Robot hand doc

https://youtube.com/shorts/aEagpjvMI6s
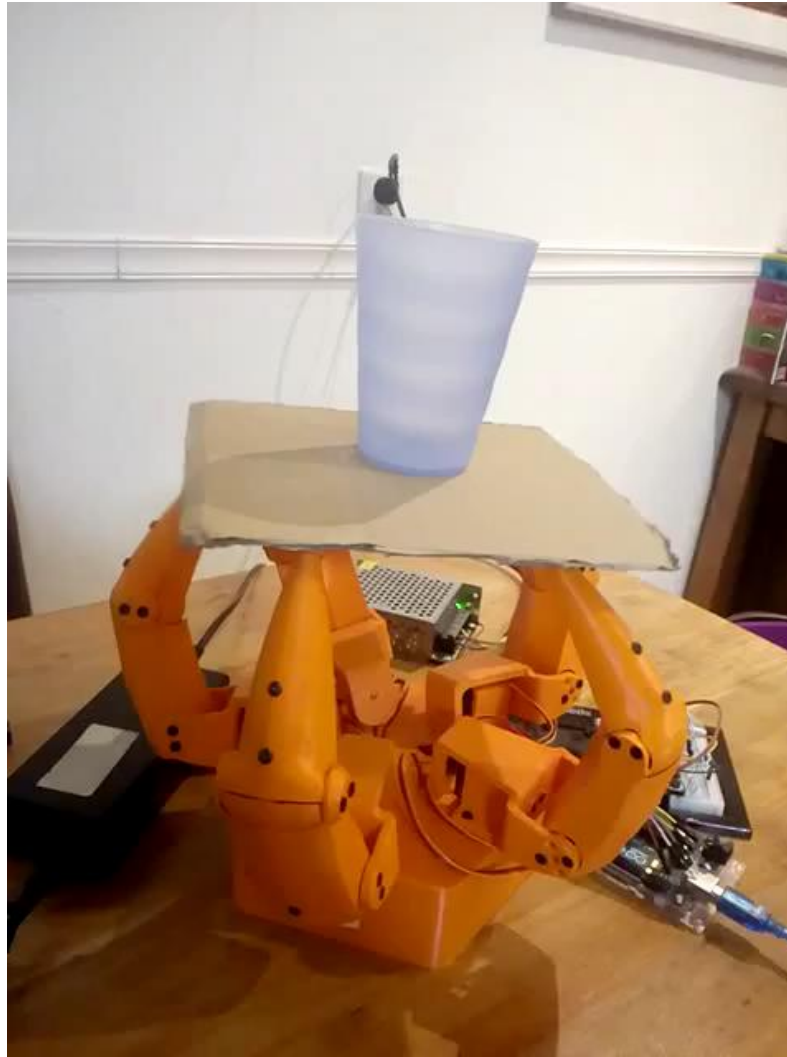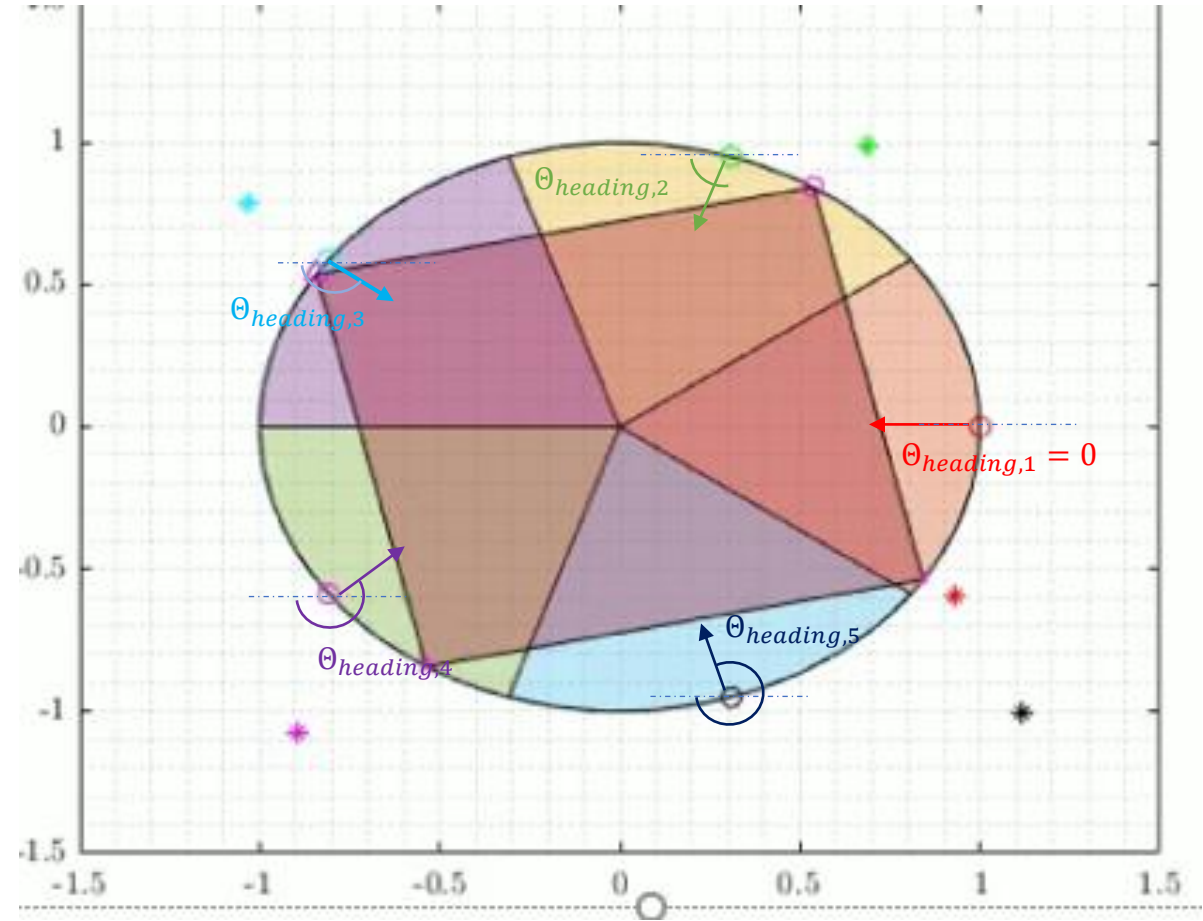
Content

- Robot hand layout

- Gait generation
  - 4 support fingers
  - 3 support fingers (slower movement of non-supporting fingers -> less noise)

- Inverse kinematics
  - Joint 1
  - Joint 2
  - Joint 3

- Putting it together (simulation)

Robot hand layout

- $\Theta_{heading}$ describes in which direction the finger is pointing when the lower joint in in the neutral position
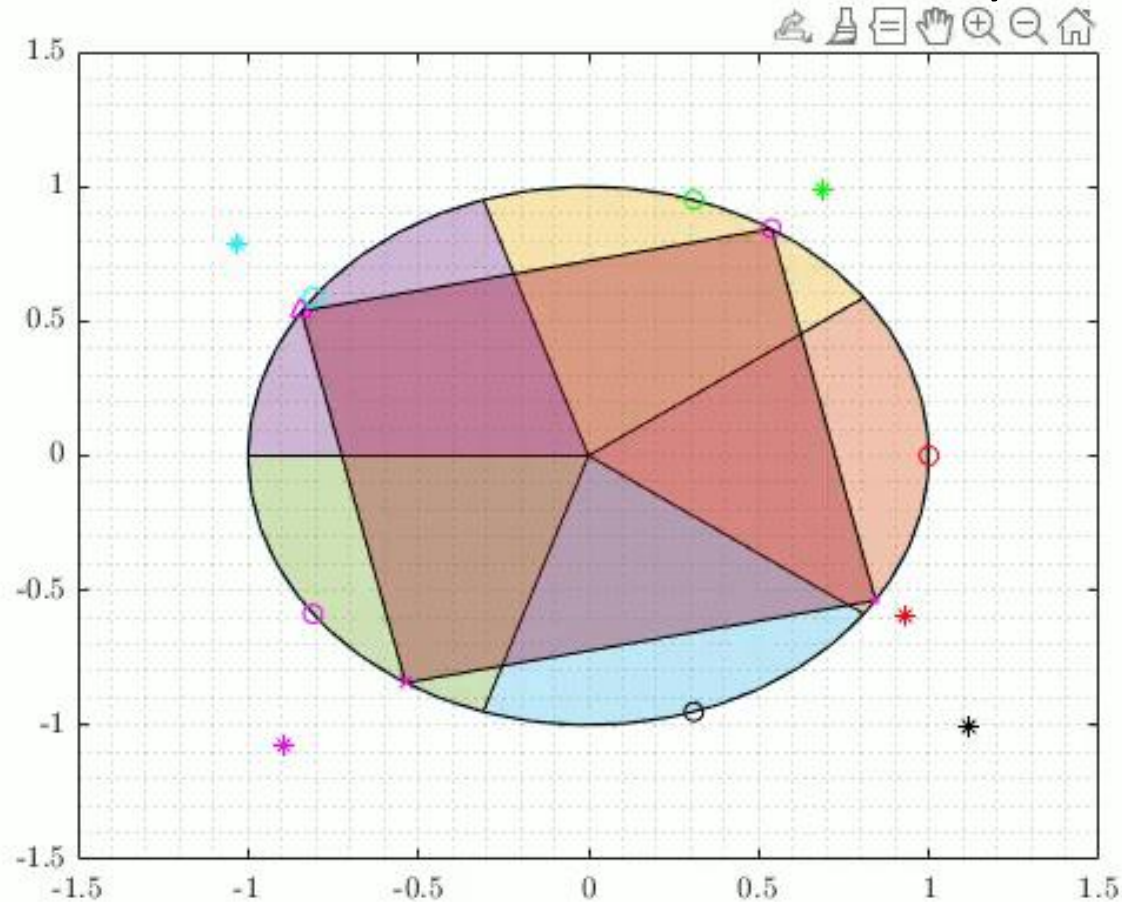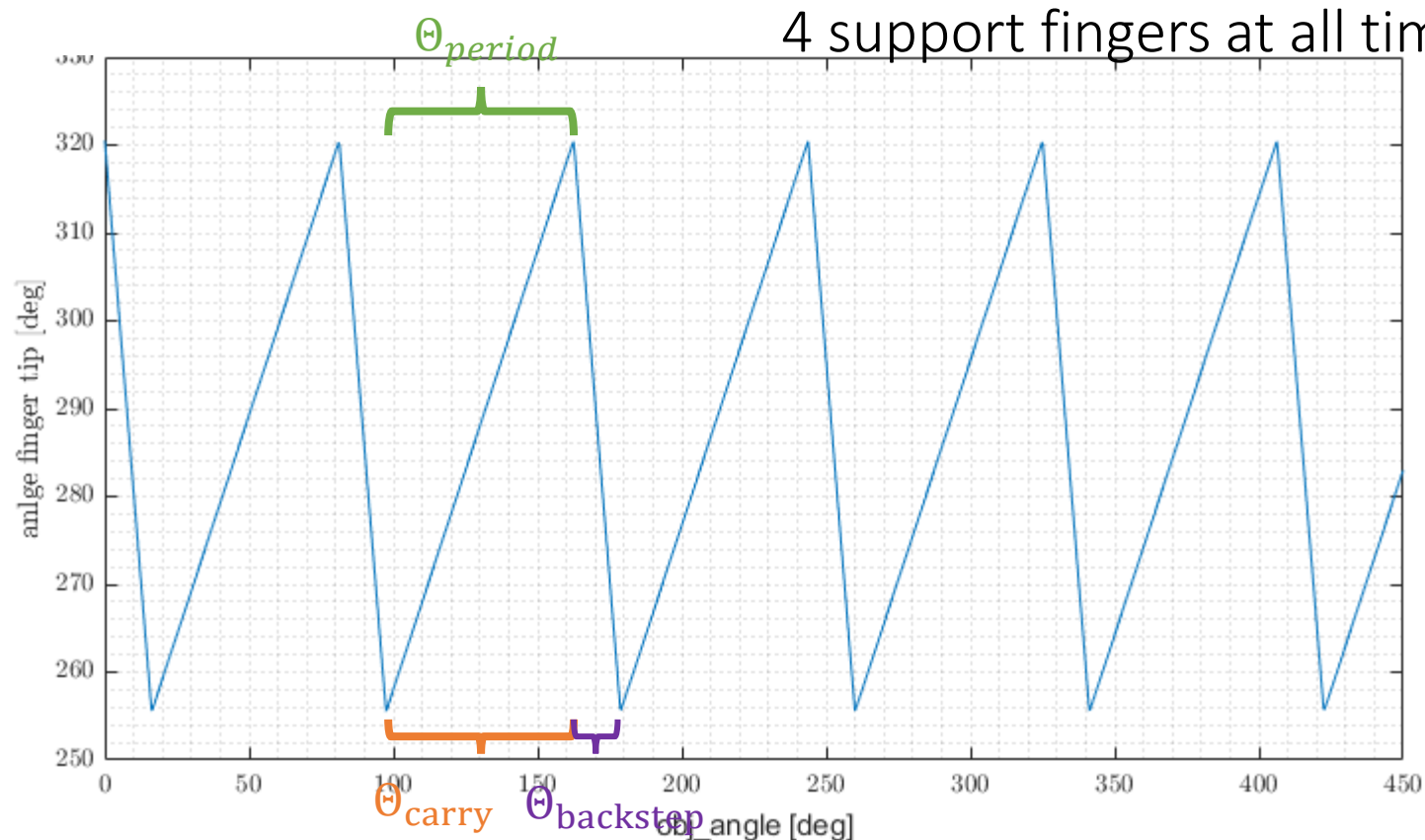
Gait generator doc
4 finger support

Goal: describe this mathematically

Gait generator doc

4 support fingers at all times

Observations

- Gait must be periodic
- Slope during the carry phase = 1
- Each finger has the same gait (with a different phase offset)
- The retraction phase of different fingers can not overlap

Analysis

Because retraction phases for fingers can not overlap
(always want 4 support points)

$$5\Theta_{backstep} < \Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

Thus

$$4\Theta_{backstep} \leq \Theta_{carry}$$

$$\Theta_{backstep} = f_{backstep} \frac{1}{4}\Theta_{carry}$$

$$f_{backstep} \in [0,1] \text{ (design choise)}$$

$$\Theta_{surplus} = (\Theta_{carry} - 4\Theta_{backstep})$$

(Time (in terms of obj rotation) when all fingers are touching)
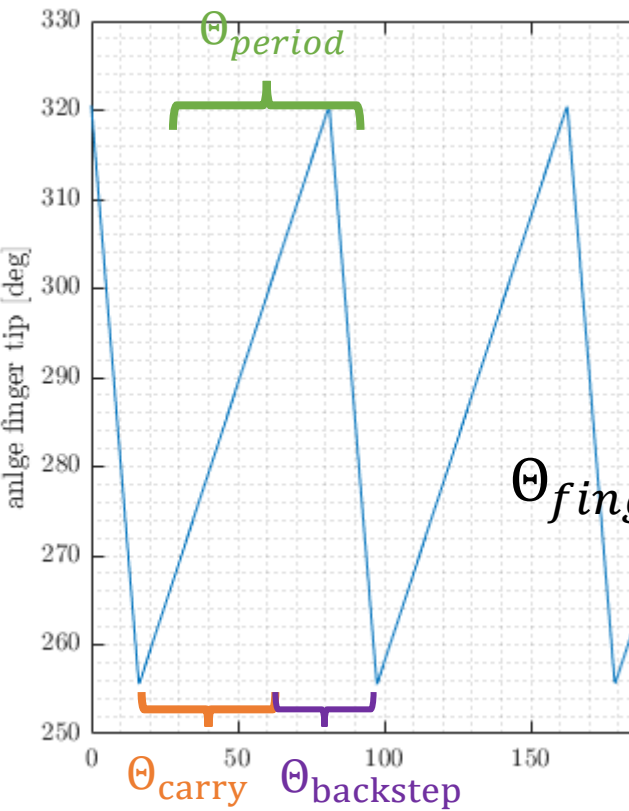
Can add a phase offset $\Theta_{phase\ offset} = f_{surplus}\Theta_{surplus}/5$
between fingers

$$f_{surplus} \in [0,1] \text{ (design choice = 1)}$$

Need $\Theta_{carry} < \frac{360}{5}[deg]$

else finger tips will come to close to each other

Radial angle equation

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
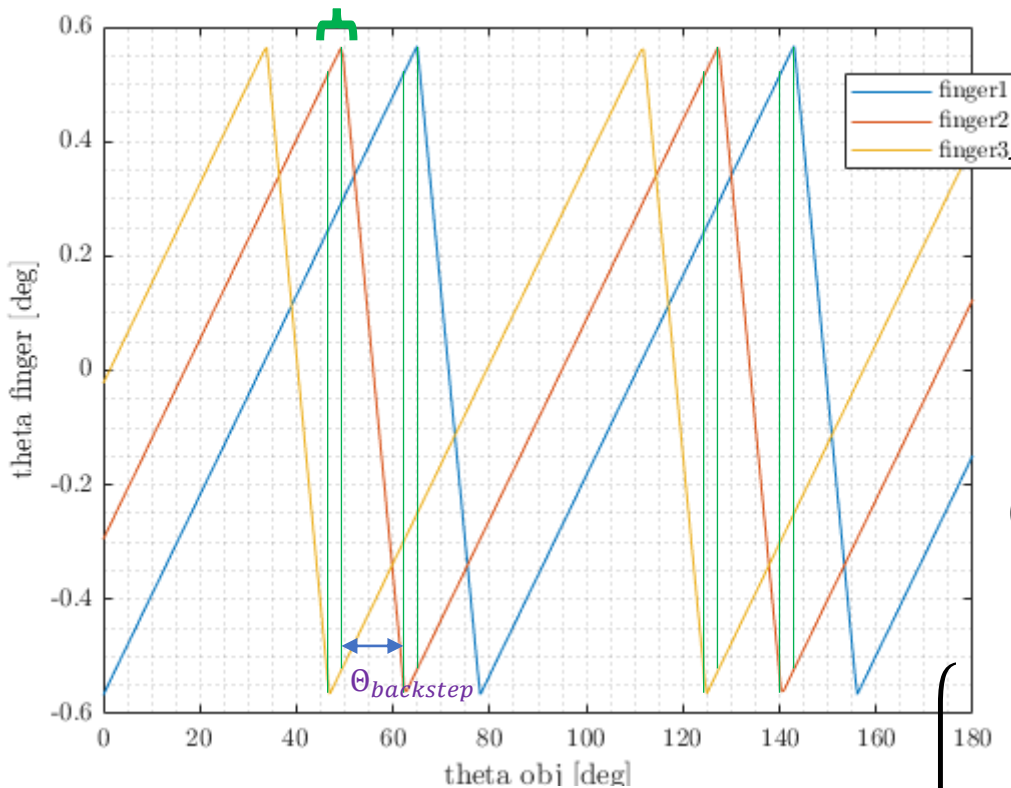$$\Theta_{m} = mod(\Theta_{obj}, \Theta_{period})$$

$$\Theta_{finger}(\Theta_{obj}) = \begin{cases} -\dfrac{\Theta_{carry}}{2} + \Theta_{m} & if \; \Theta_{m} \leq \Theta_{carry} \\ \dfrac{\Theta_{carry}}{2} - \dfrac{\Theta_{carry}}{\Theta_{backstep}}(\Theta_{m} - \Theta_{carry}) & if \; \Theta_{m} > \Theta_{carry} \end{cases}$$

$$\Theta_{finger_i}(\Theta_{obj}) = \Theta_{finger}(\Theta_{obj} + \Theta_{backstep} + (i-1)\Theta_{phase \; offset})$$

Handover (parallel)

- When 1 finger takes over a carry point from another finger, one should raise in height the other should lower

$\Theta_{handover}$

$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{4}\Theta_{carry}$$

$f_{backstep} \in [0, 1]$ (1 is fast raise/lower, 0 means slow raise lower)

- This scheme moves a finger up and down at same rate
- Take over is not at the object height…

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
$$\Theta_m = mod(\Theta_{obj}, \Theta_{period})$$

$$\Delta h_{finger}(\Theta_{obj}) = \begin{cases} -\Delta h \dfrac{\Theta_{handover} - \Theta_m}{\Theta_{handover}} & if \ \Theta_m \leq \Theta_{handover} \\ 0 & if \ \Theta_{handover} < \Theta_m \leq \Theta_{carry} - \Theta_{handover} \\ -\Delta h \dfrac{\Theta_m - (\Theta_{carry} - \Theta_{handover})}{\Theta_{handover}} & if \ \Theta_{carry} - \Theta_{handover} < \Theta_m \leq \Theta_{carry} \\ -\Delta h & if \ \Theta_{carry} < \Theta_m \end{cases}$$
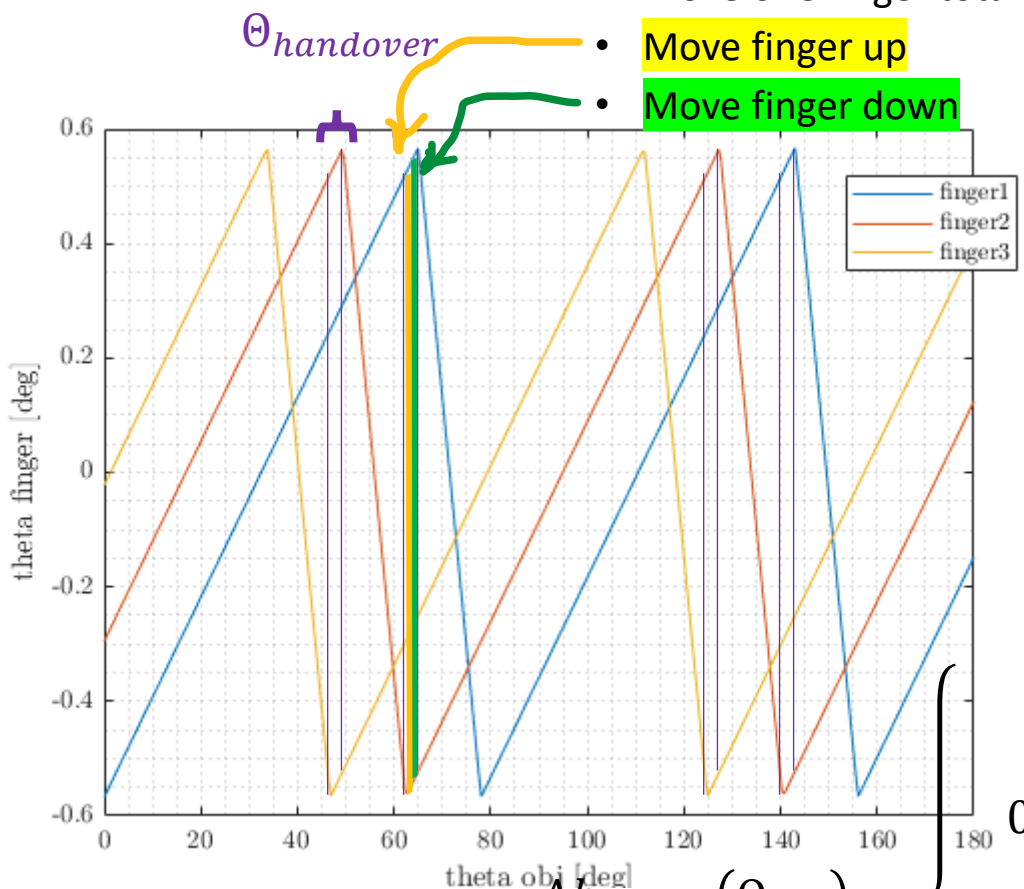
Handover (sequential)

- Want <u>handover</u> to happen <u>at the object height</u>
  - Move one finger totally up before retracting the other…
  - <mark>Move finger up</mark>
  - <mark>Move finger down</mark>

$\Theta_{handover}$

theta finger [deg] vs theta obj [deg]

Legend: finger1, finger2, finger3

$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{4}\Theta_{carry}$$

$f_{backstep} \in [0, 1]$ (design choice)

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
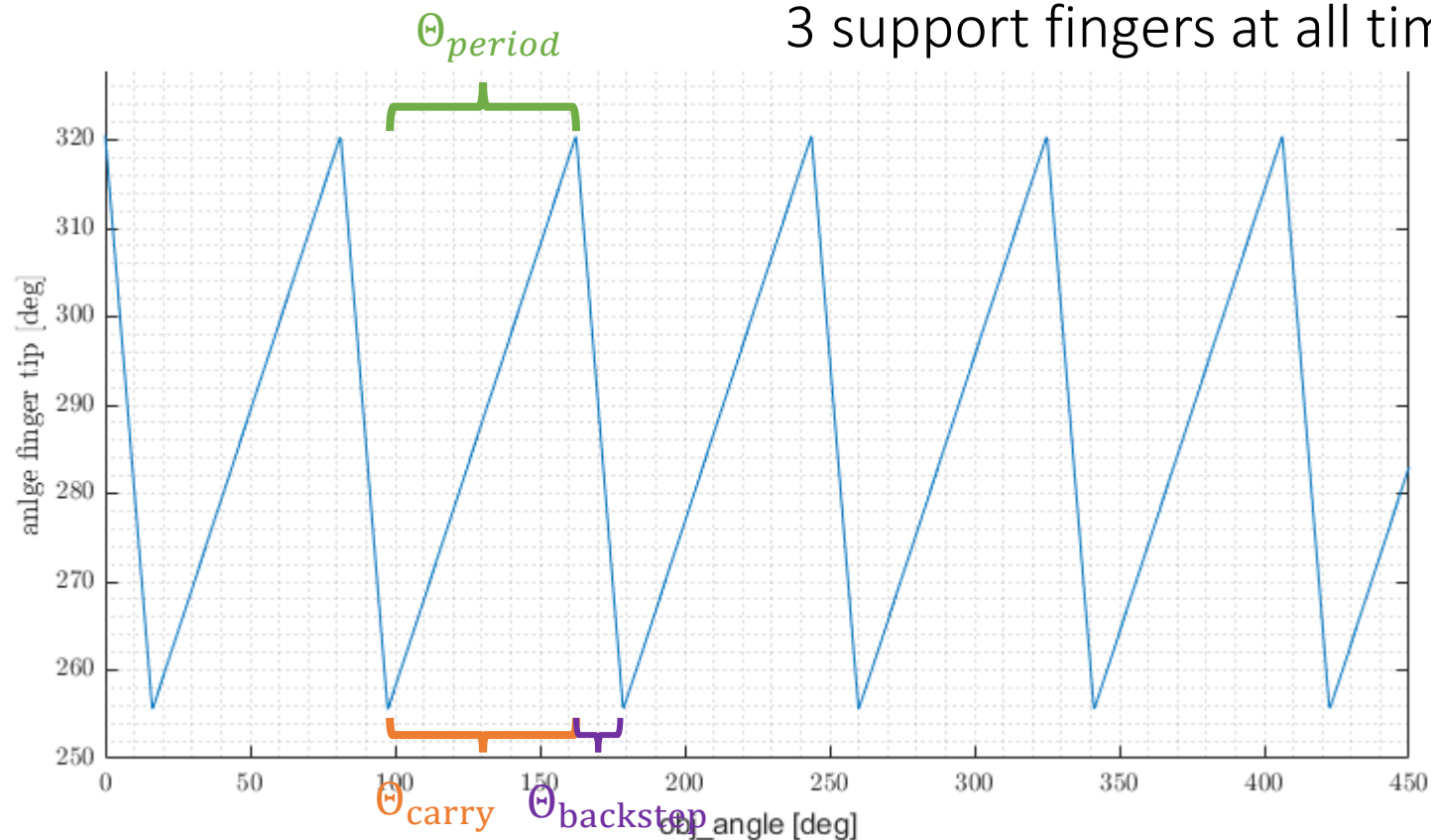$$\Theta_{m} = mod(\Theta_{obj}, \Theta_{period})$$

Linear height profile

$$\Delta h_{finger}(\Theta_{obj}) = \begin{cases} -\Delta h \dfrac{\Theta_{handover}/2 - \Theta_m}{\Theta_{handover}/2} & if\ \Theta_m \leq \Theta_{handover}/2 \\[2mm] 0 & if\ \Theta_{handover}/2 < \Theta_m \leq \Theta_{carry} - \Theta_{handover}/2 \\[2mm] -\Delta h \dfrac{\Theta_m - \left(\Theta_{carry} - \dfrac{\Theta_{handover}}{2}\right)}{\dfrac{\Theta_{handover}}{2}} & if\ \Theta_{carry} - \Theta_{handover}/2 < \Theta_m \leq \Theta_{carry} \\[2mm] -\Delta h & if\ \Theta_{carry} < \Theta_m \end{cases}$$

Gait generator doc

3 support fingers at all times

Θ_{period}

Θ_{carry} $\Theta_{backstep}$ obj_angle [deg]

anlge finger tip [deg]

Observations
- Gait must be periodic
- Slope during the carry phase = 1
- Each finger has the same gait (with a different phase offset)
- The retraction phase of different fingers can not overlap

Analysis

Because retraction phases for fingers can not overlap (always want 4 support points)

$$4\Theta_{backstep} < \Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

Thus

$$3\Theta_{backstep} \leq \Theta_{carry}$$

$$\Theta_{backstep} = f_{backstep} \frac{1}{3}\Theta_{carry}$$

$$f_{backstep} \in [0,1]$$

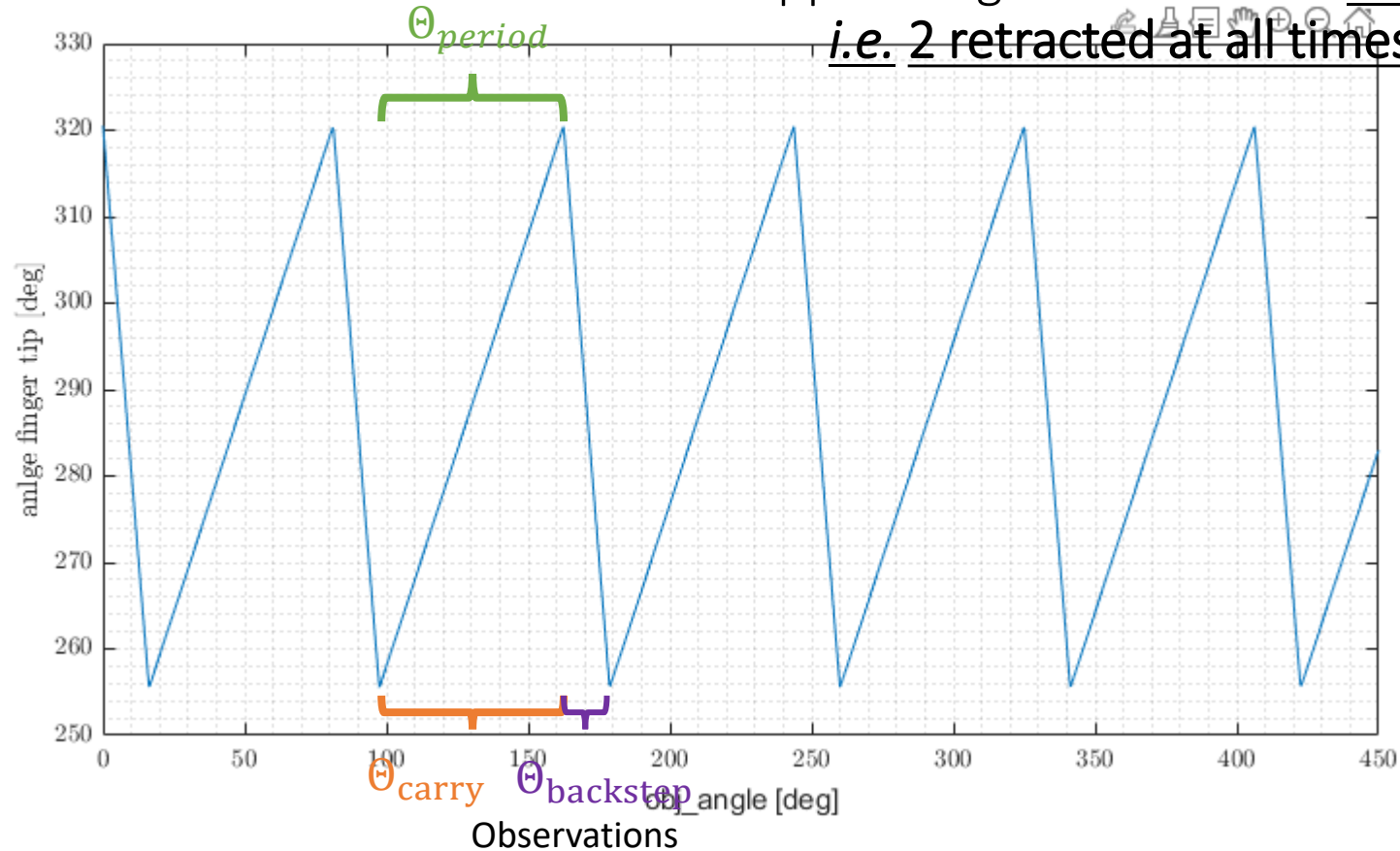By choosing $f_{backstep} < 1$, there are times where obj is supported by more than 3 points

$$\Theta_{phase\ offset} = (\Theta_{carry} - 3\Theta_{backstep})/5$$

Need $\Theta_{carry} < \dfrac{360}{5}[deg]$

else finger tips will come to close to each other

Gait generator doc

3 support fingers at all times _improved_
_i.e._ 2 retracted at all times

Analysis

Because retraction phases for fingers can not overlap
(always want 4 support points)

$$4\Theta_{backstep} < \Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

Thus
$$3\Theta_{backstep} \leq \Theta_{carry}$$
$$\Theta_{backstep} = f_{backstep}\frac{1}{3}\Theta_{carry}$$
$$f_{backstep} \in [0, 1]$$

By choosing $f_{backstep} < 1$, there are times where obj is supported by more than 3 points

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
$$\Theta_{phase\ offset} = \Theta_{period}/5$$

Observations

- Gait must be periodic
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- Each finger has the same gait (with a different phase offset)
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Need $\Theta_{carry} < \frac{360}{5}[deg]$

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Handover (parallel)

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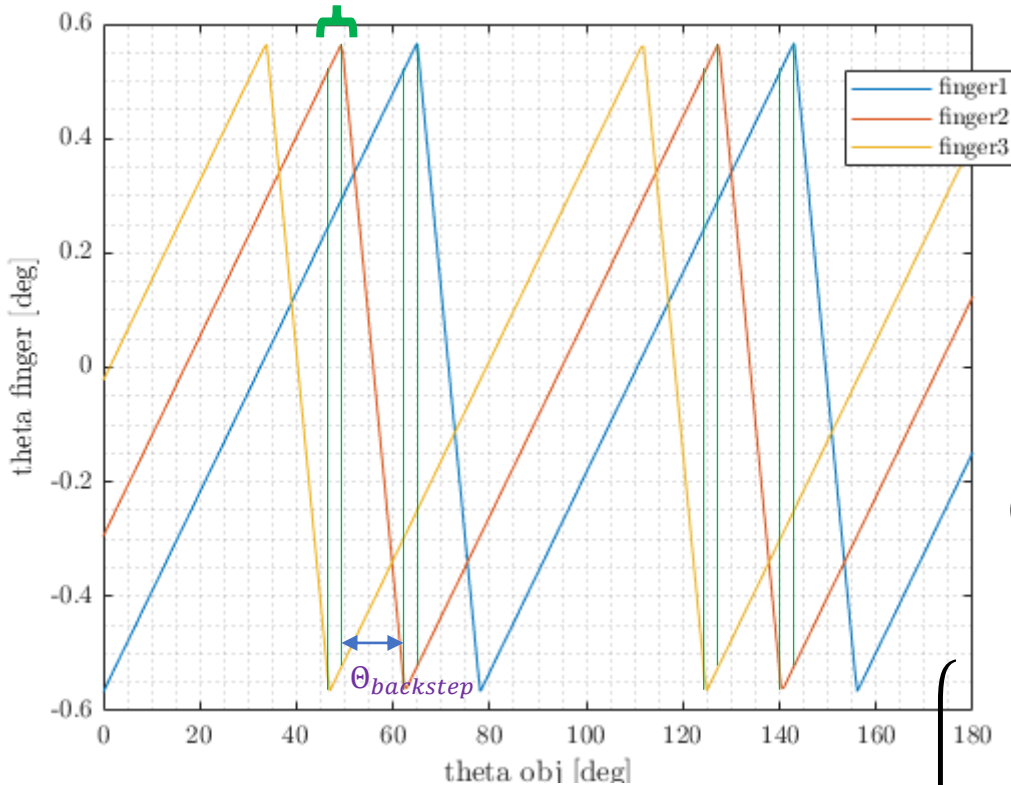
$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{3}\Theta_{carry}$$
$$f_{backstep} \in [0, 1]$$

- This scheme moves a finger up and down at same rate
- Take over is not at the object height…

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
$$\Theta_m = mod(\Theta_{obj}, \Theta_{period})$$

$$\Delta h_{finger}(\Theta_{obj}) = \begin{cases} -\Delta h \dfrac{\Theta_{handover} - \Theta_m}{\Theta_{handover}} & if\ \Theta_m \le \Theta_{handover} \\ 0 & if\ \Theta_{handover} < \Theta_m \le \Theta_{carry} - \Theta_{handover} \\ -\Delta h \dfrac{\Theta_m - (\Theta_{carry} - \Theta_{handover})}{\Theta_{handover}} & if\ \Theta_{carry} - \Theta_{handover} < \Theta_m \le \Theta_{carry} \\ -\Delta h & if\ \Theta_{carry} < \Theta_m \end{cases}$$
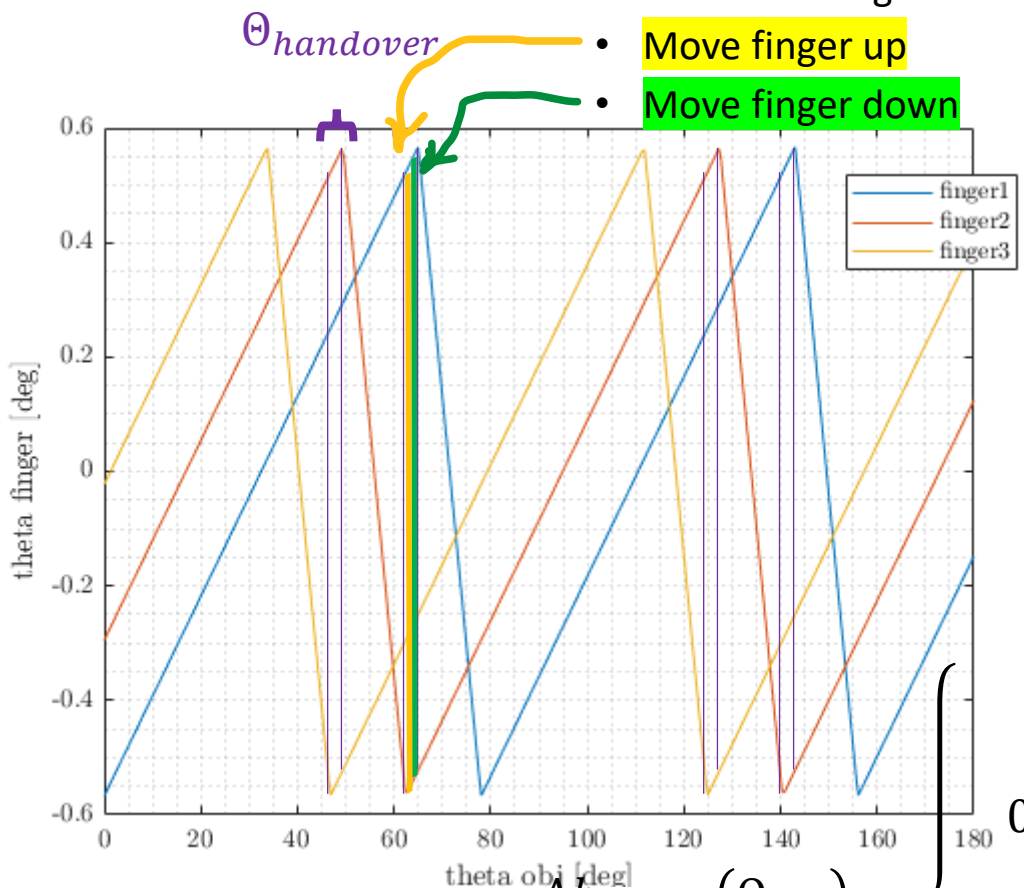
Handover (sequential)

- Want <u>handover</u> to happen <u>at the object height</u>
  - Move one finger totally up before retracting the other...
  - $\Theta_{handover}$ Move finger up
  - Move finger down

$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{3}\Theta_{carry}$$

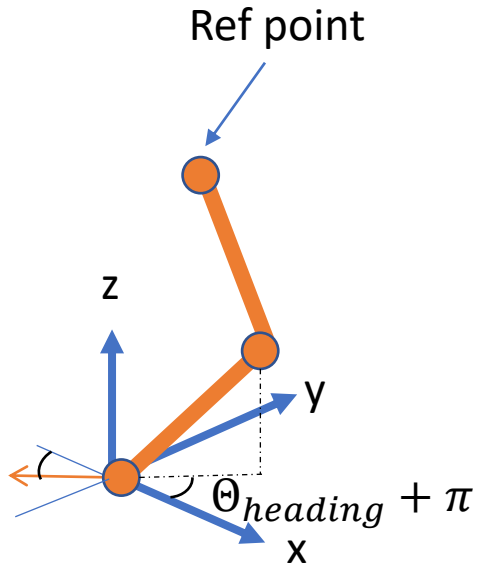$$f_{backstep} \in [0, 1] \text{ (design choice)}$$

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$
$$\Theta_{m} = mod(\Theta_{obj}, \Theta_{period})$$

theta finger [deg] / theta obj [deg]

finger1
finger2
finger3

Linear height profile

$$\Delta h_{finger}(\Theta_{obj}) = \begin{cases} -\Delta h \dfrac{\Theta_{handover}/2 - \Theta_{m}}{\Theta_{handover}/2} & if \ \Theta_{m} \leq \Theta_{handover}/2 \\ 0 & if \ \Theta_{handover}/2 < \Theta_{m} \leq \Theta_{carry} - \Theta_{handover}/2 \\ -\Delta h \dfrac{\Theta_{m} - \left(\Theta_{carry} - \dfrac{\Theta_{handover}}{2}\right)}{\dfrac{\Theta_{handover}}{2}} & if \ \Theta_{carry} - \Theta_{handover}/2 < \Theta_{m} \leq \Theta_{carry} \\ -\Delta h & if \ \Theta_{carry} < \Theta_{m} \end{cases}$$
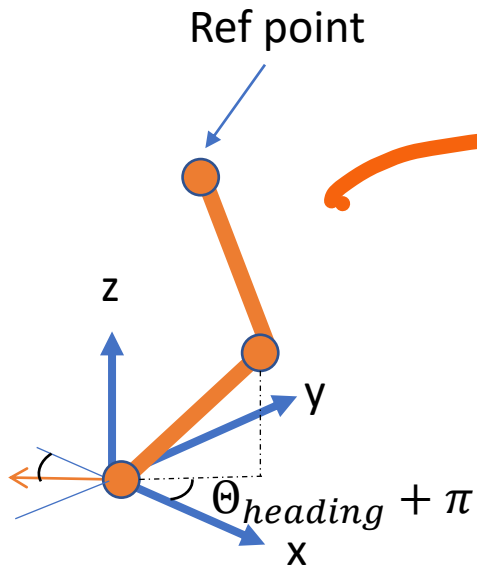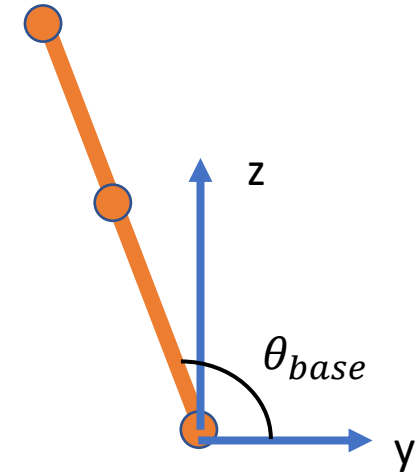
Inverse kinematics

Goal

Given a reference point (generated by the gait generator) find the joint angles such that the finger reaches this point

Inverse kinematics
base angle

Ref point

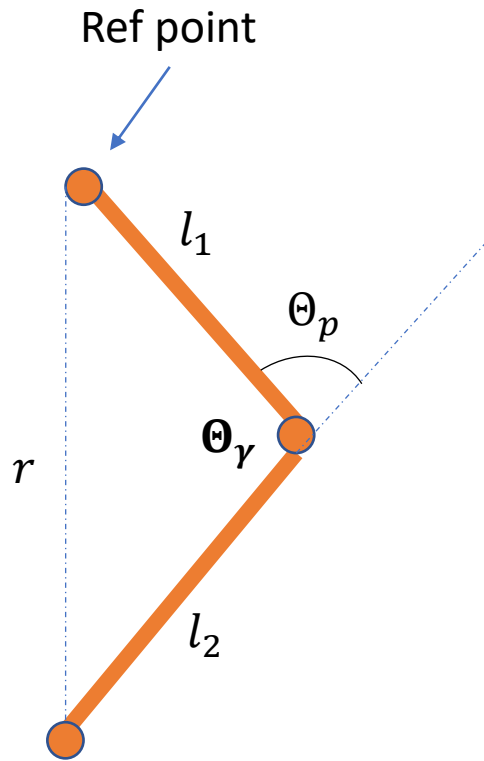- 1. Rotate (ref point-base coordinate) by the heading
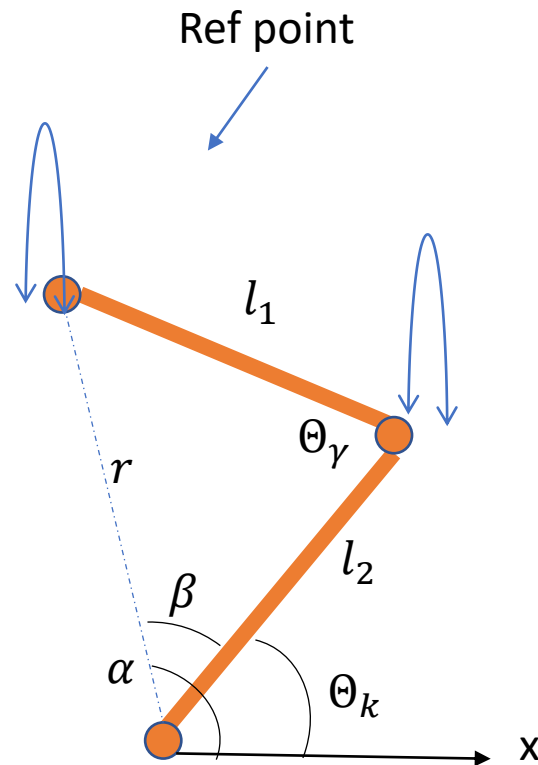
- 2. Project into 2D (ignore x coordinate)

- Apply sine/cosine rule

- $\Theta_{base} = \text{asin}(\frac{z}{\sqrt{(z^2+y^2)}})$

- $\Theta_{base} = \text{acos}(\frac{y}{\sqrt{(z^2+y^2)}})$

z

y

$\Theta_{heading} + \pi$

x

z

θ_{base}

y

Inverse kinematics phalanx

- $\Theta_p = \pi - \boldsymbol{\theta_\gamma}$

- (Θ_p can be set by a servo in the finger)

- Law of cosines

- $r^2 = l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta_\gamma)$

- $\boldsymbol{\theta_\gamma} = \mathrm{acos}(\frac{l_1^2 + l_2^2 - r^2}{2l_1 l_2})$

Inverse kinematics phalange

- Observation
  - Rotation of the base angle does not change the x coordinates

- $\Theta_k = \alpha - \beta$
- (Θ_k can be set by a servo in the finger)

- Trig + law of cosines
- $\alpha = \text{acos}(\frac{x}{r})$
- $\beta = \text{acos}(\frac{r^2 + l_2^2 - l_1^2}{2rl_2})$

Putting it together

- Can use the simulation to tune (to look good)
  - Finger lengths
  - Radius around which the fingers are distributed
  - The radius around which the object is carried
    - In reality the controlled joint angles are not perfect and the finger has some play. This causes the finger to not reach the reference point. In practice a large object carry radius was necessary such that the object does not fall/slide off.

https://youtu.be/Ng2OiVzpYz4