Deep learning HW2

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Q3:

3.a:

Starting off with the definitions:

$$T(Im) : T(Im(i,j)) = Im(i-l, j-k)$$
$$f(i,j) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n) * Im(i-m, j-n)$$

Now, we shall prove that $T(f(Im)) = f(T(Im))$:

$f(T(Im)) = f(Im(i-k, j-l) \forall i, j) =$
$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n) * Im(i-l-m, j-k-n) =$

We substitute m and n by $m' = m + l$ and $n' = n + k$:
$= \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} h(m'-l, n'-k) * Im(i-m', j-n')$

On the other hand, since f is defined by h, we can show:
$T(f(Im)) = T(\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n) * Im(i-m, j-n)) =$
$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} T(h(m,n)) * Im(i-m, j-n) =$
$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m-l, n-k) * Im(i-m, j-n)$

To sum, because the only difference is the names of the parameters, we receive:

$$T(f(Im)) = f(T(Im))$$

3.b:

3.c:

let us look at the following matrix, A:

$$A = \begin{matrix} 1 & 2 & 3 & 1 \\ 15 & 4 & 3 & 2 \\ 3 & 6 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{matrix}$$

while MAX-POOLING 3x3 will give us an answer of 15 for the top square, a one pixel shift would make the answer 6 instead. Thus, we have given a counter-example for MAX-POOLING 3x3 being invariant to one pixel shift.

3.d:

It is easy to notice that if all weights are equal, a fully connected layer will not change for any ordering of $x_1, x_2, ..., x_n$. therefore, equal weights for all inputs is a sufficient condition for the layer to be permutation invariant.