+00  -3.41060763e-02   2.03234971e+01   2.15758256e-01]

print regr.intercept_
76.9490920195

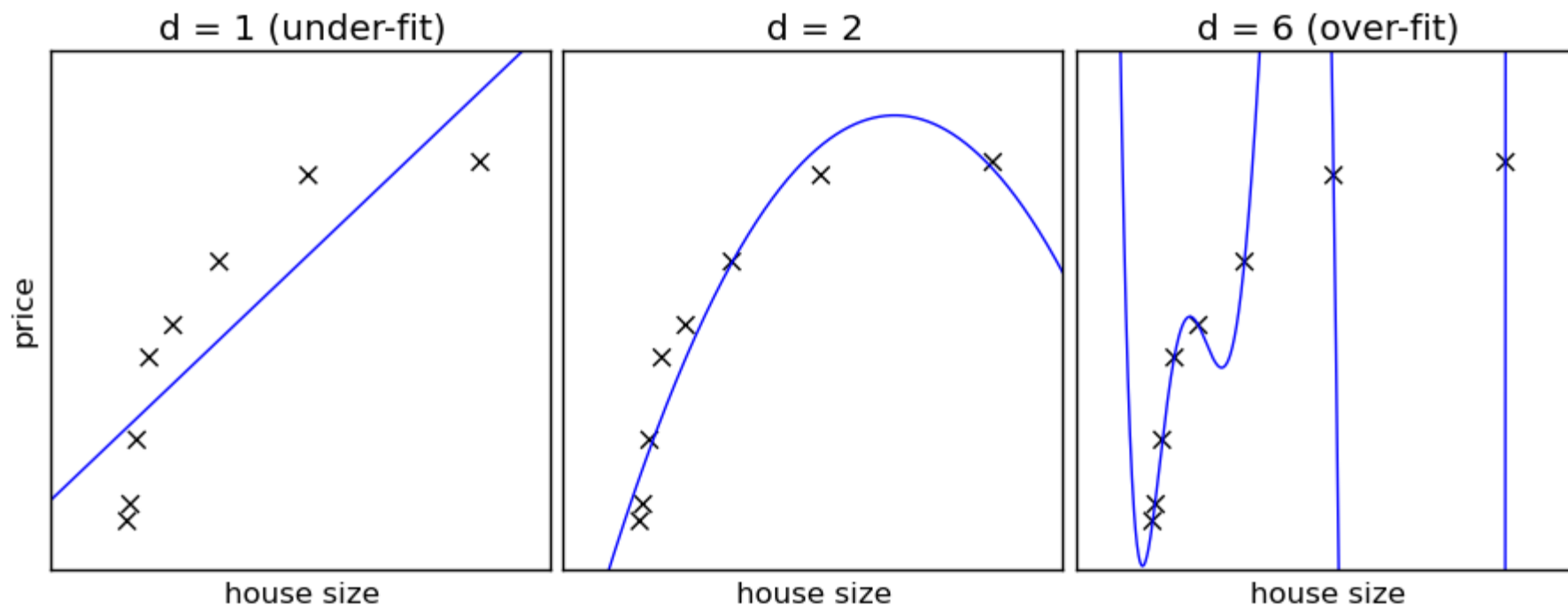
print mean_squared_error(y_test, regr.predict(X_test))
7.92744075579

regr.score(X_test, y_test)  # r2_score(y_test, regr.predict(X_test))
0.92521397739317868
```

Lab: Linear Regression

Sklearn & MLlib

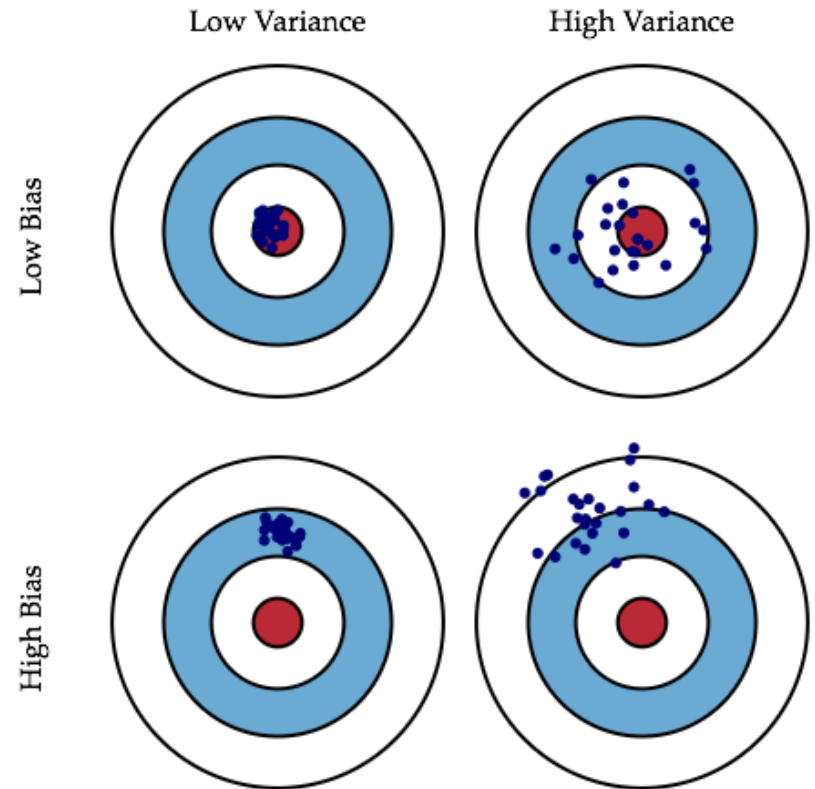
What Can Go Wrong With Simple Linear Models?



Error: Bias vs Variance

Bias: the difference between expected (average) prediction of the model and the correct value.

Variance: how the predictions for a given point vary between different realizations for the model.



<http://scott.fortmann-roe.com/docs/BiasVariance.html>

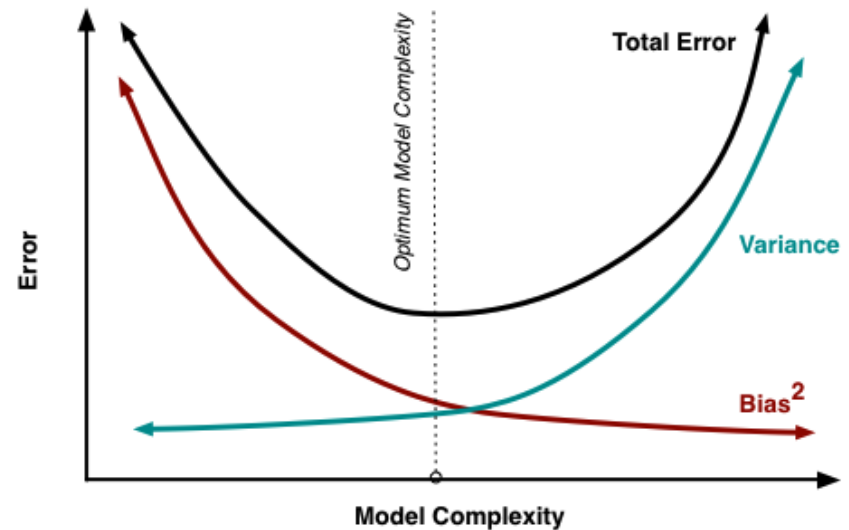
Bias vs. Variance Trade-Off

Related to model complexity:

The more parameters added to the model (the more complex), Bias is reduced, and variance increased.

Sources of complexity:

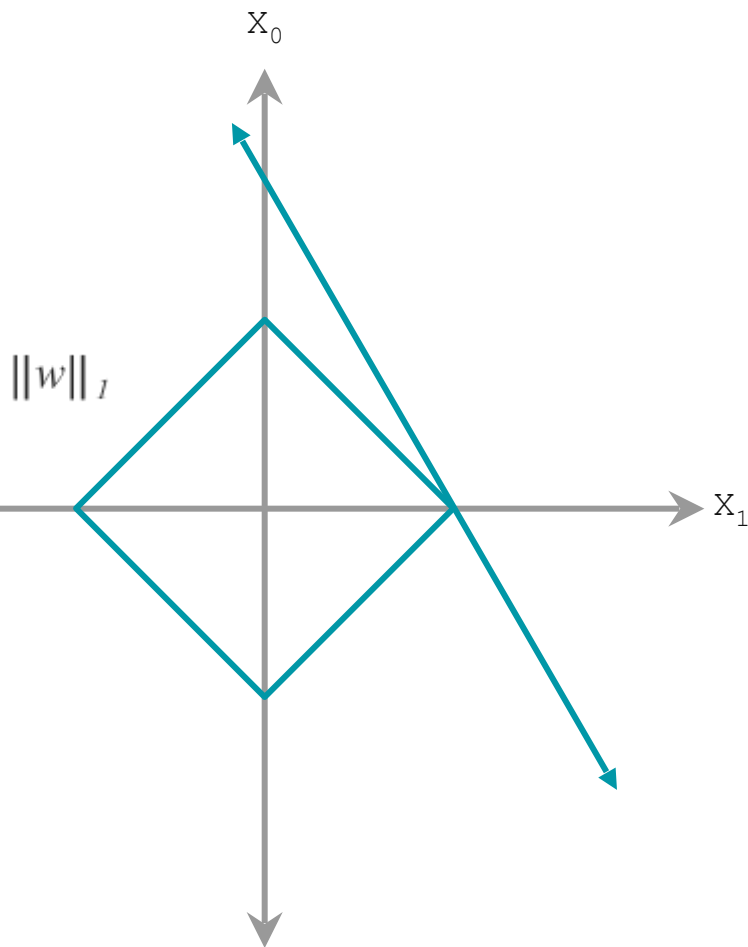
- k (nearest neighbors)
- # of features
- epochs (neural nets)
- learning rate



<http://scott.fortmann-roe.com/docs/BiasVariance.html>

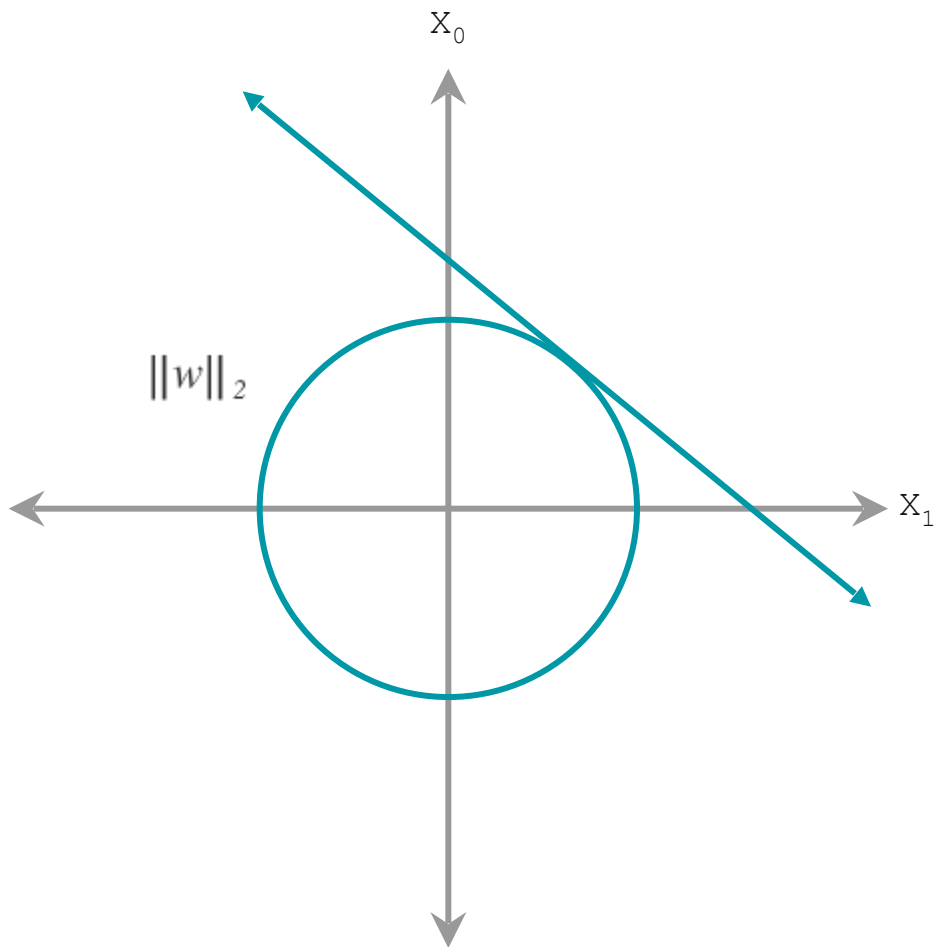
Regularization

- As we increase the complexity of the model we reduce bias but increase variance, which can lead to overfitting.
- Regularization introduces a parameter to penalize the magnitude of coefficients of features and minimize the error between predicted and actual observations.
 - Generally, the size (magnitude) of coefficients increases exponentially with an increase in model complexity, which leads to overfitting.
- OLS does not include regularization



L1 Regularization

Possibility that a feature is eliminated by setting its coefficient equal to zero.



L2 Regularization

Features are kept balanced by minimizing the relative change of coefficients during learning.

Lasso

- Uses L1 regularization
- Prefers fewer parameters attempting to reduce the number of variables the solution depends on.

Lasso Regression

```
clf = linear_model.Lasso(alpha=0.5)
clf.fit(X_train, y_train)
```

```
Lasso(alpha=0.5, copy_X=True, fit_intercept=True,
      max_iter=1000, normalize=False, positive=False,
      precompute='auto', tol=0.0001, warm_start=False)
```

```
print mean_squared_error(y_test, clf.predict(X_test))
18.84667821
```

```
clf.score(X_test, y_test)
0.82870491763341947
```

Lab: Lasso Regression

Sklearn & MLlib

Ridge Regression

- Prevent overfit/collinearity by penalizing the size of coefficients - minimize the penalized residual sum of squares:
- Said another way, shrink the coefficients to zero.

$$\min_w \sum (Xw - y)^2 + \alpha \sum w^2$$

- Where $\alpha > 0$ is complexity parameter that controls shrinkage. The larger α , the more robust the model to collinearity.
- Alpha influences the bias/variance tradeoff: the larger the ridge alpha, the higher the bias and the lower the variance.

Ridge Regression

```
clf = linear_model.Ridge(alpha=0.5)
clf.fit(X_train, y_train)
```

```
Ridge(alpha=0.5, copy_X=True, fit_intercept=True,
      max_iter=None, normalize=False,
      solver='auto', tol=0.001)
```

```
print mean_squared_error(y_test, clf.predict(X_test))
8.34260312032
```

```
clf.score(X_test, y_test)
0.92129741176557278
```

Lab: Ridge Regression

Sklearn & MLlib

Choosing alpha

We can search for the best parameter using the RidgeCV which is a form of Grid Search, but uses a more efficient form of leave-one-out cross-validation.

```
import numpy as np
n_alphas = 200
alphas = np.logspace(-10, -2, n_alphas)
clf = linear_model.RidgeCV(alphas=alphas)
clf.fit(X_train, y_train)
```

```
print clf.alpha_
0.0010843659686896108
```

```
clf.score(X_test, y_test)
0.92542477512171173
```

Error As a Function of Alpha

```
clf = linear_model.Ridge(fit_intercept=False)
errors = []

for alpha in alphas:
    splits = tts(dataset.data, dataset.target('Y1'), test_size=0.2)
    X_train, X_test, y_train, y_test = splits
    clf.set_params(alpha=alpha)
    clf.fit(X_train, y_train)
    error = mean_squared_error(y_test, clf.predict(X_test))
    errors.append(error)

axe = plt.gca()
axe.plot(alphas, errors)
plt.show()
```

And More Models

Listed only from the Documentation (not API):

- ElasticNet
- Multi-Task Lasso
- Least Angle Regression
- LARS Lasso
- Orthogonal Matching Pursuit (OMP)
- Bayesian Regression
- Automatic Relevance Determination (ARD)
- Logistic Regression
- Stochastic Gradient Descent
- Perceptron
- Random Sample Consensus (RANSAC)

ElasticNet

- Combines L1 and L2 regularization
- Works well with a large number of features
- Forms a group of independent, correlated variables. If any of the variables are a strong predictor, EN includes the entire group in model building.

Lab: ElasticNet Regression

Sklearn & MLlib

Pipelines (Steps)

`sklearn.pipeline.Pipeline`

- Sequentially apply *repeatable* transformations to final estimator that can be validated at every step.
- Each step (except for the last) must implement `Transformer`, e.g. `fit` and `transform` methods.
- Pipeline itself implements both methods of `Transformer` and `Estimator` interfaces.


```
from sklearn.pipeline import Pipeline
from sklearn.cross_validation import KFold

pipeline = Pipeline([
    ('extract_essays', EssayExtractor()),
    ('counts', CountVectorizer()),
    ('tf_idf', TfidfTransformer()),
    ('classifier', MultinomialNB())
])

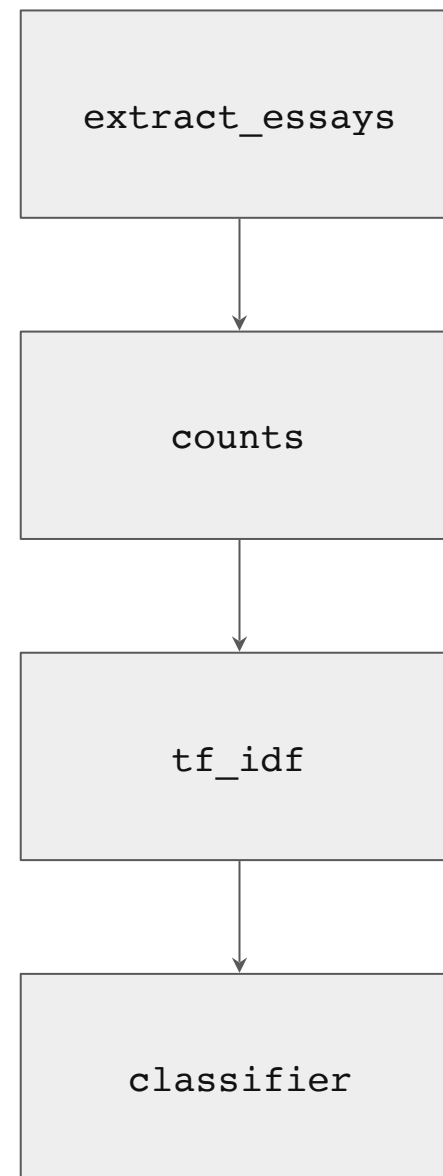
scores = []
folds = KFold(

    n = dataset.data.shape[0], n_folds=12, shuffle=True

)

for tid, cid in folds:
    pipeline.fit(dataset.data[tid], dataset.target[cid])
    score = pipeline.score(dataset.data[cid],
dataset.target[cid])
    scores.append(score)

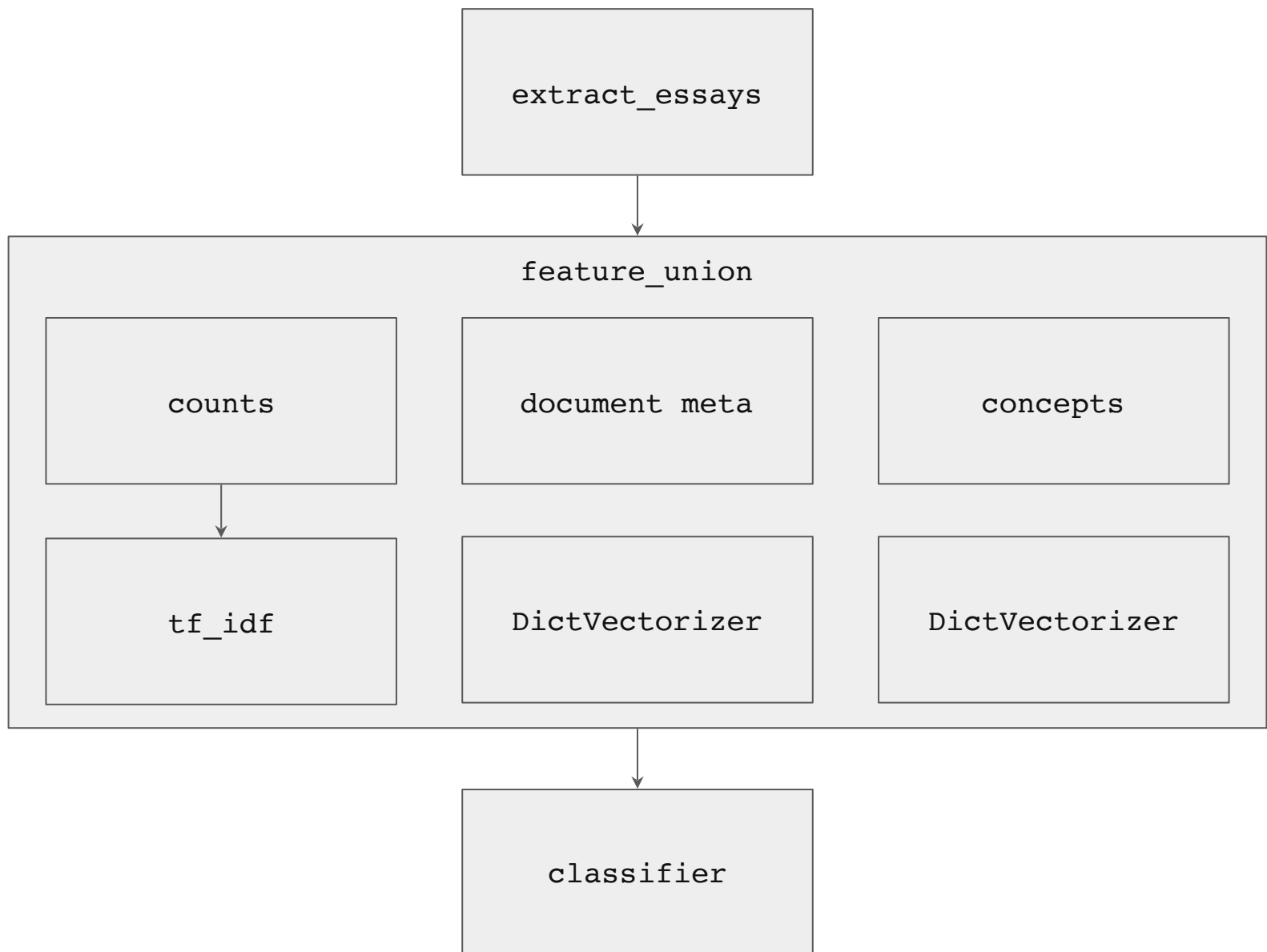
print("Score: {}".format(np.mean(scores)))
```



Pipelined Feature Extraction

The most common use for the Pipeline is to combine multiple feature extraction methodologies into a single, repeatable processing step.

- FeatureUnion
- SelectKBest
- TruncatedSVD
- DictVectorizer



Feature unions example from Zac's post

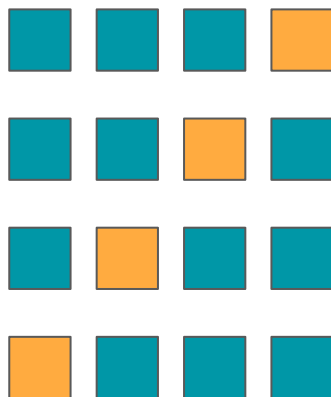
Model Evaluation

Cross-Validation and Evaluation

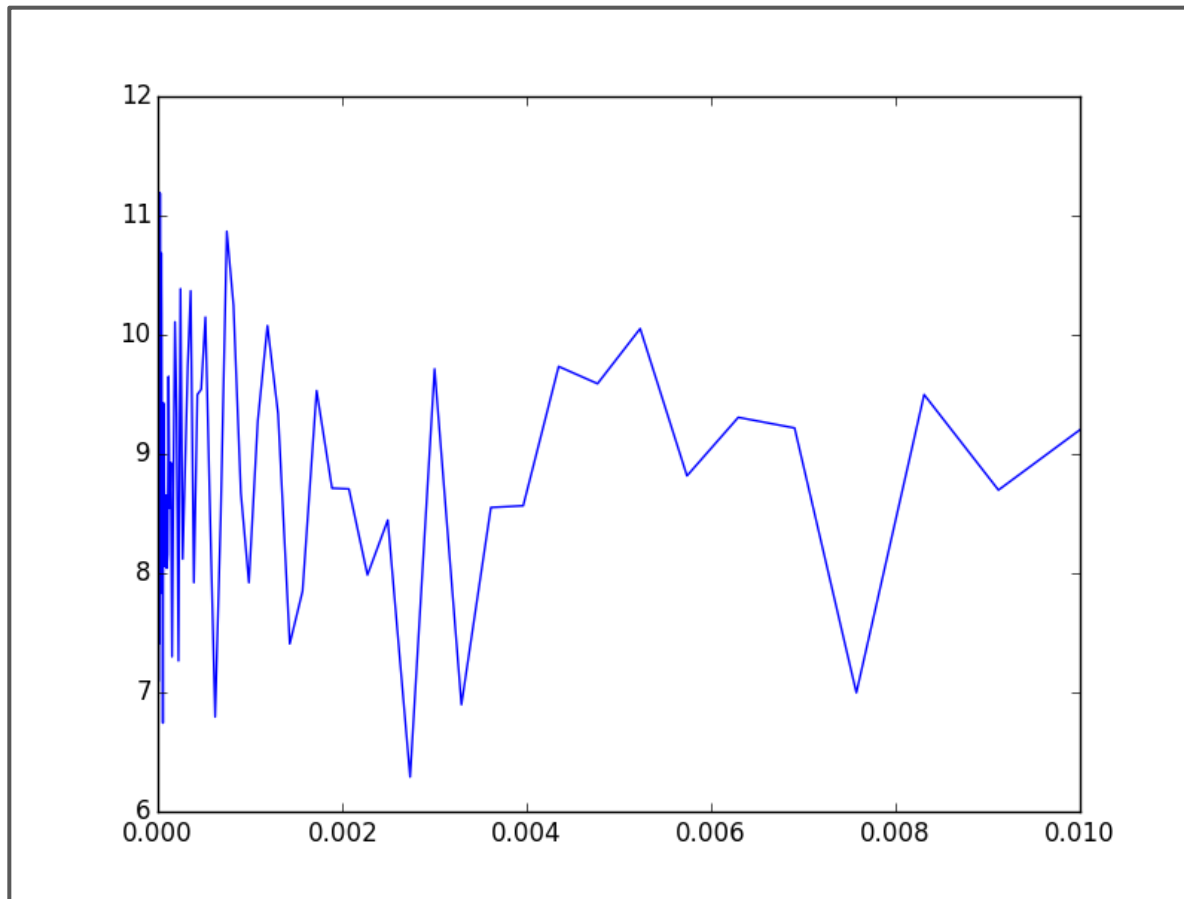
- In regressions we can determine how well the model fits by computing the mean square error and the coefficient of determination.
 - $MSE = np.mean((predicted - expected)^2)$
 - R^2 is a predictor of “goodness of fit” and is a value $\in [0,1]$ where 1 is perfect fit.

Cross-Validation and Evaluation

In order to prevent overfit and be assured of generalizability, cross-validation fits the model on a portion of the data set and evaluates it on an unseen portion of the data set. Shuffle data, split into a large train set and smaller test set. This can be done $K=12$ times, and scores averaged.



Resulting Plot



How to pick the right parameters?



Search/Tuning

Search Requires

- Estimator
- Parameter Space
- Method for sampling
- Cross validation scheme
- A score function

Search Types

- Exhaustive
- Randomized
- Parallel
- Leave One Out
- Model Specific

Lab: Pipelines & Feature Extraction with Sklearn

Model Selection



A Venn diagram consisting of three overlapping circles. The leftmost circle is light blue and contains the text 'Feature Analysis'. The middle circle is a medium blue and contains the text 'Algorithm Selection'. The rightmost circle is a dark blue and contains the text 'Hyperparameter Tuning'. The circles overlap in a way that creates a central region where all three intersect, and two other regions where two circles intersect. The text is white and centered within each circle.

Feature Analysis

Algorithm Selection

Hyperparameter Tuning

The Model Selection Triple



Feature Analysis

- **Define** a bounded, high dimensional feature space that can be effectively modeled.
- **Transform** and manipulate the space to make modeling easier.
- **Extract** a feature representation of each instance in the space.

The Model Selection Triple



Algorithm Selection

- Select a **model family** that best/correctly defines the relationship between the variables of interest.
- Define a **model form** that specifies exactly how features interact to make a prediction.
- Train a **fitted model** by optimizing internal parameters to the data.

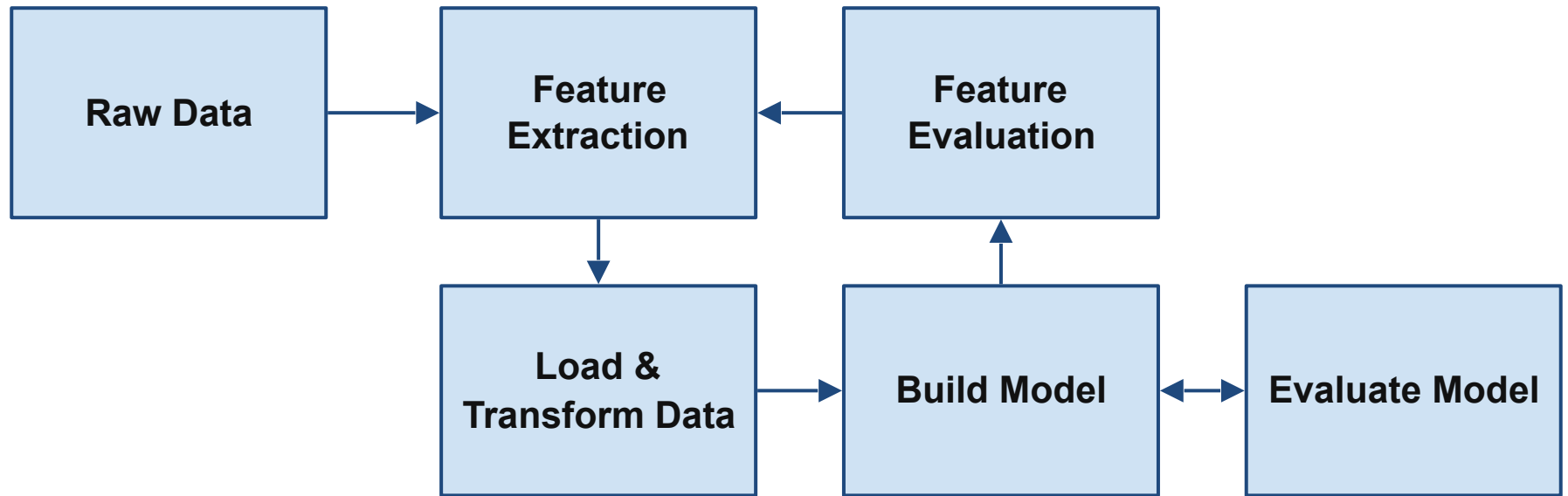
The Model Selection Triple



Hyperparameter Tuning

- **Evaluate** how the model form is interacting with the feature space.
- **Identify** hyperparameters (parameters that affect training or the prior, not prediction)
- **Tune** the fitting and prediction process by modifying these params.

Preliminary Workflow



Choosing the Right Estimator



