Algebra Elementary To Advanced

Doruk Aytekin

January 2024

Contents

Part I

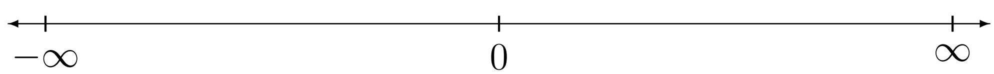
Equations And Inequalities

Chapter 1

Real Numbers

1.1 Number Line

we mostly use number line for indicates number at the middle we write 0 at the left $-\infty$ at right ∞ and it look like this but i think this diagram is not a good way to indicates real numbers this picture hide so many informantions



1.2 Number Sets

In mathematic we have many numbers and for better understand the number we grouped them and we assign symbols like Whole Numbers $\mathbb{W}$ or Intergers $\mathbb{Z}$ and in this section i will use them mathematicly to you

$$\mathbb{N} = \{1, 2, 3, \ldots, \infty\}$$

$$\mathbb{W} = \{0, 1, 2, 3, \ldots, \infty\}$$

$$\mathbb{Z} = \{-\infty, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, \infty\}$$

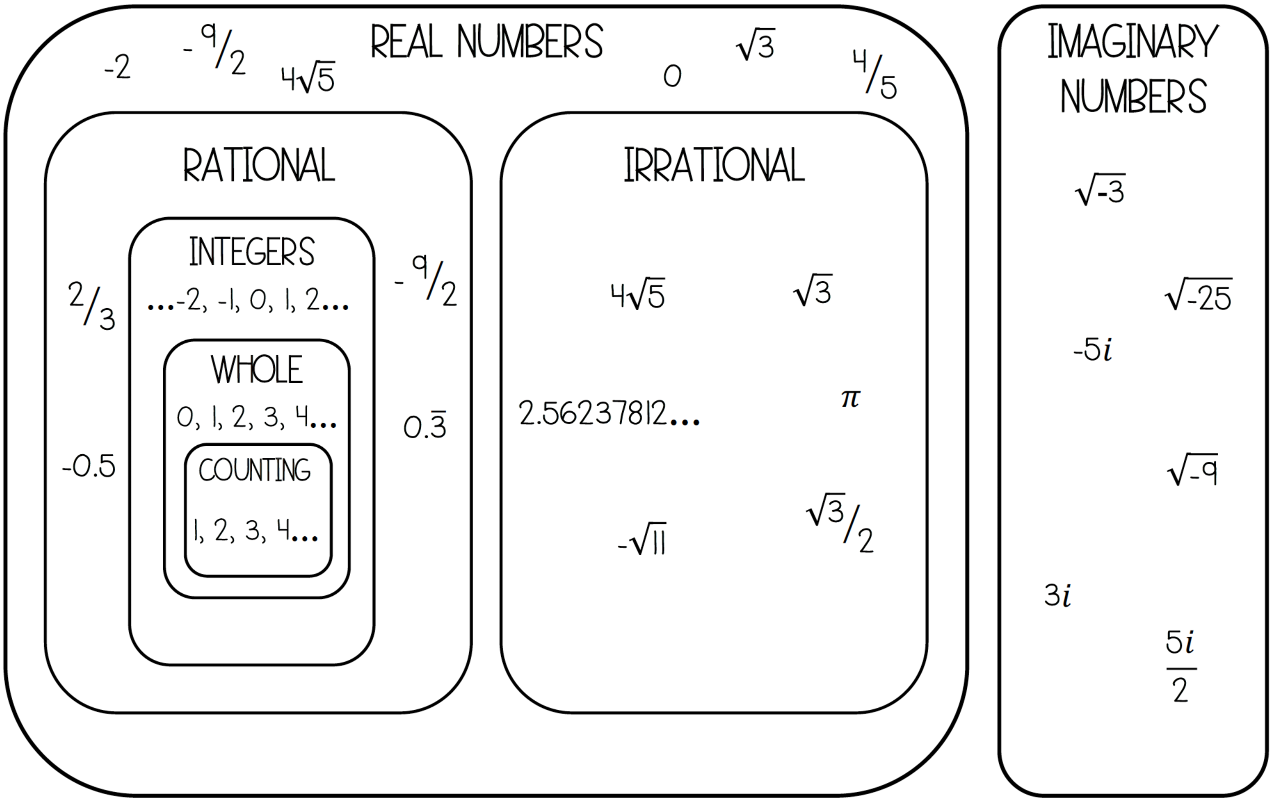
$$\mathbb{Q} = \{-\infty, \ldots, -3/2, -2/5, -1/4, 0, 1/4, 2/5, 3/2, \ldots, \infty\}$$

$$\mathbb{R} \backslash \mathbb{Q} = \{e, \pi\sqrt{2}\}$$

$$\mathbb{R} = \{-\infty, \infty\}$$

1.3 The Structure Of Real Numbers

as I said before that number line which at above is hiding a lot of informations about real number like complex numbers so for your‘s better understand I will draw another number line instead of that at the above



as you see every time we go to the up layer we inherit previous layer and if you don’t understand this with a pictures we can write this use mathematic symbol‘s

$$\mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

1.4 Rational And Irrational Numbers

If $\mathbb{R}$ = Any number with decimal expansion we have two options

Repetion (Rational)
$\frac{1}{3} = 0.3333... = 0.\overline{3} = (infinite)$
$\frac{1}{2} = 0.5000... = 0.5\overline{0} = (finite)$

No repetion (Irrational)
$\pi \approx 3.14159$
$\sqrt{2} \approx 1.414$

1.5 Square Roots

Definition: x ≥ 0 square root of x denote as $\sqrt{x}$, is the non-negative number y $\in \mathbb{R}$ such that $y^2 = x$

$$\sqrt{4} = 2$$
$$\sqrt{25} = 5$$
$$\sqrt{1} = 1$$
$$\sqrt{0} = 0$$
$$\sqrt{2} \approx 1.41421$$
$$\sqrt{3} \approx 1.732$$

Chapter 2

Properties of Real Numbers

2.1 Closure Prop

let $x, y \in \mathbb{R}$ then $x + y \in \mathbb{R}$,$x \cdot y \in \mathbb{R}$

Not Closed : $\mathbb{W} = \{0, 1, 2, 3, ...\}$

subt: $3 - 2 = 1 \in \mathbb{W}$

$2 - 3 = -1 \notin \mathbb{W}$

2.2 Commutative Prop

$$x + y = y + x$$
$$x \cdot y = y \cdot x$$
$$x - y \neq y - x$$
$$x/y \neq y/x$$

2.3 Associative Prop

$$x + (y + z) = z + (y + x)$$
$$x \cdot (y \cdot z) = z \cdot (y \cdot x)$$

2.4 Identity

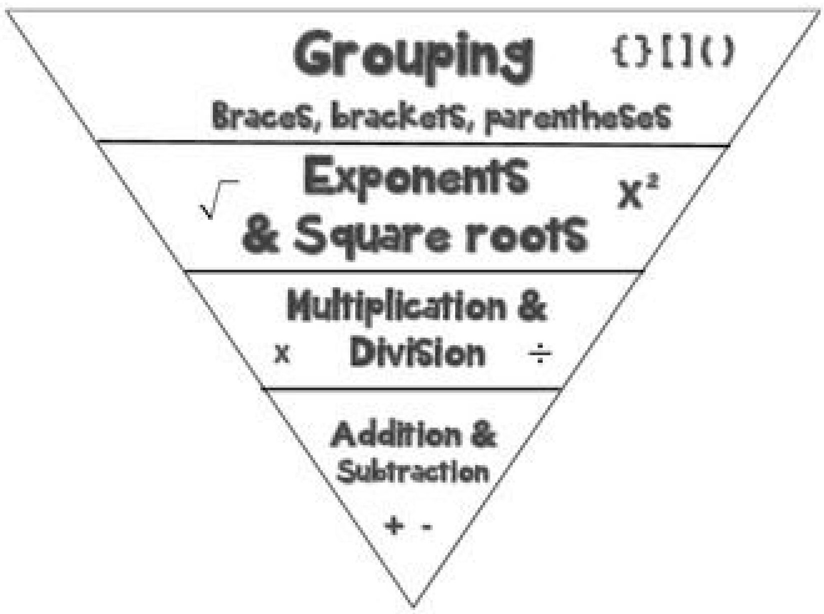
$$0 + x = x + 0 = x$$
$$0 \cdot x = x \cdot 0 = x$$

2.5 Distribution

$$x \cdot (y + z) = xy + xz$$

2.6 Order Of Operations

while calculating math expressions we track a specific ordered named order of operations firstly we make math expressions in the groups parantheses subsequently we make roots and exponentials after that we make multiple and divide finally subtraction and addition and if two expression have same weight we firstly make which one is at the left



Chapter 3

Variables And Equations

3.1 Equations

In mathematics, an equation is a mathematical formula that expresses the equality of two expressions, by connecting them with the equals sign = and they can be various type like quadratic cubic or 2 variable

$$w + x = 7$$

and if you make a expression to any side you have to this experssion to oppsite site

$$\begin{gathered} if\ 7 = 7 \\ 7 + x = 7 + x \\ 7 - x = 7 - x \\ 7x = 7x \\ \frac{7}{x} = \frac{7}{x} \end{gathered}$$

3.2 Properties Of Equations

equations have 3 properties symnetric reflexive transitive

$$\begin{gathered} x = y \\ \text{if } a = b \text{ then } b = a \text{ (symnetric)} \\ \text{if } a \in \mathbb{R}\ a = a \text{ (reflexive)} \\ \text{if } a = b, b = c \text{ then } a = c \text{ (transitive)} \end{gathered}$$

Example

find x value of equation $3x + x = \sqrt{10}$

$$3x + x = \sqrt{10}$$
$$4x = \sqrt{10}$$
$$\frac{4x}{4} = \frac{\sqrt{10}}{4}$$
$$x = \frac{\sqrt{10}}{4}$$

find x value of equation $4(x + 3) + 5(x + 2) = 84$

$$4x + 12 + 5x + 10 = 84$$
$$9x + 22 - 22 = 84 - 22$$
$$9x = 72$$
$$\frac{9x}{9} = \frac{72}{9}$$
$$x = 8$$

3.3 Absolute Value

absolute value show us to a number how many units far away from 0 I mean $|x|$ = distance from x to 0

if $x \geq 0, |x| = x$

if $x < 0, |x| = -x$ (mult by -1)

Example

find sum of possible x values $|x - 6| = 6$

if $x - 6 \geq 0$

$$x - 6 = 6$$
$$x = 12$$

if $x - 6 < 0$

$$-x + 6 = 6$$
$$-x = 0$$
$$x = 0$$

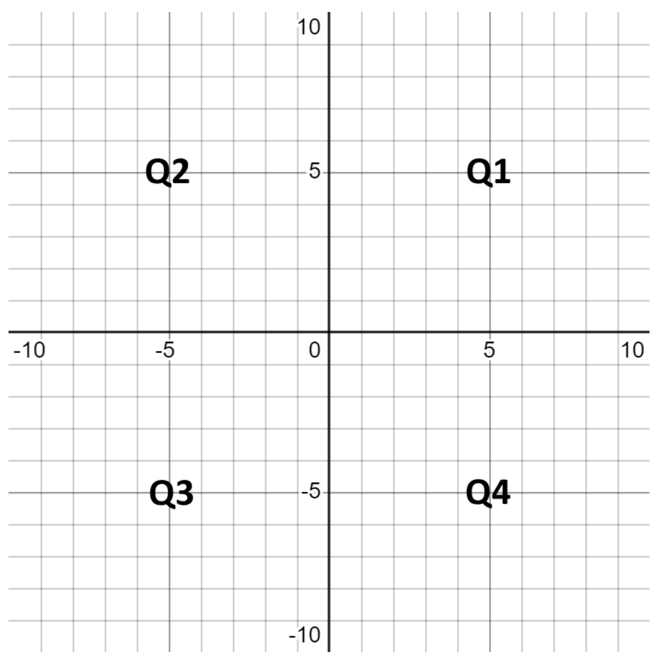
result = 12 + 0 = 12

Chapter 4

Linear Equations in Two Variables

4.1 Cartesian Plane

Probably you draw this million times and you know what is this but i want to indicate this also this is know as x-y Plane we draw with two coincide perpendicular lines and we named origin where line coinceded $0, 0$ and the x and y axis partition the plane into four a area your naming way going counter clock-wise

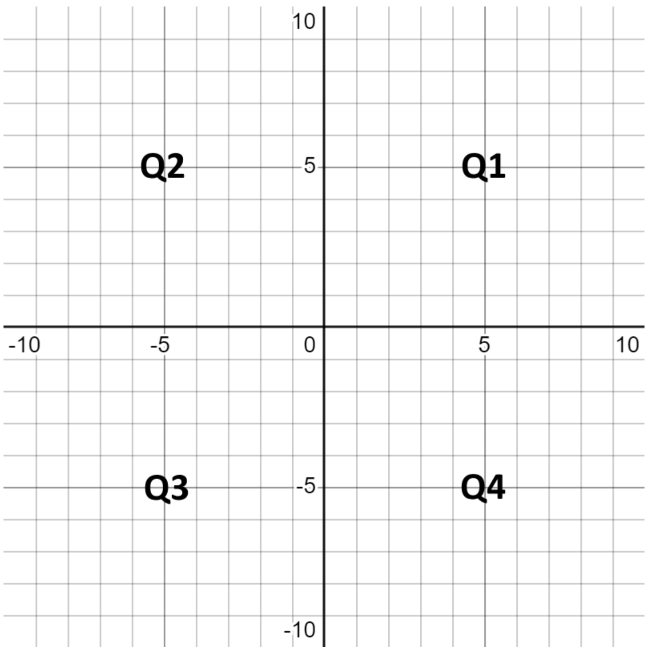


4.2 Linear Equations

i think you have seen this but i still dive this but dont worry not dively if i asked what is the equation of line you certainly imagine$y = mx + b$ m is slope and b is y intercept

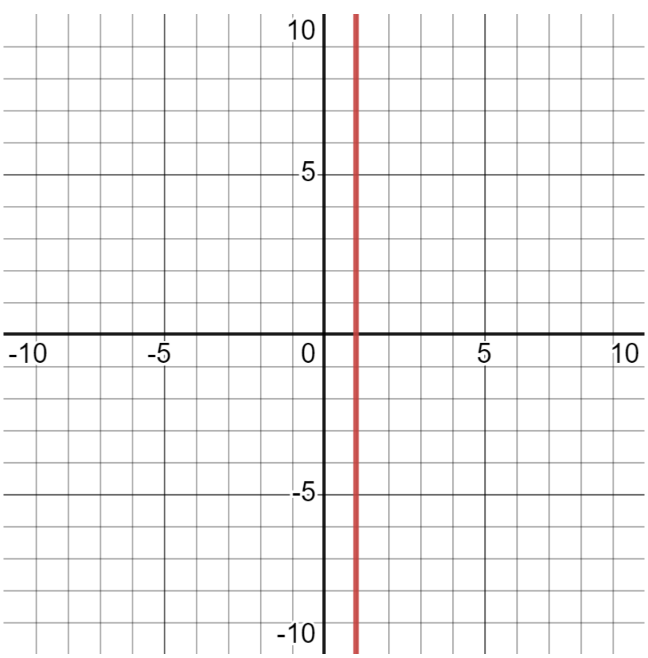
y = mx + b the point slope equation
m = slope
b = y-intercept

Example draw graph and find slope and y-intercept of equation y - 3x = 6



$$y = 3x - 6$$
$$m = 3$$
$$b = -6$$
$$(0, -6) = y - intercept$$

4.3 Standart Form Of A Line



$Ax + By + C = 0$ standart form
$$By = -Ax - C$$
$$y = \frac{-Ax}{B} - \frac{C}{B}$$
so $B \neq 0$

4.4 Finding Slope Of Line

Definition:m = slope of line and

$$
\begin{aligned}
&= \frac{rise}{run} \\
&= \frac{\Delta y}{\Delta x} \\
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
m &= \frac{\Delta y}{\Delta y} = \frac{y_2 - y_1}{x_2 - x_1}
\end{aligned}
$$

Question 1:What is the slope of the horizontal line

m = 0

Question 2:What is the slope of the vertical line

m = undefined

4.5 Finding Equation Of Line Given Two Points

find slope of line pass through (1,2) and (3,4) points
actually we can make solve this problem with 2 ways i solve with all of them but if your learn with memorization if you select second is be better for you

Way 1

$$
\begin{gathered}
m = \frac{\Delta y}{\Delta x} = \frac{4-2}{3-1} = \frac{2}{2} = 1 \\
y = mx + b \\
2 = 1x + b \\
b = 1 \\
y = x + 1
\end{gathered}
$$

Way 2
this way uses a formula named point-slope equation

$$
\begin{gathered}
y - y_1 = m(x - x_1) \\
y - 2 = 1(x - 1) \\
y = x + 1
\end{gathered}
$$

4.6 Paralel And Perpendicular Lines

Paralel Lines
paralel lines have same slope and they never coincided in a point

Perpendicular Lines
Perpendicular lines have slopes that are negative reciprocal

Example
find the equation of line through (-1,1) and $\perp$ to line y = 2x - 1

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{-1}{2}(x - (-1)) \\ y &= -\frac{x}{2} + \frac{1}{2} \end{aligned}$$

Chapter 5

Linear Inequalities In One Variable

5.1 Linear Inequalities In One Variable

let's just talk about what is the inequalities inequalities means not eqaul something so when you work with inequalities you are going to b working statements like x $\geq$ 7 or x $\leq$ 7

Definition let a,b $\in$ R a is less than write as a < b if b - a $\in \mathbb{R}$

Props

1. either a < b , b < a or a = B
2. if a < b, c $\in \mathbb{R}$ then a+c < b+c
3. if a < b, c > a then ac < bc
4. if a < b, c < 0 then ac > bc

Example

2x - 1 < 4x + 3 find the x values interval

$$\begin{aligned} 2x - 1 + 1 &< 4x + 3 + 1 \\ 2x &< 4x + 4 \\ x &< 2 \end{aligned}$$

-1 < 2x + 3 $\geq$ 5 find the x values interval

$$\begin{aligned}
-1 < 2x+3 &\quad \text{and} \quad 2x+3 \leq 5 \\
-1+-3 < 2x+3-3 &\quad \text{and} \quad 2x+3-3 \leq 5-3 \\
-4 < 2x &\quad \text{and} \quad 2x \leq 2 \\
-2 < x &\quad \text{and} \quad x \leq 1 \\
-2 < &\quad \text{x} \quad \leq 1
\end{aligned}$$

Intervals:

1. [bracket $\Rightarrow$ include end point
2. (parantheses $\Rightarrow$ not include end point $(-\infty, \infty) = \mathbb{R}$

Example (x-1)(x-3) > 0

$$(x-1)\cdot(x-3) = 0$$
$$x = 1 \text{ x } = 3$$

values for try they to can fit to inequalitie:0,2,4

$$\begin{array}{ccc}
x = 0 & x = 2 & x = 4 \\
(0-1)\cdot(0-3) > 0 & (2-1)\cdot(2-3) > 0 & (4-1)\cdot(4-3) > 0 \\
(-1)\cdot(-3) > 0 & (1)\cdot(-1) > 0 & (3)\cdot(1) > 0 \\
3 > 0 \checkmark & -1 > 0 \times & 3 > 0 \checkmark
\end{array}$$

Answer: $(-\infty, 1) \bigcup (3, \infty)$

$|2x-1| < 3$ **What is the possible values interval of x**

$$\begin{aligned}
2x-1 < 3 &\quad \text{and} \quad -2x+1 < 3 \\
2x < 4 &\quad \text{and} \quad -2x < 2 \\
x < 2 &\quad \text{and} \quad x > 1
\end{aligned}$$

Answer: -1 < x < 2

Chapter 6

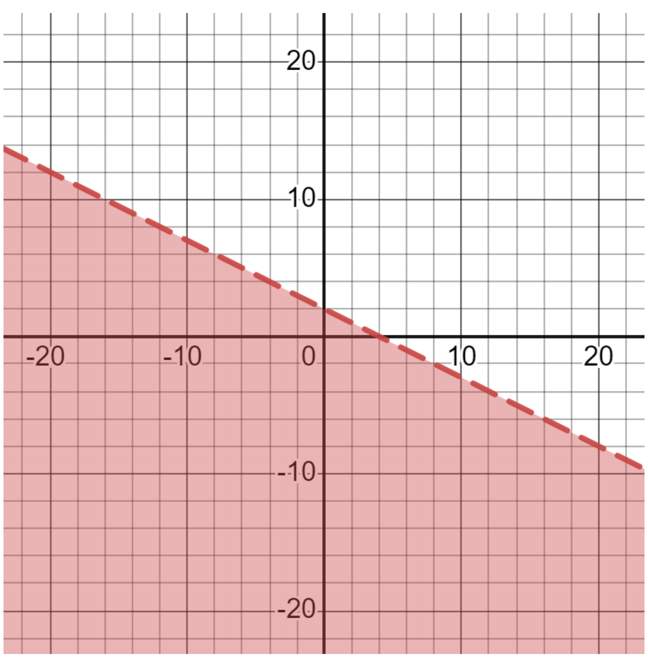
Linear Inequalities In Two Variable

6.1 Linear Inequalities In Two Variable

you might be thinking what is linear inequalities i two variable and how they are writing all of them are great quesstion firstly i start with what is linear mean linear means that all number's exponent is 1 not 2 3 or 1/2 and they writing as below equation

$$Ax + By < C \quad A, B, C \in \mathbb{R}$$

Example y < $-\dfrac{x}{2} + 2$ graph the solution section



$$y = -\frac{x}{2} + 2$$
$$x = 0 \; y = 2$$
$$x = 4 \; y = 0$$

for decide to which side will be scan it we select a random point and if this point fit to inequality we scan this side and we usually select $(0, 0)$ that point

Chapter 7

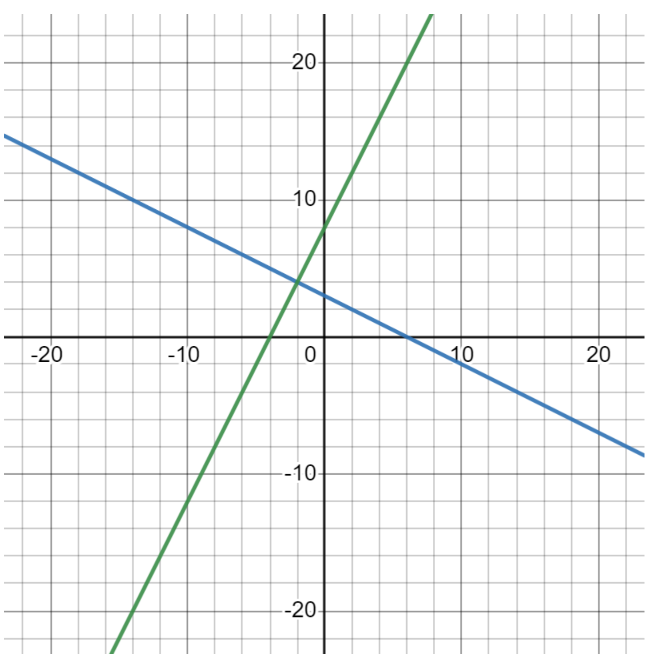
System Of Equations

7.1 System Of Equations

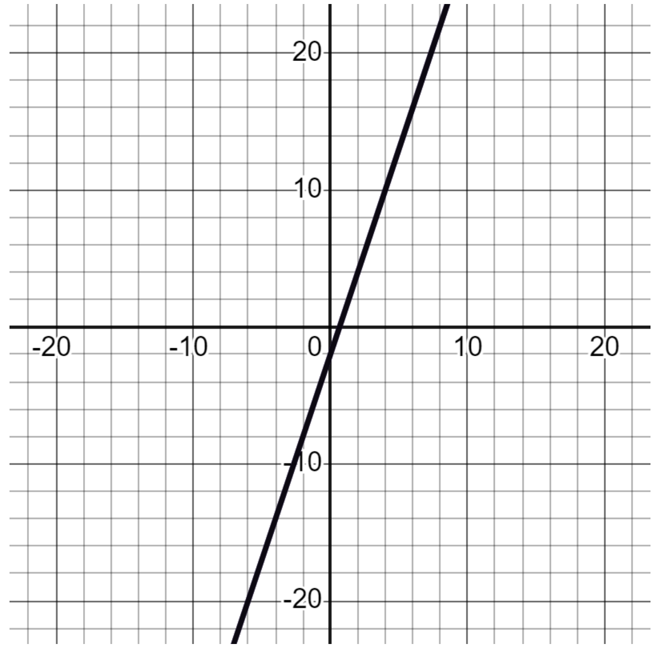
Definition: A collection of two or more equation is called system

$$
\begin{aligned}
2x + 3y &= 4 \\
3x + 4y &= 5
\end{aligned}
$$

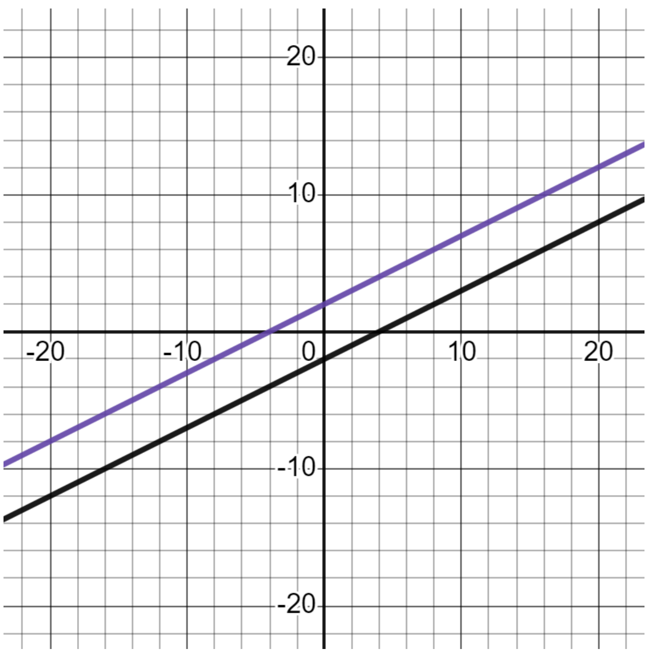
7.2 Solve By Graphing



$$
\begin{aligned}
x + 2y &= 6 \\
2x - y &= -8
\end{aligned}
$$



$$3x - y = 2$$
$$2y - 6x = -4$$



$$y = \frac{1x}{2} + 2$$
$$x - 2y = 4$$

7.3 Addition Method

$5x + 6y = 8$
$8x + 2y = 27$
find x variable

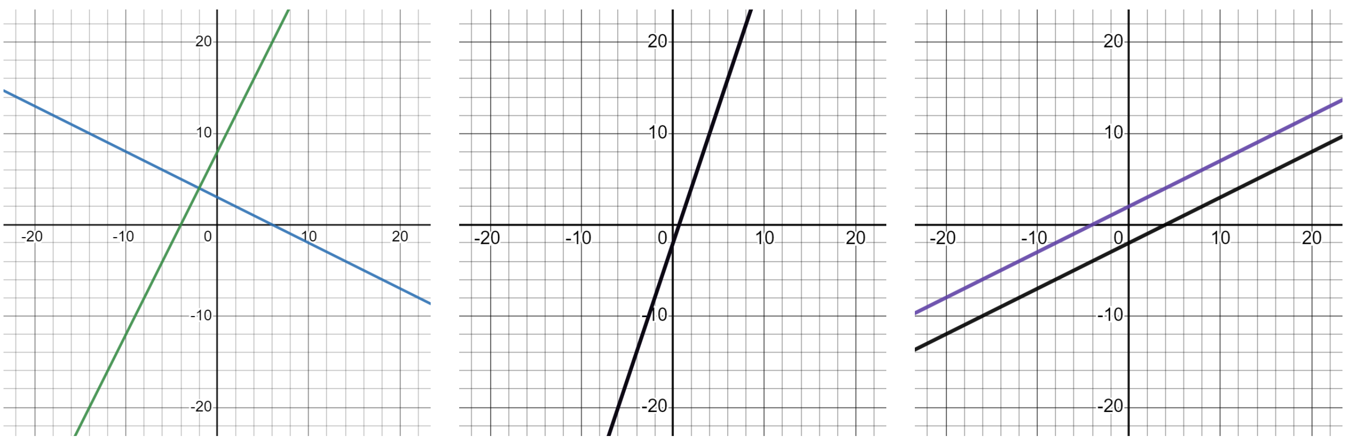
$$5x + 6y = 8$$
$$-3(8x + 27) = 27$$
$$5x + 6y = 8$$
$$-24x - 6y = -81$$
$$-19x = -73$$
$$x = 73/19$$

7.4 Substitution Method

$3x - y = 6$
$6x + 5y = -23$
find x value

$$y = 3x - 6$$
$$6x + 5(3x - 6) = -23$$
$$6x + 15x - 30 = -23$$
$$21x = 7$$
$$x = \frac{1}{3}$$

7.5 Type Of Systems



consistent
1 solution
independent

consistent
∞ solution
dependent

inconsistent
no solution

7.6 Inconsistent And Dependent Systems

Inconsistent
$3x - y = 9$
$2y - 6x = 7$

find x value

$$2(3x - y) = 9$$
$$2y - 6x = 7$$
$$6x - 2y = 18$$
$$2y - 6x = 7$$
$$18 = 7$$
$$x = \emptyset$$

Dependent
$\frac{1x}{2} - \frac{2y}{3} = -2$
$4y = 3x + 12$
find x value

$$6(3x - y) = -2$$
$$4x = 3x + 12$$
$$3x - 4y = -12$$
$$4x - 3x = 12$$
$$0 = 0$$
$$x = \infty$$

Chapter 8

System Of Linear Inequalities

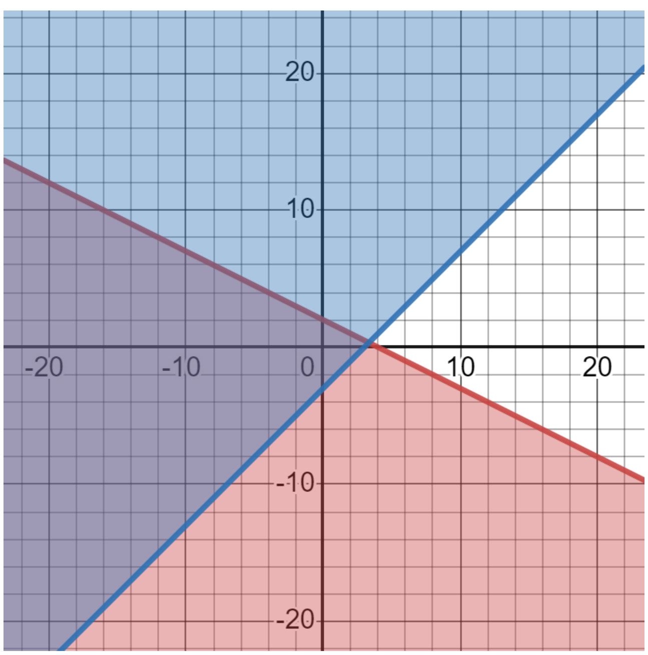
8.1 System Of Linear Inequalities

it is almost same thing system of equations only difference is now we have < > instead of =

$$x + y > 5$$
$$x - y < 7$$

8.2 Solve By Graphing

draw graph of system



$$x + 2y \leq 4$$
$$y \leq x - 3$$

8.3 Type Of Systems

consistent systems	inconsistent systems
has solutions	no solutions
$x + y > 7$	$x > 0$
$x - y < 5$	$x < 0$

Part II

Functions And Applications

Chapter 9

Functions

9.1 Functions

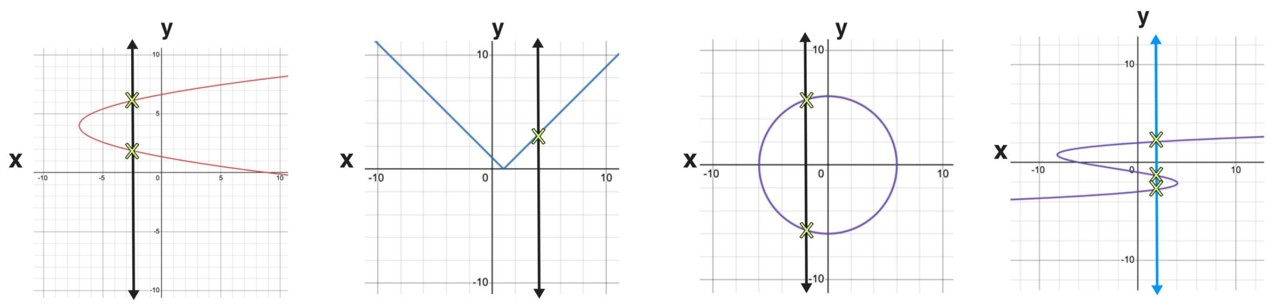
Definition: A function is a rule that assigns each element in a set to a unique element in another function actually we thick fuctions as a machine and you give him instructions and and its produce a thigns that he have to make

$$f(x) = \pi \cdot x^2$$

9.2 Vertical Line Test

we can write fuctions all day but sometimes we want to visualize them but is all graphs representing a function what do you think about that if you wanting answer of question absolutely NO therefore how can i find out a graph is represent a function for this we use vertical line test and this works like that We draw vertical lines on each x-axis and if this lines coincided more than one on the graph this graph is not representing any if this lines coincided no more than one on the graph this graph is representing a function

Example A **Example B** **Example C** **Example D**



Not a function — Function — Not a function — Not a function

9.3 Domain And Range

this machine take numbers or other symbols and give you numbers or other symbols but this machine can't take every number or can't produce every number i mean this machine have restrictions. for instance machine only take natural numbers and only produce whole numbers and we call this actions Domain and Range

Domain: the set of numbers that allowed to be used in the Functions

Range: the set of outputs of the function (set of y value)

9.4 Linear Functions

Definition:

$$f(x) = ax + b \qquad a, b \in \mathbb{R}$$

Example
(3,1) is perpendicular to y = 2x -1 so what is the equation of line through (3,1)

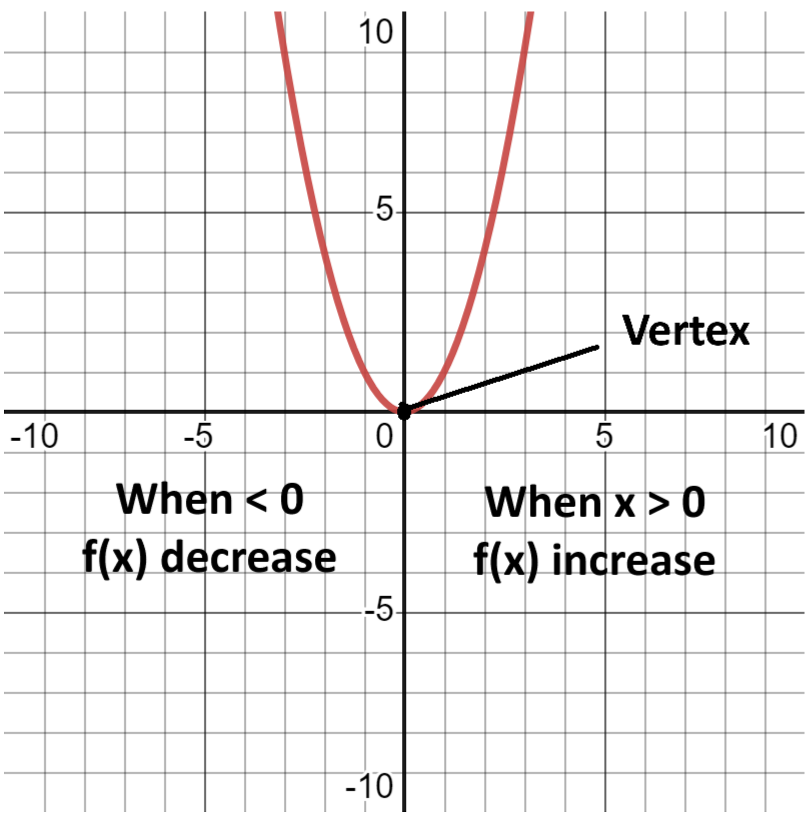
$$m = -\frac{1}{2}\ x_1 = 3\ y_1 = 1$$
$$y - y_1 = -\frac{1}{2}(x - x_1)$$
$$y - 1 = -\frac{x}{2} + \frac{3}{2}$$
$$y = -\frac{x}{2} + \frac{5}{2}$$

Chapter 10

Quadratic Functions

10.1 Quadratic Functions

$$f(x) = x^2 + bx + c \qquad a, b, c \in \mathbb{R}$$



$$f(x) = x^2$$
$$a = 1, b = 0, c = 1$$

End Behaviour
as $x \to +\infty, f(x) = x^2 \to +\infty$
as $x \to -\infty, f(x) = x^2 \to +\infty$

Horizontal Shifts
start with c > 0
y = f(x - c) $\Rightarrow$ shifted right c units
y = f(x + c) $\Rightarrow$ shifted left c units

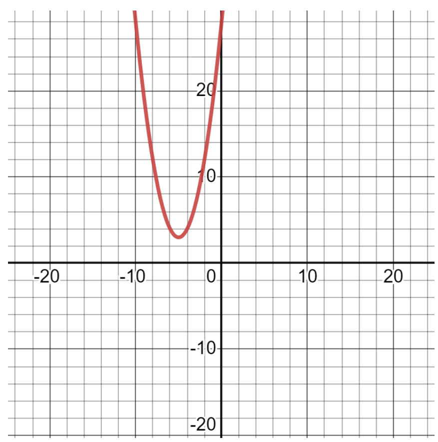
10.2 Standart Form Of A Quadratic Function

$$
\begin{aligned}
f(x) &= ax^2 + bx + c \\
&= a(x^2 + \frac{bx}{a} + \frac{c}{a}) \\
&= a(x^2 + \frac{bx}{a} + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 + \frac{c}{a}) \\
&= a(x^2 + \frac{bx}{a} + (\frac{b}{2a})^2) - a((\frac{b^2}{4a^2}) + \frac{c}{a}) \\
&= a(x^2 + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a}
\end{aligned}
$$

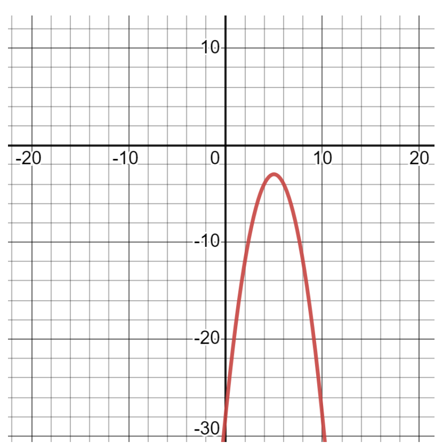
vertex at x = $\frac{b}{2a}$

vertex at y = $-\frac{4ac - b^2}{4a}$

when a > 0 graph is upward and vertex is min

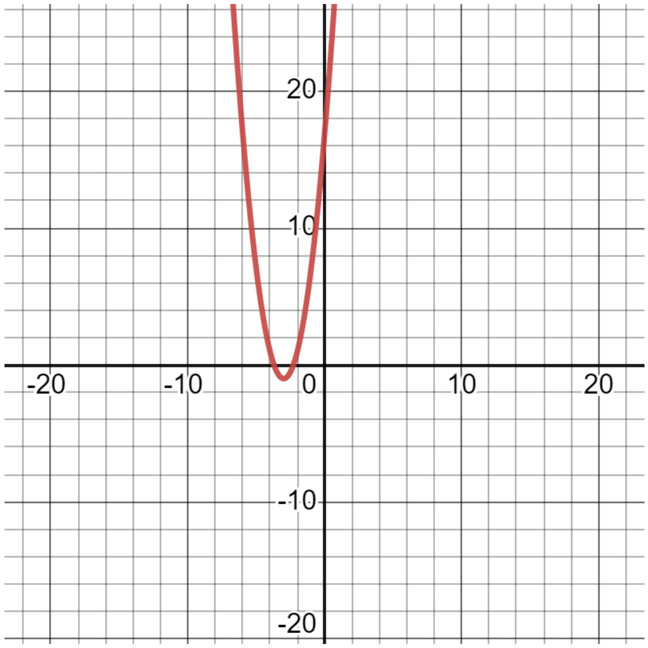


when a < 0 graph is downward and vertex is max



Examples

$f(x) = 2x^2 + 12x + 17$ draw graph the equation



$$(\frac{b}{2a})^2 = (\frac{12}{4}) = 3^2 = 9$$
$$f(x) = 2(x^2 + 6x + 9 - 9) + \frac{17}{2}$$
$$f(x) = 2(x+3)^2 - 18 + 17$$
$$f(x) = 2(x+3)^2 - 1$$

find equation of parabola which vertex at (2,-1) and contains (4,2) points

$$f(x) = a(x-2)^2 - 1$$
$$f(4) = a(4-2) - 1 = 2$$
$$2 = a(2)^2 - 1$$
$$4a = 3$$
$$a = \frac{3}{4}$$
$$f(x) = \frac{3}{4}(x-2) - 1$$

10.3 Quadratic Formula

$$ax^2 = bx + c = 0 \qquad \text{x intercept (set y} = 0)$$
$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition: $b^2 - 4ac$ id called discrement

1. if $b^2 - 4ac > 0 \Rightarrow$ 2 x-intercepts
2. if $b^2 - 4ac = 0 \Rightarrow$ 1 x-intercept
3. if $b^2 - 4ac < 0 \Rightarrow$ no solution

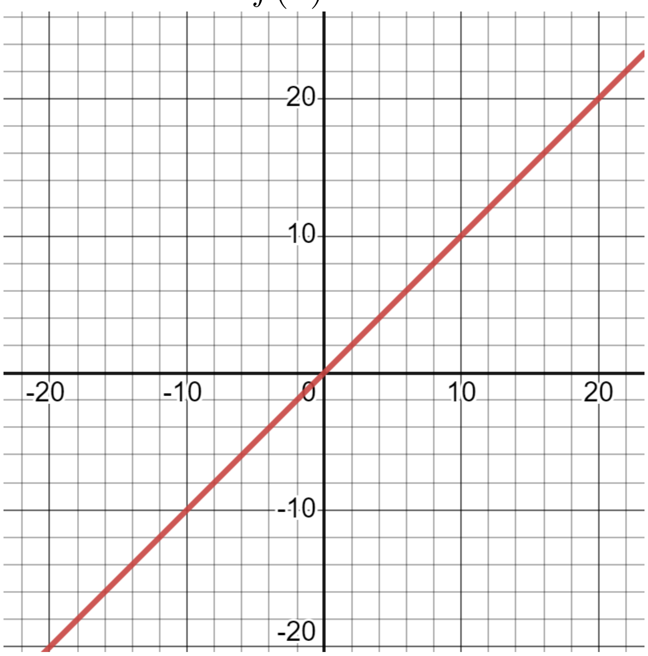
Chapter 11

Common Functions

11.1 Linear Functions

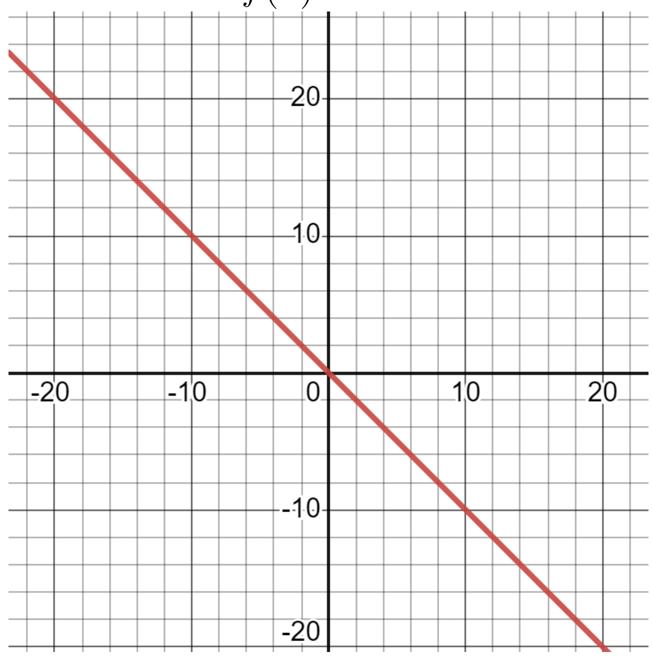
$mx + b = 0$

$f(x) = x$



D:$\mathbb{R}$
R:$\mathbb{R}$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\to \infty$,f(x) $\to \infty$
as x $\to -\infty$,f(x) $\to -\infty$

$f(x) = -x$

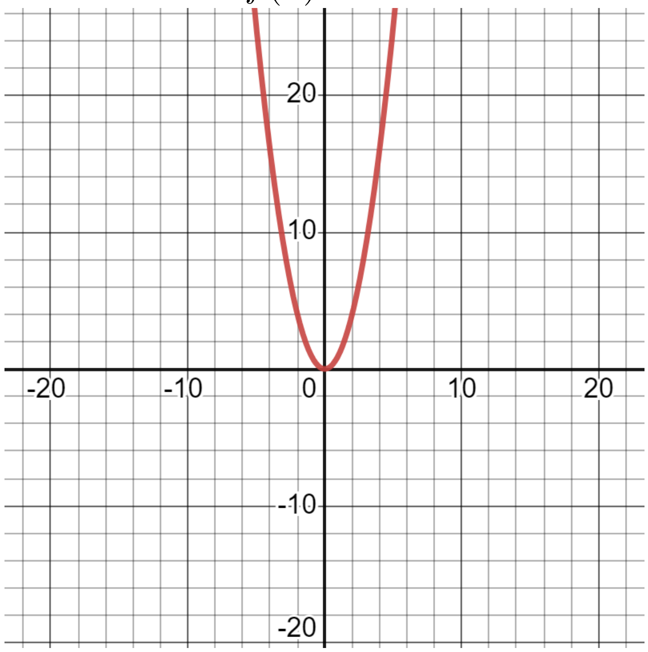


D:$\mathbb{R}$
R:$\mathbb{R}$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\to \infty$,f(x) $\to -\infty$
as x $\to -\infty$,f(x) $\to \infty$

11.2 Quadratic Functions

$ax^3 + bx^2 + cx + d = 0$

$f(x) = x^2$

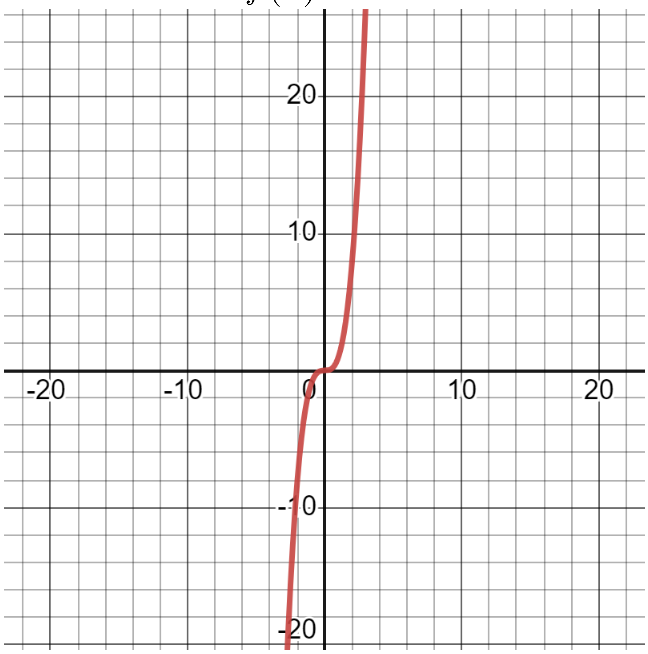


D:$\mathbb{R}$
R:$[0, \mathbb{R}]$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\rightarrow \infty$,f(x) $\rightarrow \infty$
as x $\rightarrow -\infty$,f(x) $\rightarrow \infty$

11.3 Cubic Functions

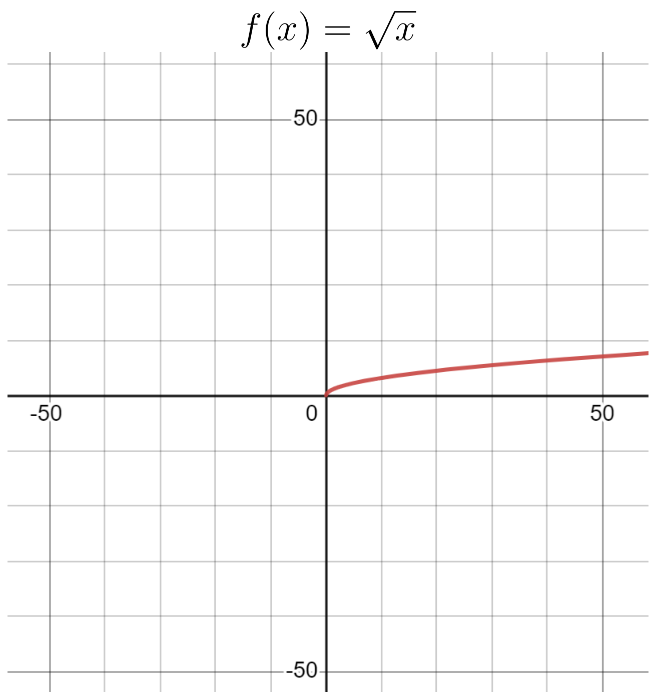
$ax^3 + bx^2 + cx + d = 0$

$f(x) = x^3$



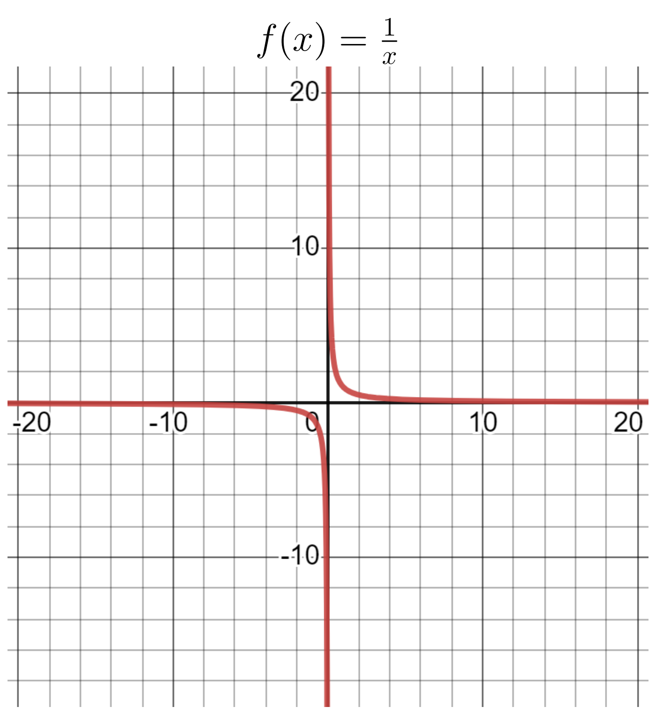
D:$\mathbb{R}$
R:$\mathbb{R}$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\rightarrow \infty$,f(x) $\rightarrow \infty$
as x $\rightarrow -\infty$,f(x) $\rightarrow -\infty$

11.4 Square Root Functions



D:$[0, \mathbb{R}]$
R:$[0, \mathbb{R}]$
x-intercept:$($0,0$)$
y-intercept:$($0,0$)$
as x $\to \infty$,f(x) $\to \infty$

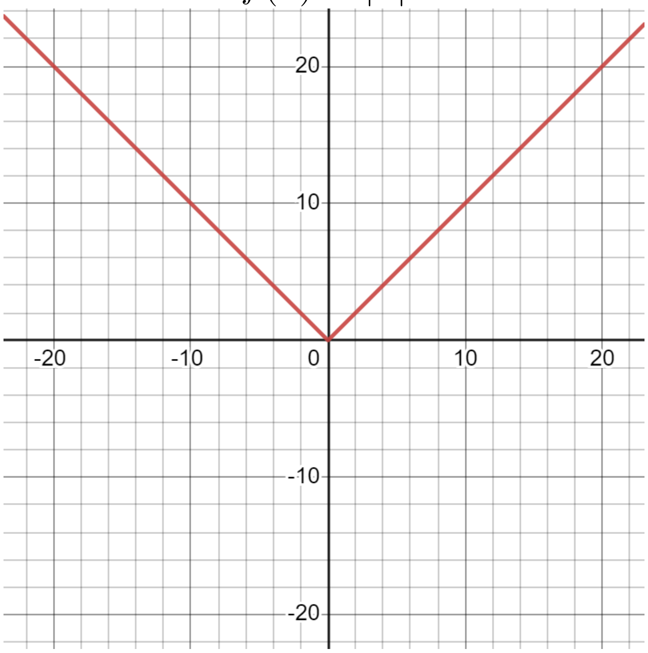
11.5 $\frac{1}{x}$ Functions



D:$\mathbb{R} \setminus 0$
R:$\mathbb{R} \setminus 0$
x-intercept:$\emptyset$
y-intercept:$\emptyset$
as x $\to \infty$,f(x) $\to 0$
as x $\to -\infty$,f(x) $\to 0$

11.6 Absolute Value Functions

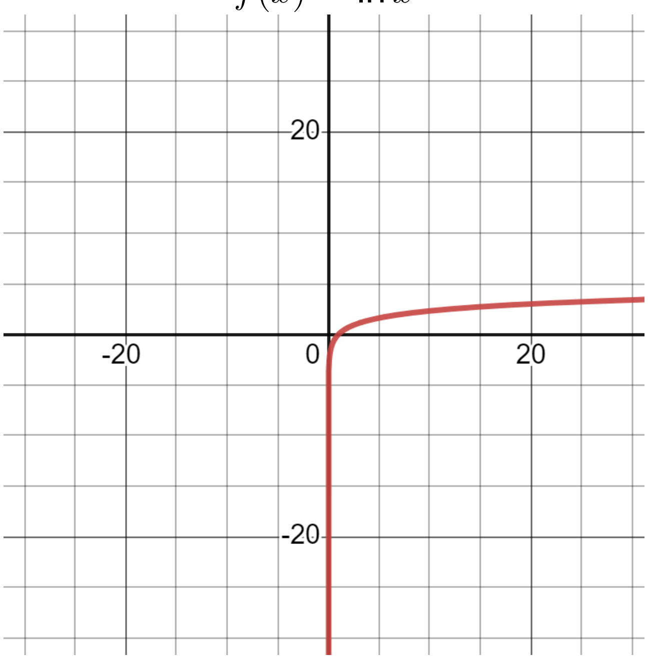
$f(x) = |x|$



D:$\mathbb{R}$
R:$[0,)R$
x-intercept:$($0,0$)$
y-intercept:$($0,0$)$
as x $\rightarrow \infty$,f(x) $\rightarrow \infty$
as x $\rightarrow -\infty$,f(x) $\rightarrow \infty$

11.7 Logarithmic Functions

$f(x) = \ln x$



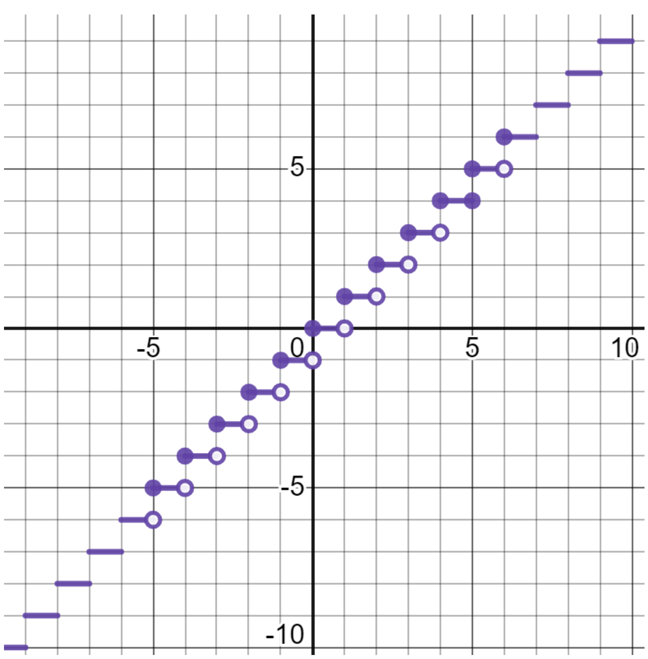
ln(1) = 0

Chapter 12

Less Common Functions

12.1 Greatest Integer Functions

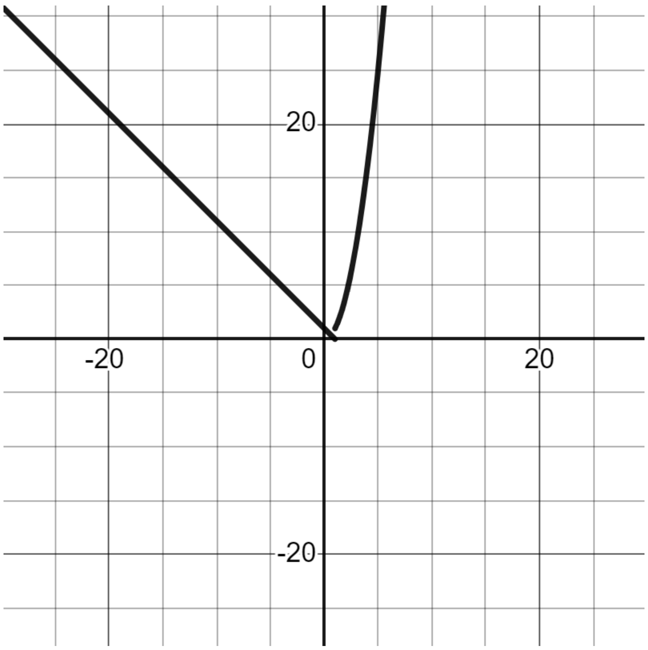
$f(x) = ax^2 + bx + c$ this function also know as floor function i mean we take integer value of given number like
$\lfloor 3.4 \rfloor = 3$
$\lfloor 3.99 \rfloor = 3$
$\lfloor \pi \rfloor = 3$



D:$\mathbb{R}$
R:$\mathbb{Z}$
x-intercept:$($0,-1$)$
y-intercept:$($0,0$)$

12.2 Piece Wise Functions

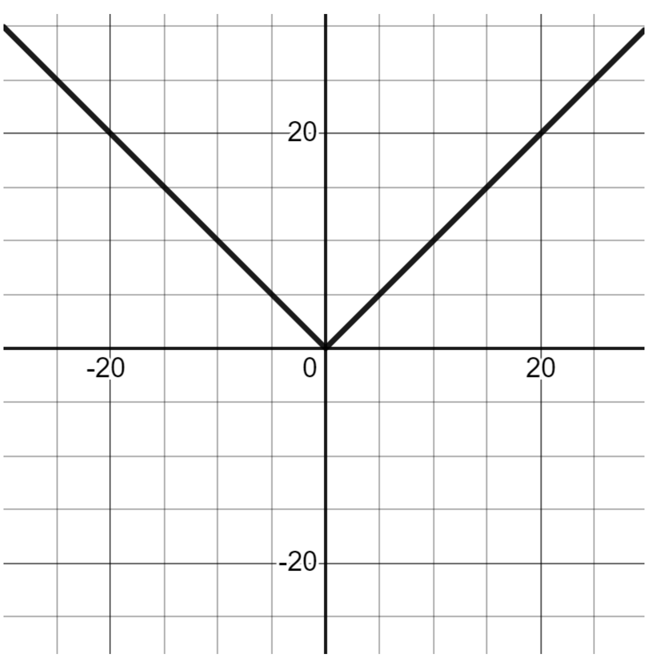
$$f(x) = \begin{Bmatrix} 1-x & x \leq 1 \\ x^2 & x \geq 1 \end{Bmatrix}$$



D:$\mathbb{R}$
R:$\mathbb{R}$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\to \infty$,f(x) $\to \infty$
as x $\to -\infty$,f(x) $\to \infty$

12.3 Absolute Piece Wise Functions

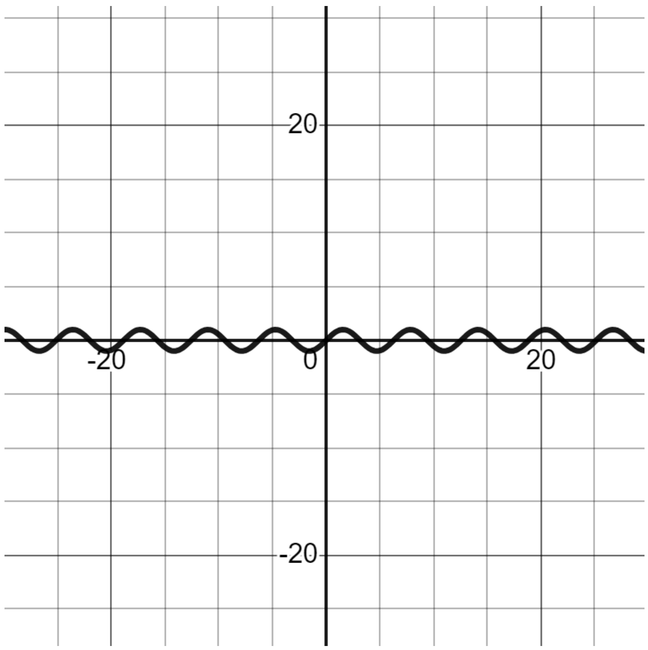
$$|x| = \begin{Bmatrix} x & x \geq 0 \\ -x & x < 0 \end{Bmatrix}$$



D:$\mathbb{R}$
R:$\mathbb{R}$
x-intercept:(0,0)
y-intercept:(0,0)
as x $\to \infty$,f(x) $\to \infty$
as x $\to -\infty$,f(x) $\to \infty$

12.4 Sin x Functions

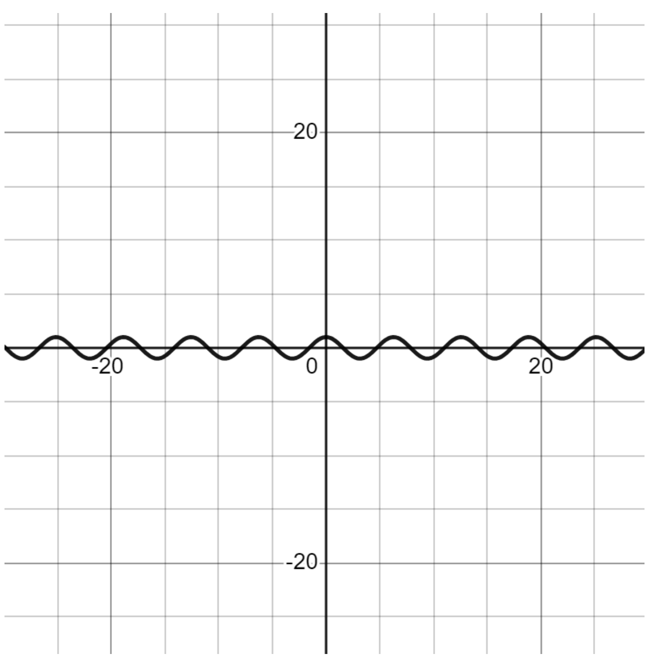
$f(x) = \sin x$



D:$\mathbb{R}$
R:$[-1, 1]$

12.5 Cos x Functions

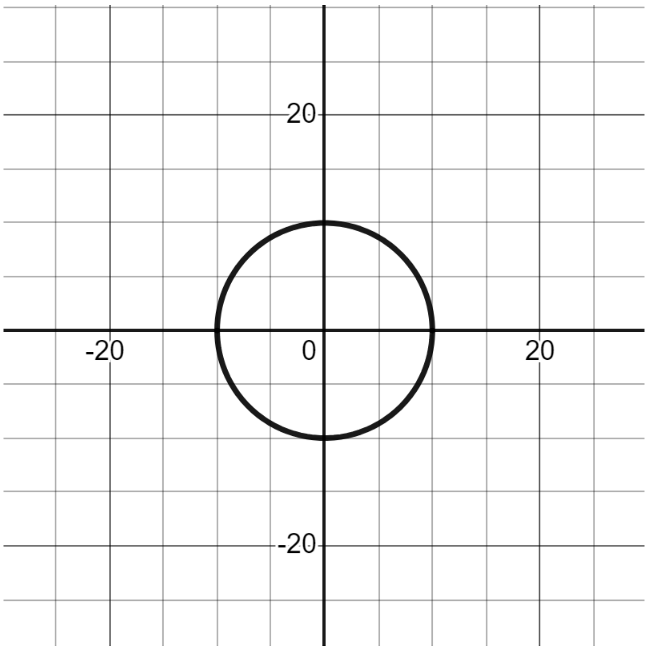
$f(x) = \cos x$



D:$\mathbb{R}$
R:$[-1, 1]$

12.6 The Unit Circle

$x^2 + y^2 = 1$



Radius:r=1
Center:$(0, 0)$

Note: this graph is not function because it can't pass Vertical Line Test

Chapter 13

Function Composition

13.1 Function Composition

this is an operation that unique to functions you can't think of this as multiplacation, divison, addition and subtraction I repeat this composition operation is unique to functions therefore You can't apply this to numbers or other topics. only you can apply to functions

$$
\begin{aligned}
&f(x) = x^2 \\
&g(x) = x + 1 \\
&(f \circ g)(x) = f(g(x)) = f(x+1) = (x+1)^2 = x^2 + 2x + 1 \\
&(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 + 1
\end{aligned}
$$

by the way when you applied this to functions order is matter

$$g \circ f \neq f \circ g$$

Example

$$
\begin{aligned}
f(x) &= 3x - 2 \\
g(x) &= x^2 \\
h(x) &= x^{\frac{1}{3}}
\end{aligned}
$$

what is the $f(g(h(x)))$

$$
\begin{aligned}
(f \circ g \circ h) &= f(g(h(x))) \\
(f \circ g \circ h) &= f(g(x^{\frac{1}{3}})) \\
(f \circ g \circ h) &= f((x^{\frac{1}{3}})^2) \\
(f \circ g \circ h) &= f(x^{\frac{2}{3}}) \\
(f \circ g \circ h) &= 3x^{\frac{2}{3}} - 2
\end{aligned}
$$

Part III

Polynomials and Roots

Chapter 14

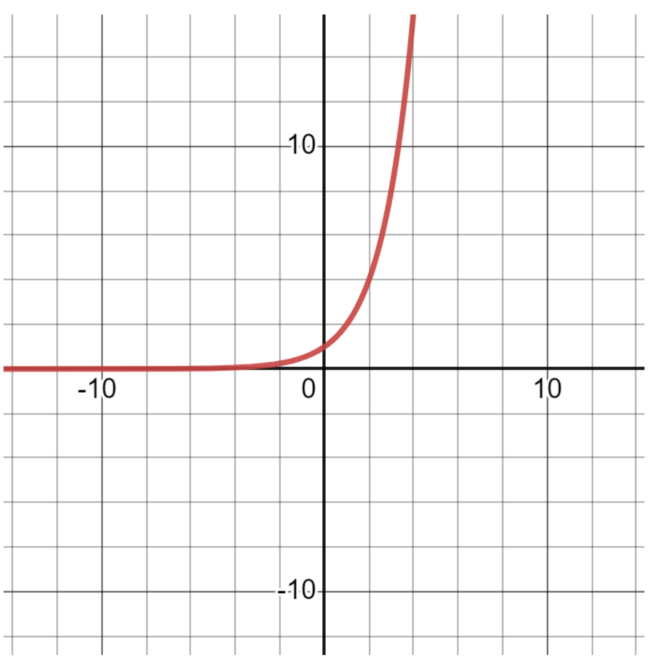
Exponential Functions

14.1 Exponential Functions

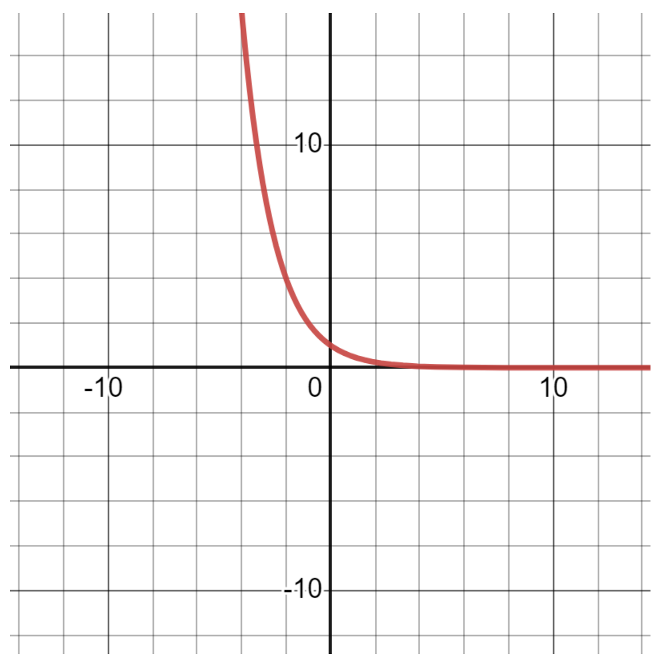
Definition: An exponential function with base a is of $f(x) = a^x$ where a > 0 and a $\neq$ 1

$$f(x) = 2^x$$
$$g(x) = 3^x$$
$$h(x) = \pi^x$$

$f(x) = 2^x$

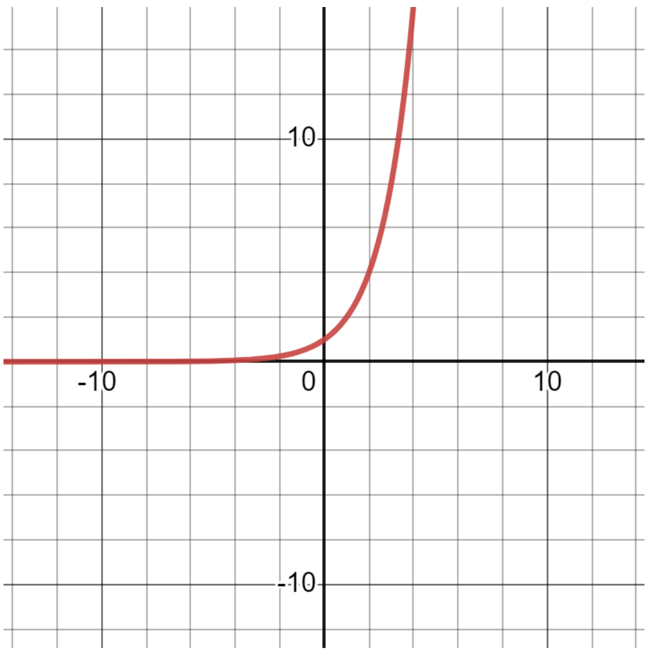


$f(x) = (\frac{1}{2})^x$



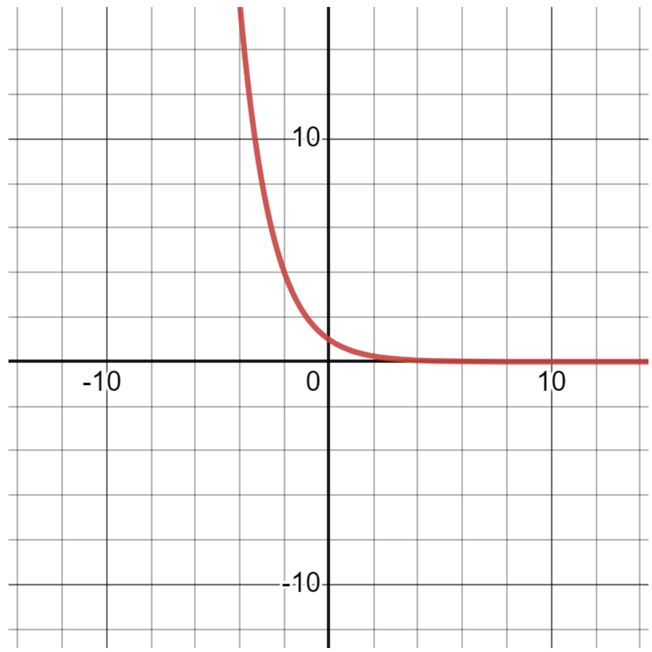
14.2 Domain Of Exponential Functions

$a > 1$



D:$\mathbb{R}$
R:y>0 = (0,∞)

$0 < a < 1$



D:$\mathbb{R}$
R:y>0 = (0,∞)

14.3 Properties Of Exponential Functions

1. if a > 1 $f(x)$ always increase
2. if 0 < a < 1 $f(x)$ always decrease
3. the y intercept is $(0, 1)$ or 1
4. horizontal asymptote at x axis (y=0)
5. D:$\mathbb{R}$
6. R:y>0
7. $f(x)$ is one to one function

14.4 Exponential Equation

Proposition: if a^{x_1} = a^{x_2} then x_1 = x_2

Example $f(x) = 3^4 - x$ and $f(x) = \frac{1}{9}$ so what is the x value

$$3^4 - x = 3^{-2}$$
$$4 - x = -2$$
$$x = 6$$

this topic learn better solve questions so i write some questions about this

$4^{x^2} = 4^6 - x$ what is the x value

$3^x = 9^x + 5$ what is the x value

$4^5 - 9x = 8^{\frac{1}{x-2}}$ what is the x value

Chapter 15

Algebra Of Polynomials

15.1 Polynomials

Definition: n ≥ 0 | n $\in \mathbb{Z}$ $\qquad a_1, a_2, a_3 \dots a_n \in \mathbb{R}$

$$y = 7$$
$$y = 3x + 4$$
$$y = 4x^2 + 7x + 2$$
$$y = x^{10} + x^4 + 3x + 2$$

Note: for being polynomial yur exponent must be greater than or eqaul zero and must be integer so this equations which are at the bottom is not polynomials

$$y = \frac{1}{x} = x^{-1}$$
$$y = \sqrt{x} = x^{\frac{1}{2}}$$

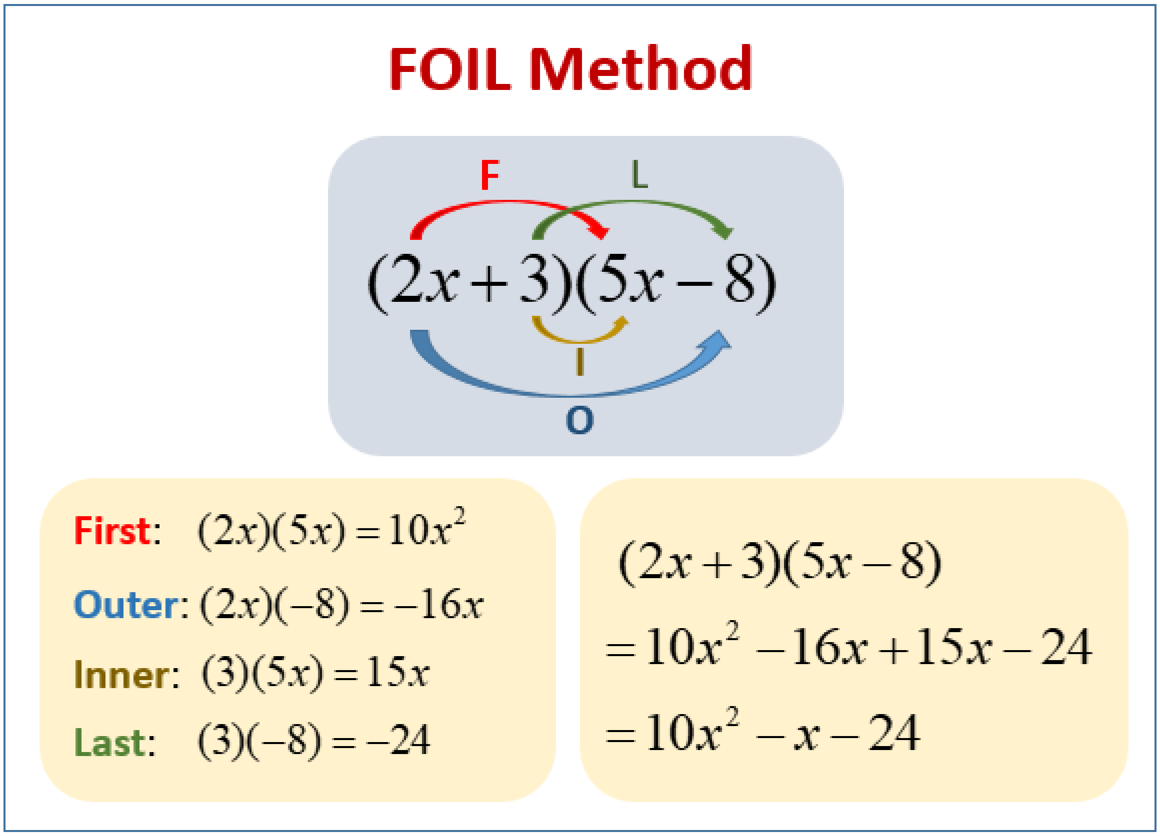
15.2 Polynomials Arithmetic

actually it is same thing that number arithmetic. we make operation among whose terms coefficient is same at the addition and subtraction. at the multiplacation and division we operation this to every term

$$3x^2 - x + 5 - 8x^2 + 3x - 9 = -5x^2 + 2x - 4$$
$$-3x(2x - 3) = 6x^2 + 9x$$

15.3 FOIL Method

when you product the two bionomials you can use foil method you might heard this but i want continue to explain firstly we product first items subsequently ouert items after inner items and subsequently we product last items and finally we simplify terms that we get from product



15.4 Rationalize A Denominator

when your denominator is irrational and you want to rationalize it you have to multiple by 1 but not this 1 our 1 = $\frac{\text{changed denominator}}{\text{changed denominator}}$ and changed denominstor is which we change positive and negative sign of irrational number in the denominator i know your head confused but you will better understand on example

$$\frac{2}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}}$$
$$\frac{2(2+\sqrt{2})}{2-\sqrt{2}\cdot 2+\sqrt{2}}$$
$$\frac{4+2\sqrt{2}}{4+2\sqrt{2}-2\sqrt{2}-2}$$
$$\frac{4+2\sqrt{2}}{2} = 2+\sqrt{2}$$

15.5 Division Polynomials

dividing polynomials is same to numbers division also we use long division too and rules for number divisin is can apply to polynomials like if the dividend $P(x)$ and divisor $D(x)$ are polynomials such that $D(x) \neq 0$ then there exist unique polynomials $Q(x)$ quotient and the remainder $R(x) such that$

$$P(x) = Q(x) \cdot D(x) + R(x) \tag{15.1}$$

$$\text{where } R(x) = 0 \text{ or } \deg R(x) < \deg D(x) \tag{15.2}$$

this topic also understand better with solve examples so i write some expales to solve

$\dfrac{8x^5}{2x^3}$ simplify divison

$\dfrac{x^3 - 8}{x - 2}$ simplify divison

$\dfrac{x^2 - 3x + 2}{x - 1}$ simplify divison

15.6 Solving Polynomials

i write some polynomials to you don't forget to solve them

$6x^5 + 24x^3 = 0$

$x^3 - x^2 - 9x + 5 = 0$

$x^3 + 3x^2 - 4x - 12 = 0$

$x^3 - 7x + 6 = 0$

Chapter 16

Square Cube Roots

16.1 Square Roots

Definition: if a $\geq$ 0 the principal sqaure root of a denoted $\sqrt{a}$ is the non negative number

$$a = S^2$$

16.2 Other Roots

$$S^4 = 25 \Rightarrow S = \sqrt[4]{25}$$
$$x^n = 25 \Rightarrow x = \sqrt[n]{25}$$

Example

$$x^3 = -8$$
$$x = \sqrt[3]{-8}$$
$$\sqrt[3]{-8} = -2$$

the odd root of a negative number is negative number this is normal to have negative numbers when you have odd rots but on the even roots we dont get real solution we get complex number slutions like -2i

$$\begin{aligned} \sqrt[4]{16} &= 2 \longleftrightarrow x^4 = 16 \\ \sqrt[4]{-16} &= -2i \longleftrightarrow x^4 = -16 \Rightarrow \text{ no real solution} \end{aligned}$$

16.3 Proporties Of Exponets

$$\sqrt{x} = x^{\frac{1}{2}}$$
$$\sqrt[4]{x} = x^{\frac{1}{4}}$$
$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$(xy)^{\frac{1}{n}} = x^{\frac{1}{n}} \cdot y^{\frac{1}{n}}$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$$

$$(\frac{x}{y})^{\frac{1}{n}} = \frac{x^{\frac{1}{n}}}{y^{\frac{1}{n}}}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$(x^r)^s = x^r s$$

$$(x^{\frac{1}{n}})^s = s^{\frac{s}{n}}$$

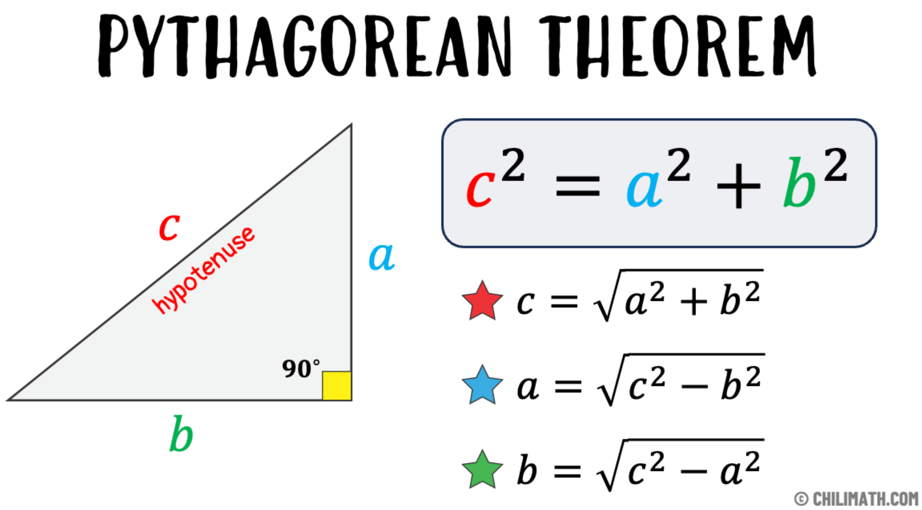
$$(\sqrt[n]{x})^s = \sqrt[n]{x^s}$$

Chapter 17

Pythogorean Theroem

17.1 Pythogorean Theroem

pythogorean theroem is a statemnt about right triangles if you don't have right triangle you can't apply this theorem reminder right angle is have 90 degress or pi over 2 either is fine



17.2 Pythogorean Theroem Proof

there is hundred ways to proof pythogorean theroem but i indicate only 1 way (algebric proof) if you want to see more proof you can look at this website https://www.cut-the-knot.org/pythagoras/index.shtml in this they indicate many proofs

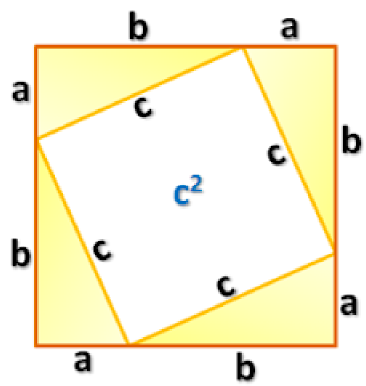


Figure 4-A

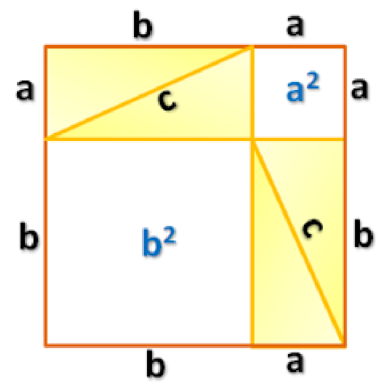
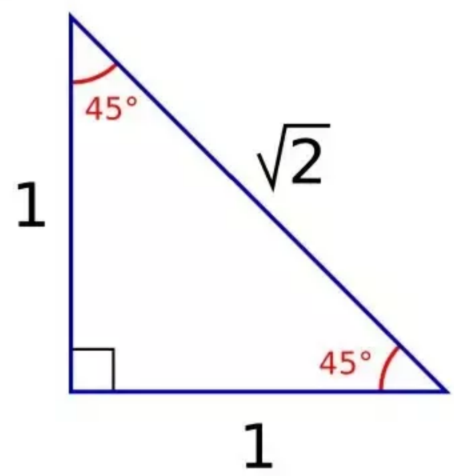


Figure 4-B

17.3 Special Right Triangles

IsosCeles Right Triangle



30 60 90 Right Triangle