PreCalculus

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Pre-Algebra

Real numbers

in mathematic we work with numbers but these numbers have some features like some of them positive or some of them have fraction even some of them is not a real number they are complex numbers therefore for make a clearlty the numbers we grouped them. in this section you will see this groups

Natural Numbers $\mathbb{N}$
Natural numbers are most primitive numbers sets of numbers sets and they include numbers 1 to ∞ and denoted with $\mathbb{N}$ symbol

Whole Numbers $\mathbb{W}$
Whole numbers is a number sets that inherit all numbers from natural in additon this include 0 beside natural numbers and this denoted with $\mathbb{W}$ symbol

Integers $\mathbb{Z}$
Integers is a number sets that include negative numbers like -1 -26 beside positive numbers and this is denoted with $\mathbb{Z}$ symbol

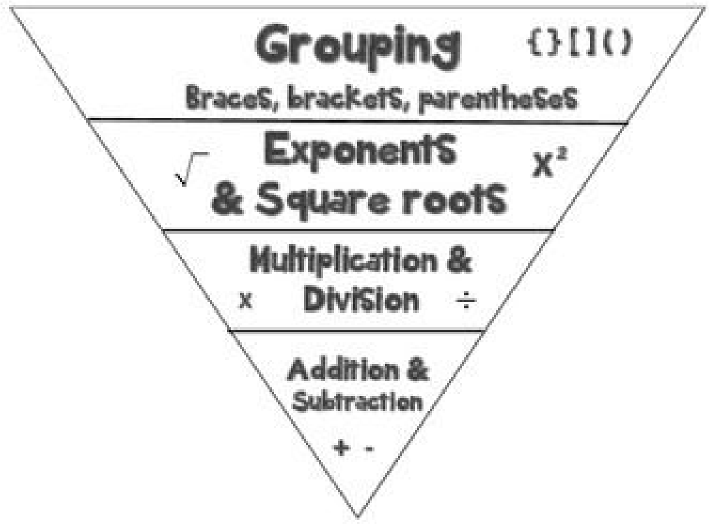
Rational Numbers $\mathbb{Q}$
Rational numbers are that have decimal expansion which are terminating with a pattern like 15.76$\overline{0}$ or 12.7$\overline{0}$ and they denoted $\mathbb{Q}$ symbol

Irrational Numbers $\mathbb{I}$
Irrrational numbers are that have decimal expansion too but not terminating with a pattern for instance π or e $\sqrt{2}$ and they denoted with $\mathbb{I}$

Real Numbers $\mathbb{R}$
Real numbers are include all sets that i mentioned before number sets is not finish here we have other numbers sets but i don't cover in this section this denoted $\mathbb{R}$

Order Of Operation

despite you are going to struggle with deritative and integrals we still have to know arithmetic so for this firstly we have to learn order of operation for more easy memorize this we use order of operation triangle and its look like thisdespite you are going to strugle with deritative and integrals we still have to know arithmetic so for this firstly we have to learn order of operation for more easy memorize this we use order of operation triangle and its look like this



in this triangle firstly we make expression in the parantheses after exponents after multiplication and division finally subtraction and addition

Interval Notation

intervals are kind of standart subset of the real numers and they pop up over all the place in calculus maybe you are writing an interval because it's the solution of inequality maybe you are writing it's domain of function. as you understand intervlas are very common topic of calculus. we have some interval types and below i indicate this

Interval Types

let $a < b$	
open:	$(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$
closed:	$[a,b] = \{x \in \mathbb{R} \mid a \leq x \geq b\}$
half-open:	$[a,b)$, $(a,b]$
infinite:	$[a,\infty) = \{x \in \mathbb{R} \mid x \geq a\}$
	$(a,\infty) = \{x \in \mathbb{R} \mid x > a\}$
	$[b,\infty) = \{x \in \mathbb{R} \mid x \leq b\}$
	$(-\infty,b) = \{x \in \mathbb{R} \mid x < b\}$
	$[-\infty,\infty] = \mathbb{R}$

Combining Intervals

$$\text{solve } x^2 > 4$$
$$x > 2 \text{ or } x < -2$$
$$(2, \infty) \text{ or } (-\infty, -2)$$
$$\text{write as } (-\infty, -2) \cup (2, \infty)$$

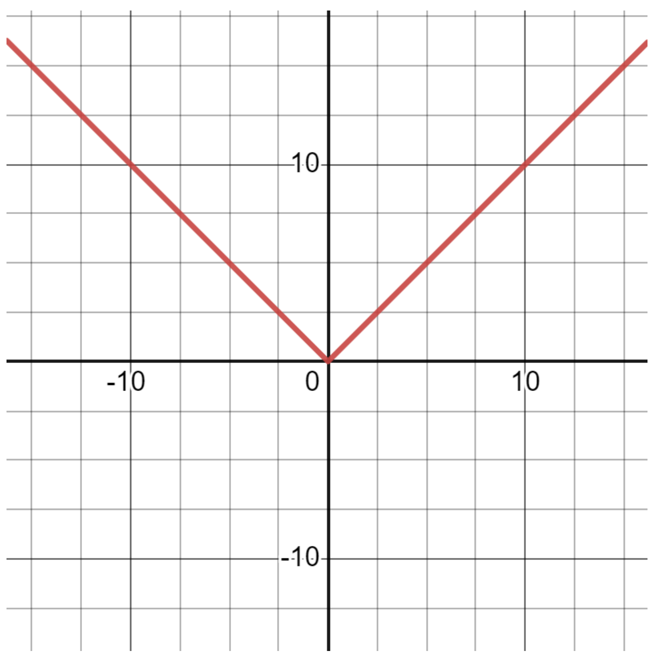
Absolute Values

Notation: $|x| (x \in \mathbb{R})$ absolute value is distance from 0 to x not a magical eraser that erase negative sign

$$|29| = 29 \text{ if } x \geq 0$$
$$|x| = x$$

$$|-7.5| = 7.5 \text{ if } x < 0$$
$$|x| = -x$$

$$y = |x|$$



$$-3x + 4 < 2$$
$$-3 + 4 - 4 < 2 - 4$$
$$\frac{-3x}{-3} < \frac{-2}{-3}$$
$$x > \frac{2}{3}$$

Basic Rules

$|a| \leq b$
same as $-b \leq a \leq b$

$$|x| < 4$$
$$-4 < x < 4$$

$|a| > b$
same as $a > b$ or $a < -b$

$$|x| > 4$$
$x > 4$ or $x < -4$
$$(-\infty, -4) \cup (4, \infty)$$

Fraction Arithmetic

probably you know make operations on the fractions so i dont cover muchly but i want to indicate fundamentals of them

Subtraction/Addition
in the subtraction and addition we equals the denominators and we add the numerators therefore we make addition and subtraction on fractions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

Multiplacation
in multiplacation we multiplacation denominators and numerators among of them

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Division
in divison we write reciprocal of second division and we multiplacation the fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Exponents

actually exponents are very simple topic of mathematic but when combining with other topics it's can be very hard but don't worry with solve questions you will be better but for make this we have to know topic so let's talk about exponents

exponents are repeatedly writing a number as a multiplacation like this

$a^3 = a \times a \times a$
$a^3 = a \times a \times a \times a$
$a^3 \times a^4 = (a \times a \times a) \cdot (a \times a \times a \times a)$

Exponents Rules
we have some exponents rules but we have many of them so i write most important rules here as i said if you wery well them you can solve many questions but don't memorize understand why they are like this

$$a^m \times a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$a^{-n} = \frac{1}{a^n}$$
$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n \cdot b^n$$
$$(\frac{a}{b})^n = \frac{a^n}{b^n}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}$$
$$a^{\frac{m}{n}} = \sqrt[n]{m} = (\sqrt[n]{a})^m$$

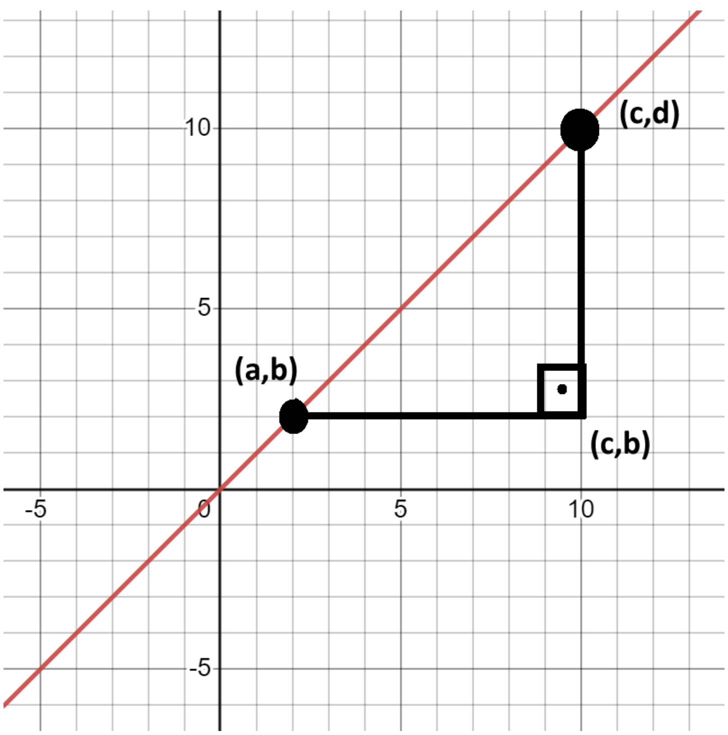
Algebra

Lines

Caution: i don’t cover what are they and how solving equations and inequalities cause if you reading this book i think you know this topic and if you learned why i mention equations we can continue

lines are very much uisng in calculus and they have very features but we mostly use slope of lines in cslculus so in this section i trach you slope of lines

Slope



slope: $\frac{\text{rise}}{\text{run}}$

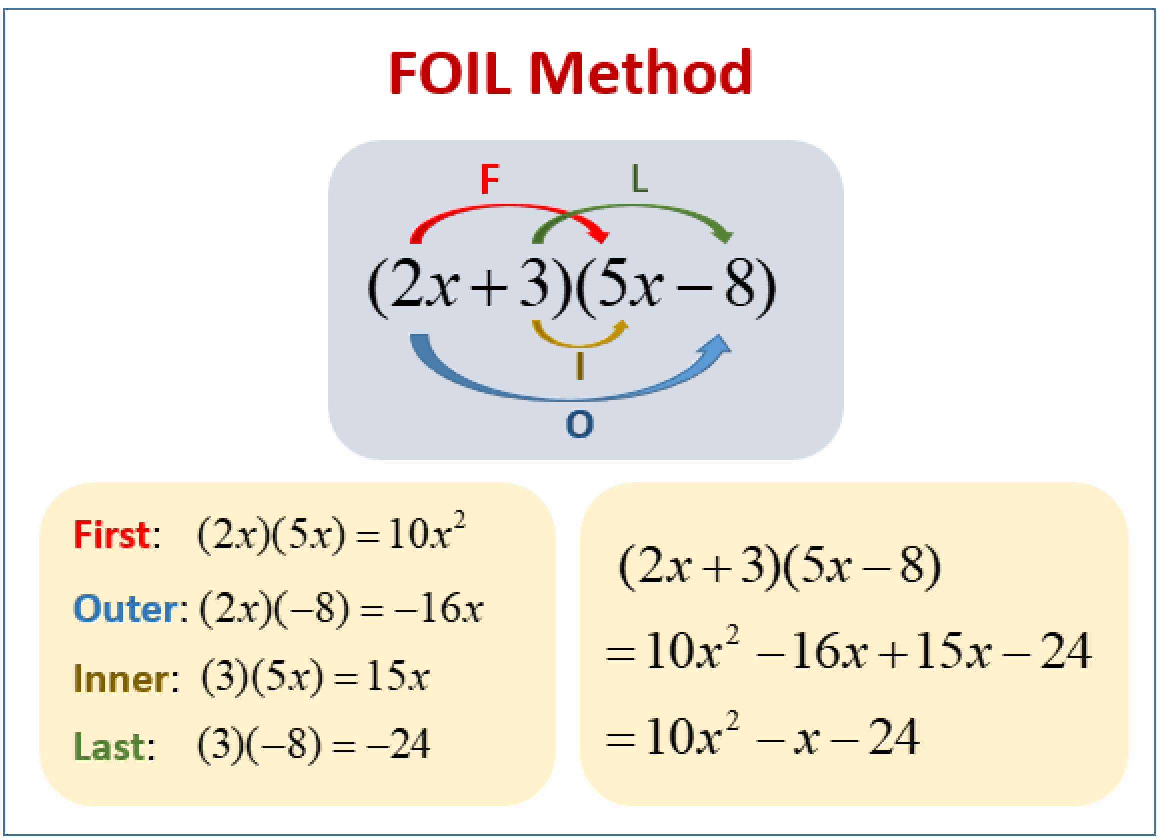
$$m = \frac{\Delta y}{\Delta x} = \frac{d-b}{c-a}$$

$$m = \frac{y-b}{x-a} = \frac{y-d}{x-c}$$

$$\begin{aligned} y-b &= m(x-a) \\ y &= mx-ma+b \\ y &= mx+\underbrace{(b-ma)}_{\text{y-intercept}} \end{aligned} \qquad \text{OR} \qquad \begin{aligned} y-d &= m(x-c) \\ y &= mx-mc+d \\ y &= mx+\underbrace{(d-mc)}_{\text{y-intercept}} \end{aligned}$$

FOIL Method

when you product the two bionomials you can use foil method you might heard this but i want continue to explain firstly we product first items subsequently ouert items after inner items and subsequently we product last items and finally we simplify terms that we get from product



Polynomials

polynomials is very imporntant topic of algebra and they have 2 imporntant features first powers of a variable and coefficients we call the the biggest power of a variable as degree and this is degree of the polynomial and coeficient of first terms is called leading coefficient we assume this not eqaul to 0

$$p(x) = \underbrace{a_n}_{\text{leading coefficient}} \cdot x^{\overbrace{n}^{\text{degree}}} + a_{n-1} \cdot x^{n+1} \ldots\ldots\ a_2 \cdot x^2 + a_1 \cdot x + a_0$$

Example

$5x^3 - 6x + 8$
$\frac{-4x^7}{3} + 2\sqrt{2}x^6 - \pi^2 x^4 - 2x + 1$
$\frac{x^2-1}{x+1}(x \neq 1)$

Not Polynomial
for beign a polynomial your powers of variable must be integers and greater than or equal to 0 so below terms are not polynomials

$\frac{1}{x}, \frac{1}{x^2}$
$\sqrt[3]{5}$

Factor And Roots

If p(a) = 0 then (x-a) is a factor of p(x)

$$p(x) = (x - a) \cdot q(x)$$

If you are trying to factor a polynomial this is key result but it still leaves you questions how do we know which number we have to plug

Rational Roots Theorem
If a is root of p(a) with a = $\frac{\text{u}}{\text{v}}$ $(u, v \in \mathbb{Z})$ in lower terms

then a_0 is divisible by u_1
and a_n is divisible by v_1

Example
$p(x) = 2x^3 - x^2 + 3x + 4$

Possible Rational Roots:
4 is divisible by $\pm$1,$\pm$2,$\pm$4
2 is divisible by $\pm$1,$\pm$2

Possible Roots are:
$\pm\frac{1}{2}$,$\pm$1,$\pm$2,$\pm$4

From now only remains trying to values and if values gives us 0 that means this value is rational roots of polynomial but if none of values can't fit to polynomial this not mean this poylnomial dont have any root this mean polynomial have irrational roots and we can't find them with thus methods

Factoring Quadratics

For factoring quadratics we can go with many solution you can choose whatever you want

Formula Way
this formula can work in many situtaions but when roots are irrational this formula won't work it will say you this quadratic don't have any real roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring With FOIL Method
we have another method to factor quadratics and this method based on foil method and actually this way understand better with examples so i dnot expalin how it's wrok i show examples

$$x^2 - 5x + 6 = (x + a) \cdot (x + b)$$
$$x^2(a + b)x + ab$$
$$ab = 6$$
$$a + b = -5b$$
$$x^2 - 5x + 6 = (x - 2) \cdot (x - 3)$$

but what if we have coefficent on x^2 term actually it's change a little bit and we have 2 way to factor i will show them all of them

Way 1
first way is divide polynomial to leading coefficent

$$2x^2 + 3x + 1 = 2(x^2 + \frac{3x}{2} + \frac{1}{2})$$
$$ab = \frac{1}{2}$$
$$a + b = \frac{3}{2}$$
$$= 2(x + 1) \cdot (x + \frac{1}{2})$$
$$= (x + 1) \cdot (2x + 1)$$

Way 2
in this way we product constant and leading coefficent and from now we make remains operations based on this

$$2x^2 + 3x + 1 = (x + a) \cdot (x + b)$$
$$ab = 2$$
$$a + b = 3$$
$$2x^2 + 2x + x + 1 = 2x(x + 1) \cdot (x + 1)$$
$$(x + 1) \cdot (2x + 1)$$

Factoring Formulas

while making factoring we have some formulas like differance of squares and others in this section i will show them

Difference of Squares:

$$x^2 - a^2 = (x - a) \cdot (x + a)$$

Caution: $x^2 + a^2$ is irreduciable

Difference of Cubes:

$$x^3 - a^3 = (x - a) \cdot (x^2 + ax + a^2)$$

Sum of Cubes:

$$x^3 + a^3 = (x + a) \cdot (x^2 - ax + a^2)$$

Examples

$$\text{DOS: } 9y^2 - 4x^2 = (3y - 2x) \cdot (3y + 2x)$$

$$\text{DOC: } 27y^3 - \frac{1}{8} = (3y - \frac{1}{2}) \cdot (9y^2 + \frac{3y}{2} + \frac{1}{4})$$

$$\text{SOC: } 27y^3 + \frac{1}{8} = (3y + \frac{1}{2}) \cdot (9y^2 - \frac{3y}{2} + \frac{1}{4})$$

Grouping
in this we simplify equation as possible and at the end we get a product of 2 binomials and with this we can find root of polynomial and master at this method get a lot of time if you are don't a genius but with hardworking you can be very good at

Example

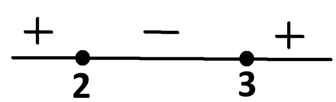
$$2u^3 - 4u^2 + 3u - 6$$
$$2u^2(u-2) \cdot 3(u-2)$$
$$(u-2) \cdot (2u+3)$$

Polynomial Inequalities

this topic is very simple when you are begin but while you advance it getting harder so instead of explaning how it done by the way probably you already know how it's doing inequalities i solve some examples and leave you many question about this topic therefore you will be more understand at topic as you solve questions

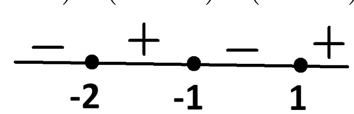
$$2x + 4 > 3$$
$$2x > -1$$
$$x > -\frac{1}{2}$$
$$x \in (-\frac{1}{2}, \infty)$$

$$x^2 > 5x - 6$$
$$x^2 - 5x + 6 > 0$$
$$(x-2) \cdot (x-3) > 0$$



$$x \in (-\infty, 2) \cup (3, \infty)$$

$$x^3 > 2 + x - 2x^2$$
$$x^3 + 2x^2 - x - 2 > 0$$
$$x^2(x+2) \cdot -1(x+2) > 0$$
$$(x+2) \cdot (x^2 - 1) > 0$$
$$(x-1) \cdot (x+1) \cdot (x+2) > 0$$



$$x \in (-2, -1) \cup (1, \infty)$$

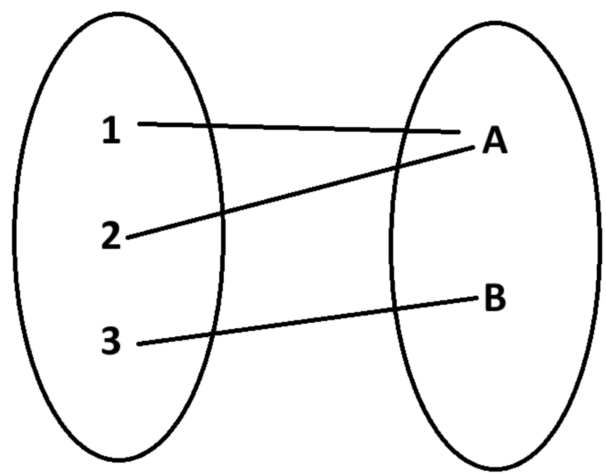
Functions

Function Definition

let A,B be sets a function f from A to B $(f : A \to B)$ is a rule that assigns each a $\in$ A to a unique element b $\in$ B

Example

$$A = 1, 2, 3$$
$$B = A, B$$



Function Notation

to notate the functions we use a letter like f,g or h but mostly we use f and we have to side input side and output which inside of the function paranteheses callled input and other side of equal called output in this sections we don’t have cool things so i don’t explained deeply

Example

$f(x) = 3x^2 - 2x + 1$

Function Domains

as you predict we don't give to every number to functions so interval that we can give to function is called domain and and output set of function callled codomain in calculus we assume domain f:$\mathbb{R} \to \mathbb{R}$

Examples

$$f(x) = \frac{1}{x^2 - 3x + 2}$$

$$f(x) = \frac{1}{(x-1)\cdot(x-2)}$$

$$(x-1)\cdot(x-2) \neq 0$$

dom(f) $= \{x \in \mathbb{R} | x \neq 1, 2\}$

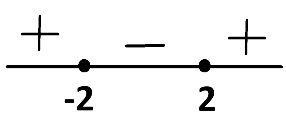
$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

$$g(x) = \sqrt{4 - x^2}$$

$$4 - x^2 \geq 0$$

$$x^2 - 4 \leq 0$$

$$(x-2)\cdot(x+2) \leq 0$$

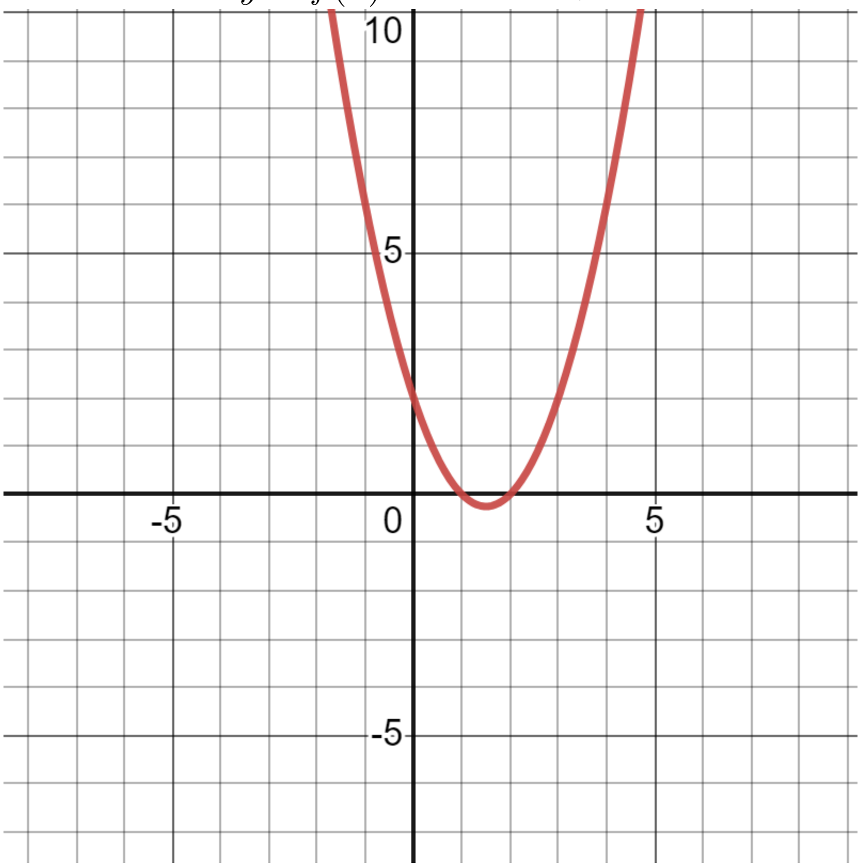


dom(g) $= [-2, 2]$

Function Graphs

every function can visualize as a graph and if we want to visualize we have to know some basic rules fisrtly while graphing our output is y-intercept and our x intercept is input and this x y combination must be in A x B set

$y = f(x) = x^2 - 3x + 2$



graph of $f : A \to B$
set of all $(a, b) \in A \times B$
such that $b = f(a)$

Function Arithmetic

in functions we can make arithmetic operations like addition divison but in functions we have domains if we make addition operation what will happen to domain of function so if you wondering let's look at that

Addition Subtraction
$(f \pm g)(x) = f(x) \pm g(x)$

Multiplication
$(f \cdot g)(x) = f(x) \cdot g(x)$

Division
$(f/g)(x) = \frac{f(x)}{g(x)}$

Domain:
independent from operation domain after operation always be intersect of f domain set g domain set but in every operation

$$\left\{ \begin{matrix} (f \pm g) \\ (f \times g) \\ (f \div g) \end{matrix} \right\} = \text{dom}(f) \cap \text{dom}(g)$$

Examples

$f(x) = \sqrt{x} \qquad g(x) = \frac{1}{x-2}$
$(f \pm g) = \sqrt{x} \pm \frac{1}{x-2}$
$(f \cdot g) = \sqrt{x} \cdot \frac{1}{x-2}$
$(f \div g) = \frac{\sqrt{x}}{\frac{1}{x-2}} = \sqrt{x} \cdot (x-2)$
dom$(f) = [0, \infty)$
dom$(g)(-\infty, -2) \cup (2, \infty)$
($dom)(f+g) = [0, 2) \cap (2, \infty)$

Function Composition

function composition is get input as output of a function
define: $(f \circ g)(x) = f(g(x))$
$g : A \to B \qquad f : B \to C$
$A \xrightarrow{\quad g \quad} B \xrightarrow{\quad f \quad} C$
$\underbrace{x \qquad\qquad g(x) \qquad\quad f(g(x))}_{f \circ g}$

Example

$$f(x) = x^2 \quad g(x) = e^x$$
$$f(g(x)) = f(e^x) = (e^x)^2 = e^{2x}$$
$$g(f(x)) = e^{f(x)} = e^{x^2}$$

Function Inverses

inverses function do opposite of what regular functions do i mean if regular functions go to B from A inverses go to A from B

Regular Functions	**Inverse Function**
$f : A \to B$	$f^{-1} : B \to A$

Identity Function
functions which is return input itsselves functions is called called identiy function

$$I(x) = x$$
$$f \circ f^{-1} = I_b \qquad f^{-1} \circ f = I_a$$
$$f^{-1}(f(x)) = x$$
$$f(f^{-1}(x)) = x$$

Caution: For being inverse function this function must be both one to one and surjective function

for being one to one fcuntion f must be for any x_1,x_2 if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Examples Find f^{-1} funtions output

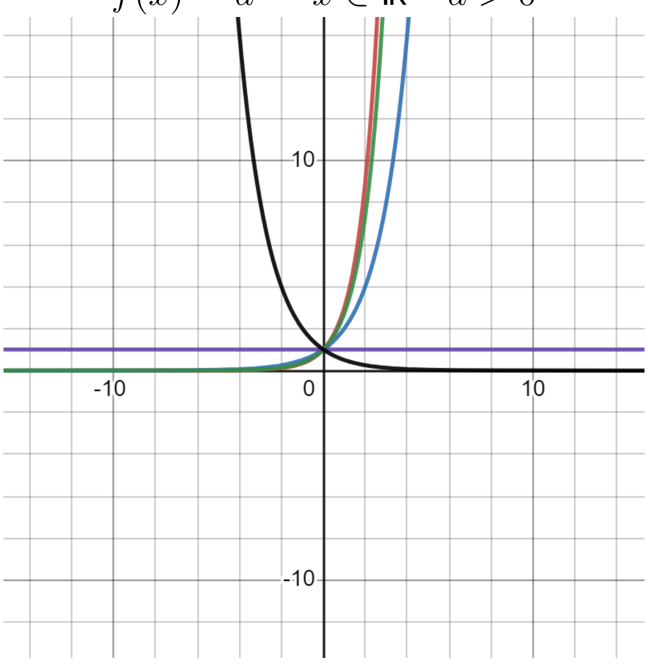
$$
\begin{aligned}
f(x) &= 3x-2\\
y &= 3x-2\\
y+2 &= 3x\\
x &= \frac{y+2}{3}\\
f^{-1} &= \frac{y+2}{3}
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= \frac{x-1}{2x+3}\\
x-1 &= y(2x+3)\\
x-1 &= 2xy+3y\\
x-2xy &= 3y+1\\
x(1-2y) &= 3y+1\\
x &= \frac{3y+1}{1-2y}\\
f^{-1}(x) &= \frac{3y+1}{1-2y}
\end{aligned}
$$

Exponential Functions

for a > 0 $a \in \mathbb{R}$
define $f(x) = a^x$ for $x \in \mathbb{N}$, $x \in \mathbb{Z}$, $x \in \mathbb{Q}$

$f(x) = a^x \quad x \in \mathbb{R} \quad a > 0$



$y = 3^x$(red)
$y = 2^x$(blue)
$y = 1^x$(purple)
$y = e^x$(green)
$y = \frac{1}{2}^x$(black)

Properties:
dom(f) = ℝ

for a > 1

as x gets bigger and is positive f(x) gets bigger

as x get bigger and is negative f(x) gets close to 0

$f(x+y) = f(x) \cdot f(y)$
$f(x \cdot y) = f(x)^y$
$f(-x) = \frac{1}{f(x)}$

Logarithms

$$y = a^x \quad \log_a(a^x) = x$$
$$\log_a(y) = a^{\log_a(x)} = x$$

if $y = a^x \Longleftrightarrow \log_a(y) = x$
if $f(x) = a^x$ then $f^{-1} = \log_a(x)$

Example
$\log_2(16) = \log_2(2^4) = 4$

Notification
$\log_e(x) = \ln(x)$

Properties

$f(x) = \ln(x)$ inverse of $g(x) = e^x$
$a = e^x \longleftrightarrow \ln(a) = x$
$b = e^y \longleftrightarrow \ln(b) = y$
$\ln(x \cdot y) = \ln(x) + \ln(y)$ why: $\ln(x \cdot y) = \ln(e^x \cdot e^y) = \ln(e^{x+y}) = x + y$
$\ln(x \div y) = \ln(x) - \ln(y)$ why: $\ln(x \div y) = \ln(\frac{e^x}{e^y}) = \ln(e^{x-y}) = x - y$
$\ln(x^y) = y \cdot \ln(x)$ why: $\ln(x^y) = \ln((e^x)^y) = \ln(e^{xy}) = xy$

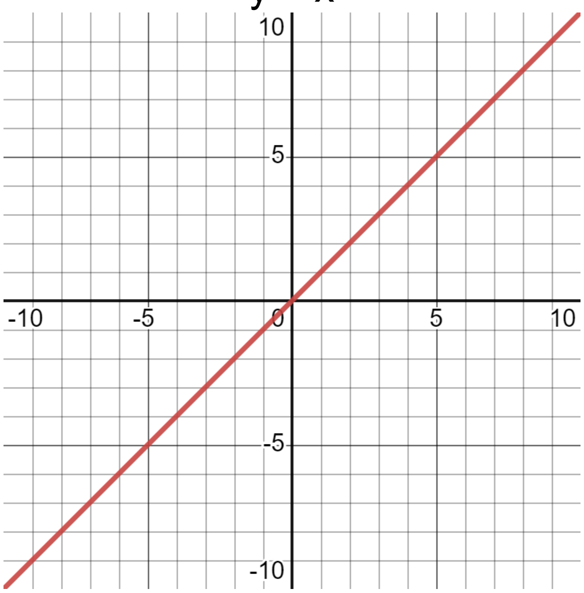
Change Of Base
let $y = \log_a(x)$
$a^y = x$
$a^y = e^{\ln(a^y)} = x$
$e^{y\ln(a)} = x$
$\ln(e^{y\ln(a)}) = \ln(x)$
$y\ln(a) = \ln(x)$
$y = \log_a(x)$
$\log_a(x) = \frac{\ln(x)}{\ln(a)}$

Graphs

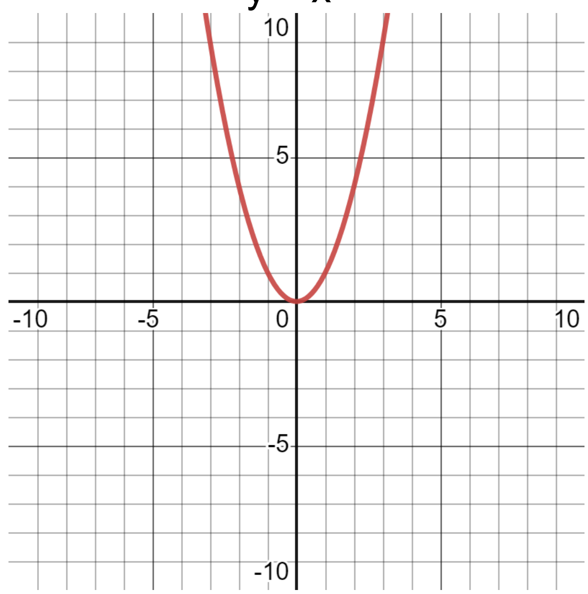
Common Graphs

while drawing graphs some functions loks like having same shape yes they have different x y coorinates but this can't change looks like same shape so we called this graphs common graphs

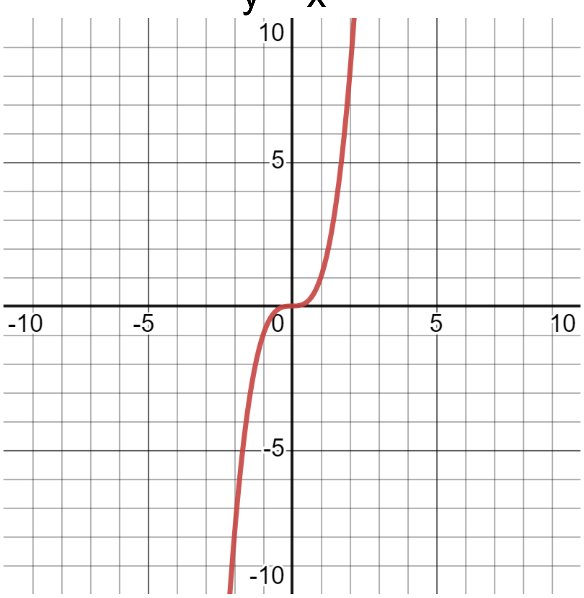
$y = x$



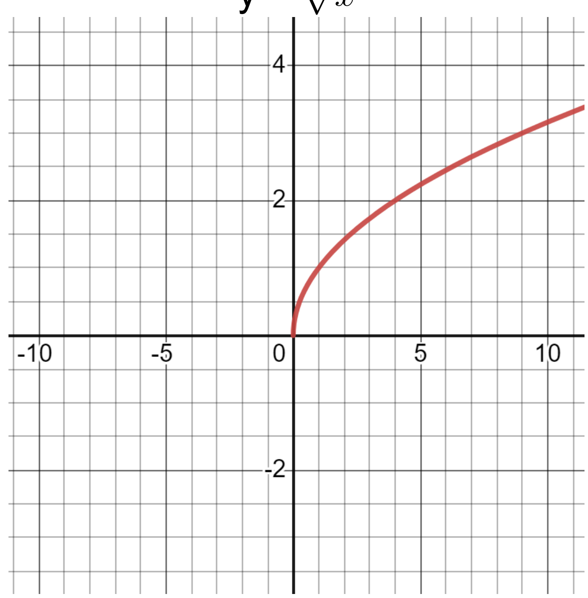
$y = x^2$



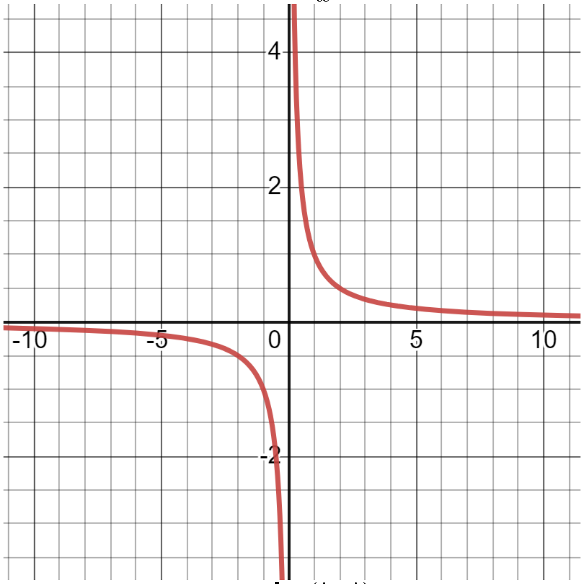
$y = x^3$



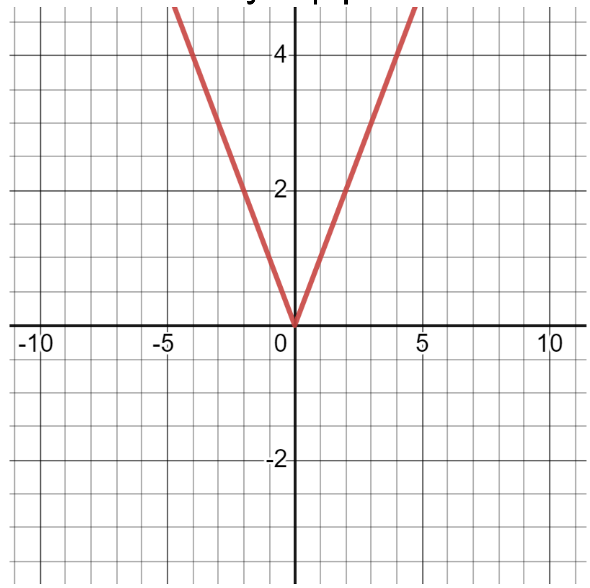
$y = \sqrt{x}$



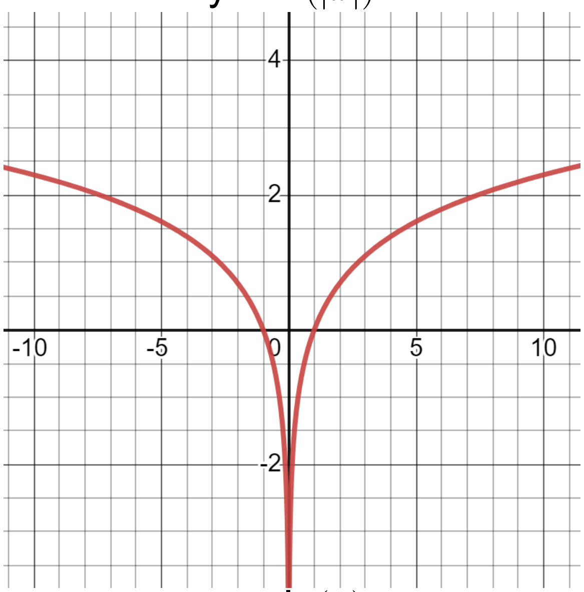
y = $\frac{1}{x}$



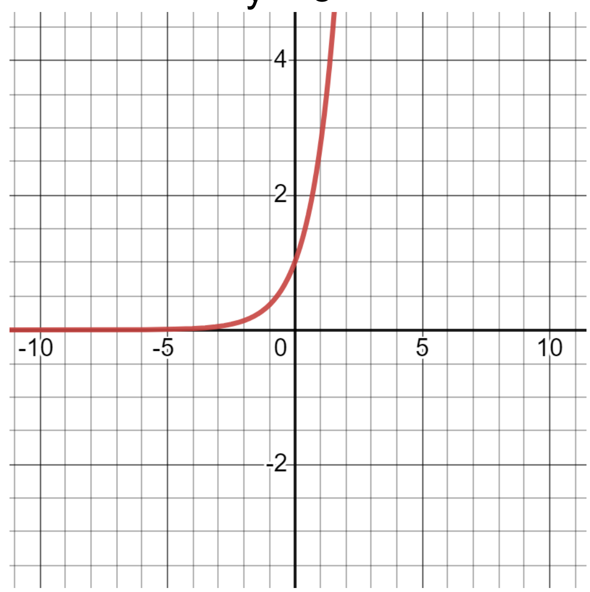
y = |x|



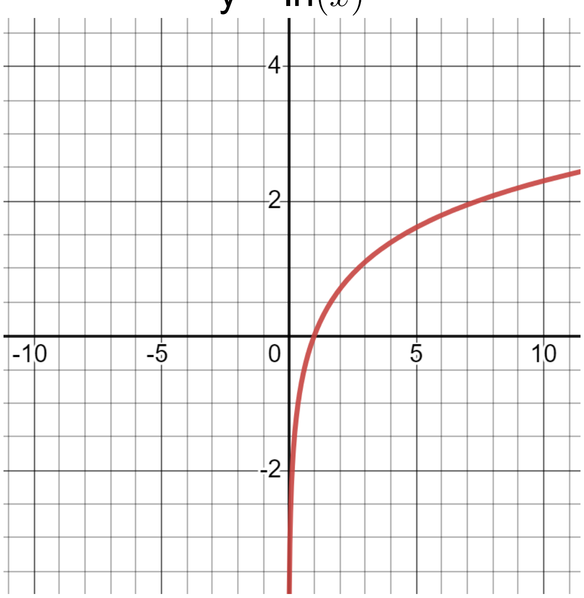
y = ln$(|x|)$



y = e^{x}



y = ln(x)



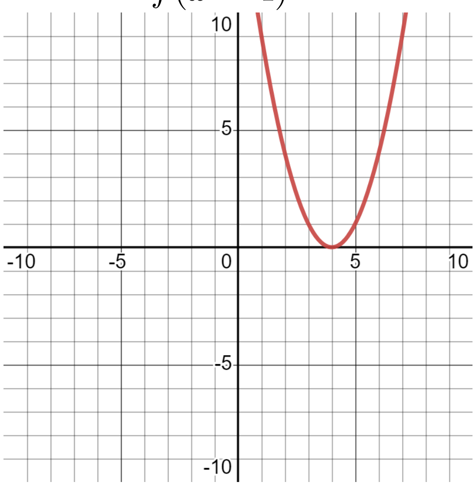
Transformation

Horizontal	Vertical
1. Translation $f(x-a)$	1. Translation $f(x)+b$
2. Strecth $f(ax)$	2. Strecth $bf(x)$
3. Reflection $f(-x)$	3. Reflection $-f(x)$

Horizontal Transformations

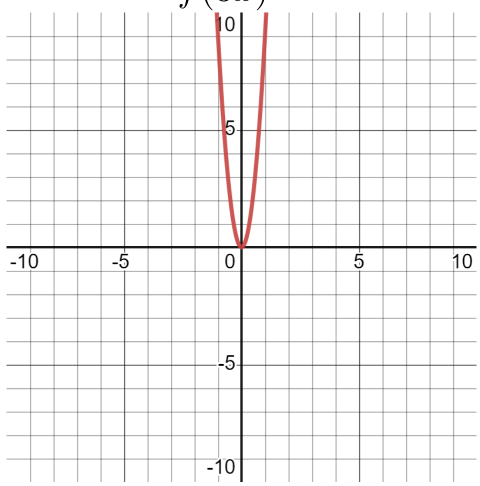
Translation

$f(x-4)^2$



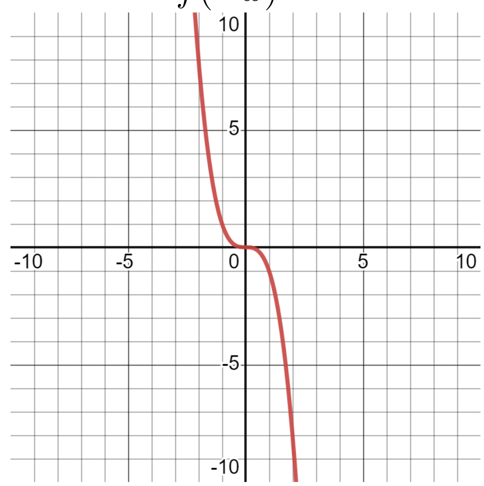
Strecth

$f(3x)^2$



Reflection

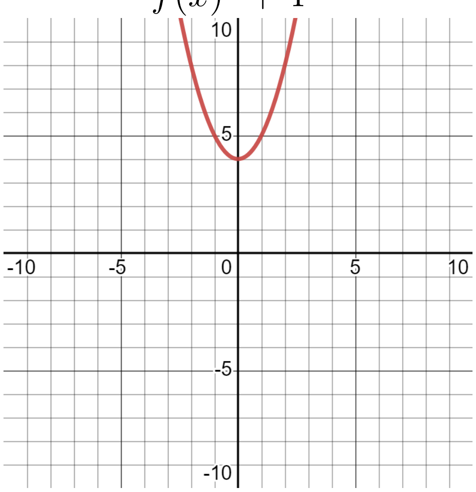
$f(-x)^3$



Vertical Transformations

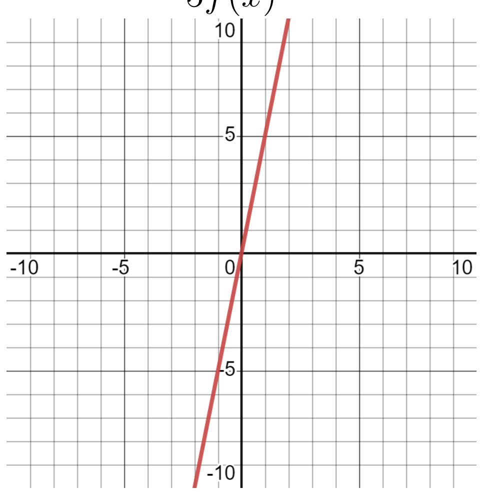
Translation

$f(x)^2+4$



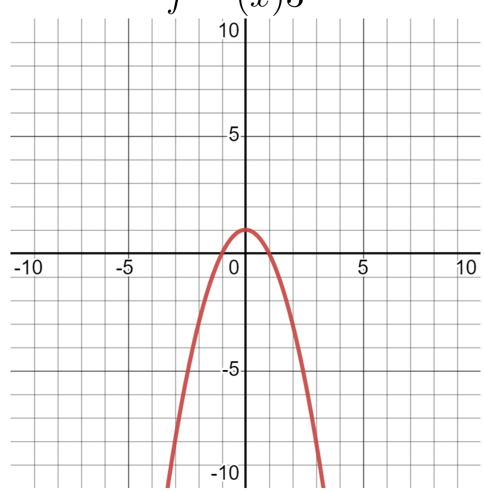
Strecth

$5f(x)$



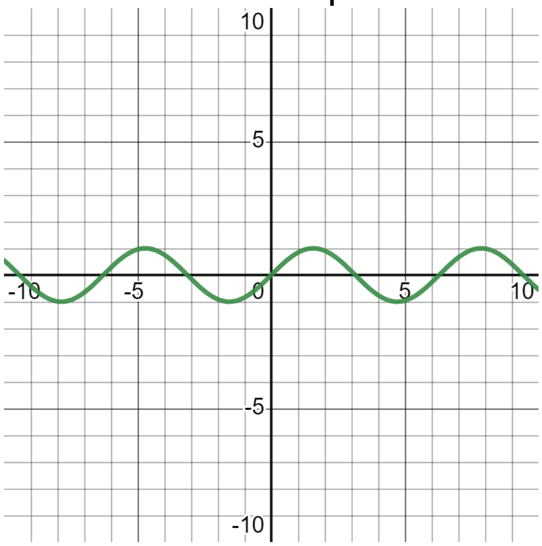
Reflection

$f-(x)3$

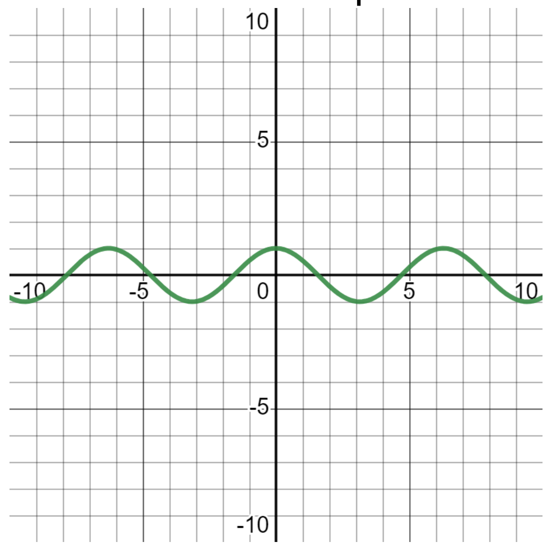


Trigonometric Functions Graphs

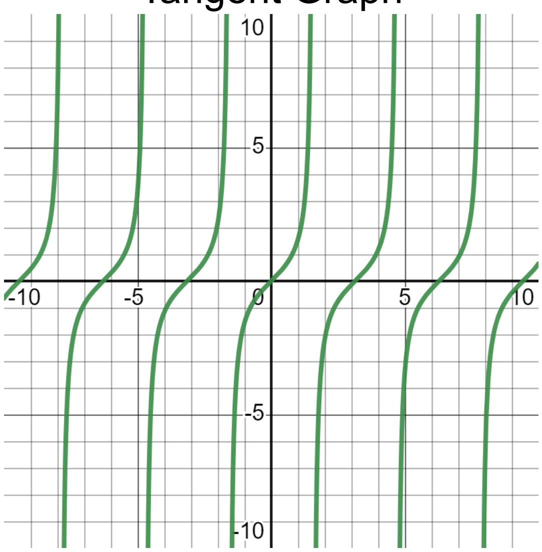
Sine Graph



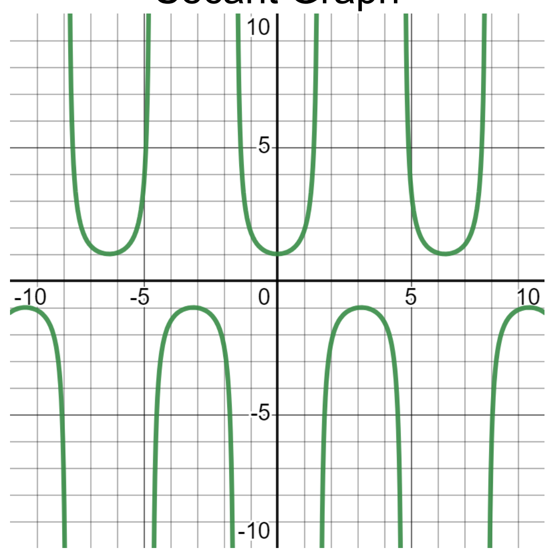
Cosine Graph



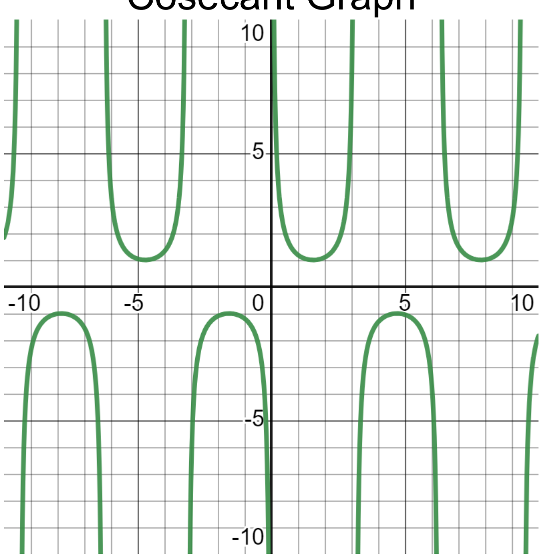
Tangent Graph



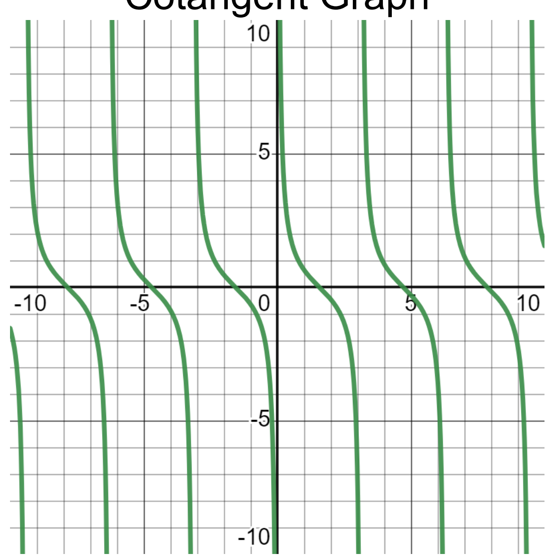
Secant Graph



Cosecant Graph



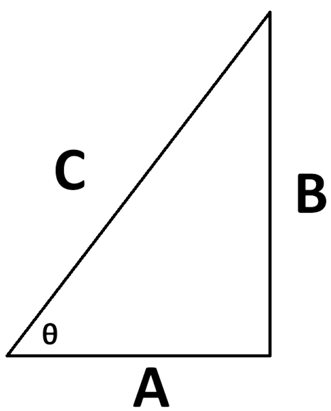
Cotangent Graph



Trigonometry

Right Triangles

in the right triangles we can find trigonometric functions with some rules so in this section i show them



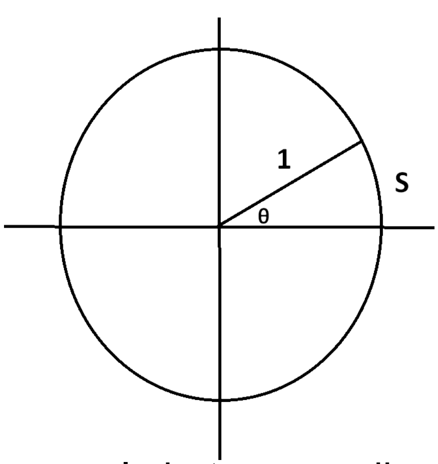
$$\cos(\theta) = \frac{a}{c}$$
$$\sin(\theta) = \frac{b}{c}$$
$$\tan(\theta) = \frac{b}{c} = \frac{\sin(\theta)}{\cos(\theta)}$$

Pythogorean Theorem

$$a^2 + b^2 = c^2$$

Unit Circle : $x^2 + y^2 = 1$

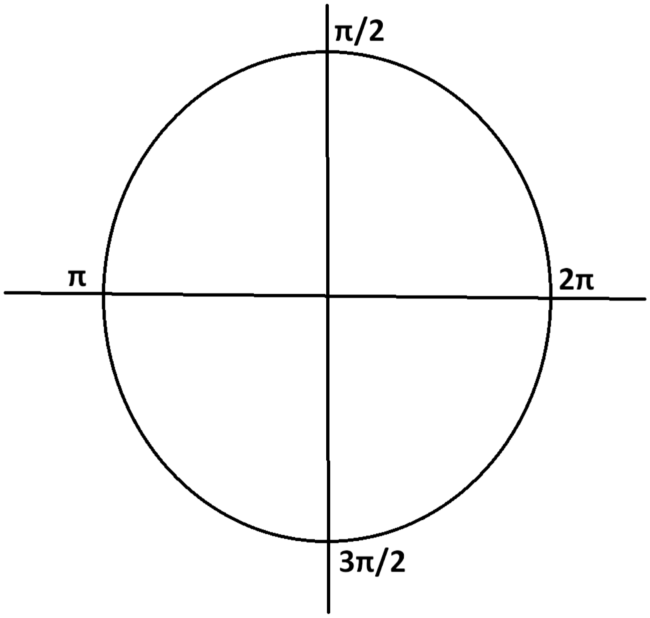
this circle is very common using tool in trigonometry because it’s provide many useful equations so in this section you will see the



$$x = \cos(\theta)$$
$$y = \sin(\theta)$$
$$s = r \cdot \theta$$

θ : angle between radius and positive x-axis
θ measured in radians (treat θ as any real number)

Radian Measurement



1 revolution(in rads) = circumference

$$C = 2\pi r = 2\pi$$

$$1\text{rev} = 2\pi = 360$$

Special Angles

$$\theta = 0: \text{ at } (1,0)$$
$$\cos(0) = 1$$
$$\sin(0) = 0$$

$$\theta = \frac{\pi}{2}: \text{ at } (0,1)$$
$$\cos(\frac{\pi}{2}) = 0$$
$$\sin(\frac{\pi}{2}) = 1$$

$$\theta = \frac{\pi}{3}: \text{ at } (\frac{1}{2}, \frac{\sqrt{3}}{2})$$
$$\sin(\frac{\pi}{3}) = \frac{1}{2}$$
$$\cos(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{4}: \text{ at } (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$
$$\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$$
$$\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{6}: \text{ at } (\frac{\sqrt{3}}{2}, \frac{1}{2})$$
$$\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$$
$$\sin(\frac{\pi}{6}) = \frac{1}{2}$$

Trigonometric Functions

between $-1, 1 \left\{ \begin{array}{ll} \sin(\theta) \text{ domain: } \mathbb{R} & \text{zeros: } 0, \pm\pi, \pm 2\pi \ldots \\ \cos(\theta) \text{ domain: } \mathbb{R} & \text{zeros: } \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots \end{array} \right\}$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \ldots$ zeros: $0, \pm\pi, \pm 2\pi \ldots$

$\csc(\theta) = \frac{1}{\sin(\theta)}$

$\sec(\theta) = \frac{1}{\cos(\theta)}$

$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\tan(\theta)}$

Identities

Basic:

$$\sin(x) = \cos(x - \frac{\pi}{2})$$

Even/Odd:

$$\sin(x) = -\sin(x) \text{ (odd)}$$
$$\cos(-x) = \cos(x) \text{ (even)}$$

Periodic:

$$k \in \mathbb{Z}$$
$$\sin(x + k \cdot 2\pi) = \sin(x)$$
$$\cos(x + k \cdot 2\pi) = \cos(x)$$
$$\tan(x + k \cdot \pi) = \tan(x)$$

Fundemental:

Pythogorean

$$\cos^2(x) + \sin^2(x) = 1$$

Addition/Subtraction:

$$\cos(x \pm y) = \cos(x) \cdot \sin(y) \pm \sin(x) \cdot \cos(y)$$
$$\sin(x \pm y) = \sin(x) \cdot \cos(y) \pm \sin(y) \cdot \cos(x)$$