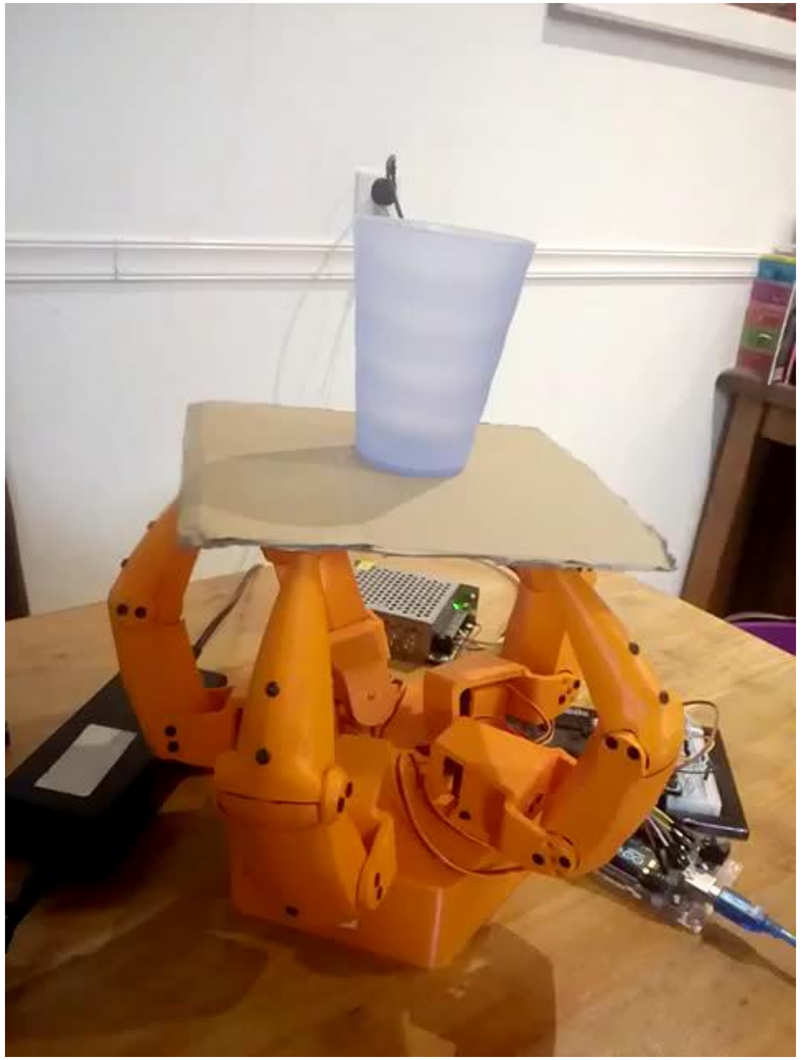
Robot hand doc



https://youtube.com/shorts/aEagpjvMI6s

Content

- Robot hand layout

- Gait generation
  - 4 support fingers
  - 3 support fingers (slower movement of non-supporting fingers -> less noise)

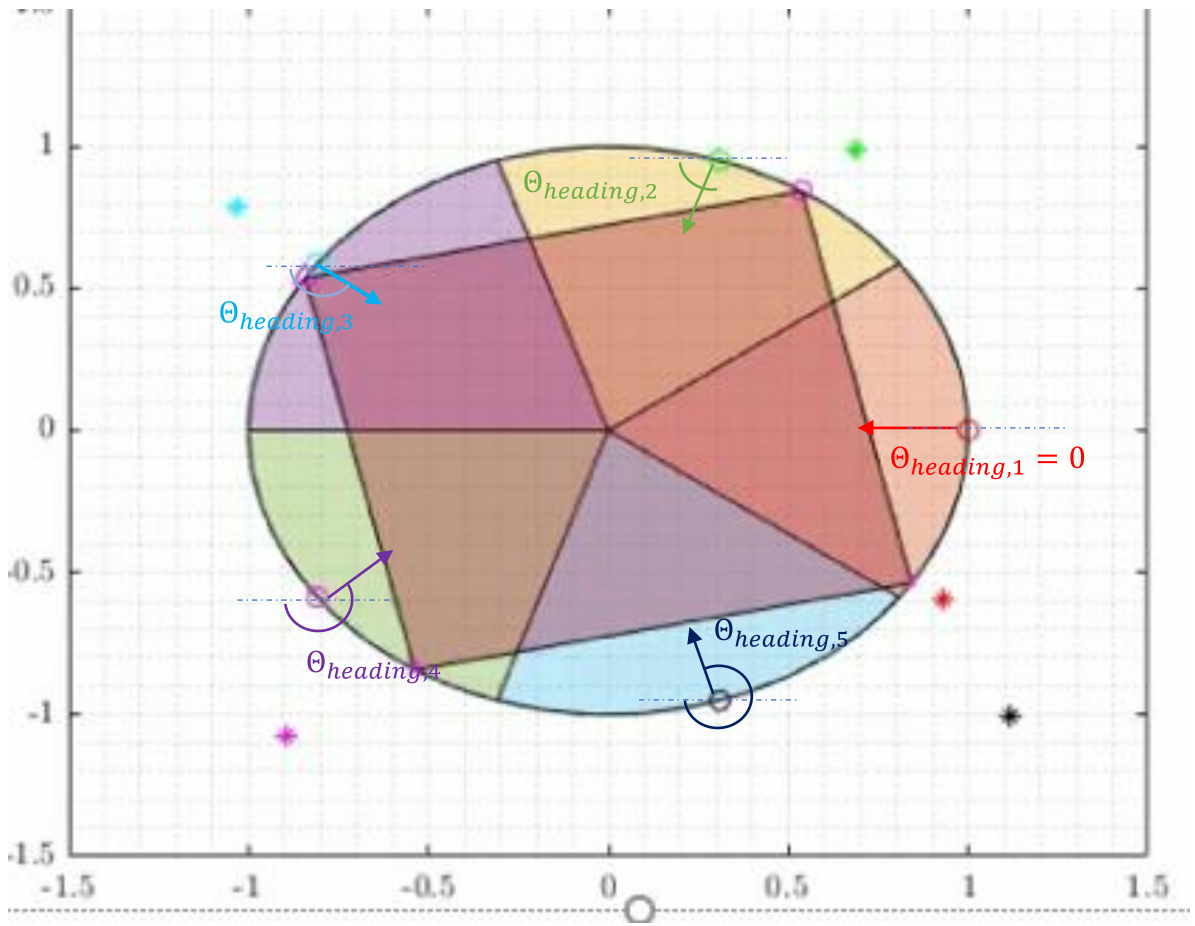
- Inverse kinematics
  - Joint base
  - Joint phalange
  - Joint knuckle

- Putting it together (simulation)

Robot hand layout

- $\Theta_{heading}$ describes in which direction the finger is pointing when the lower joint in in the neutral position

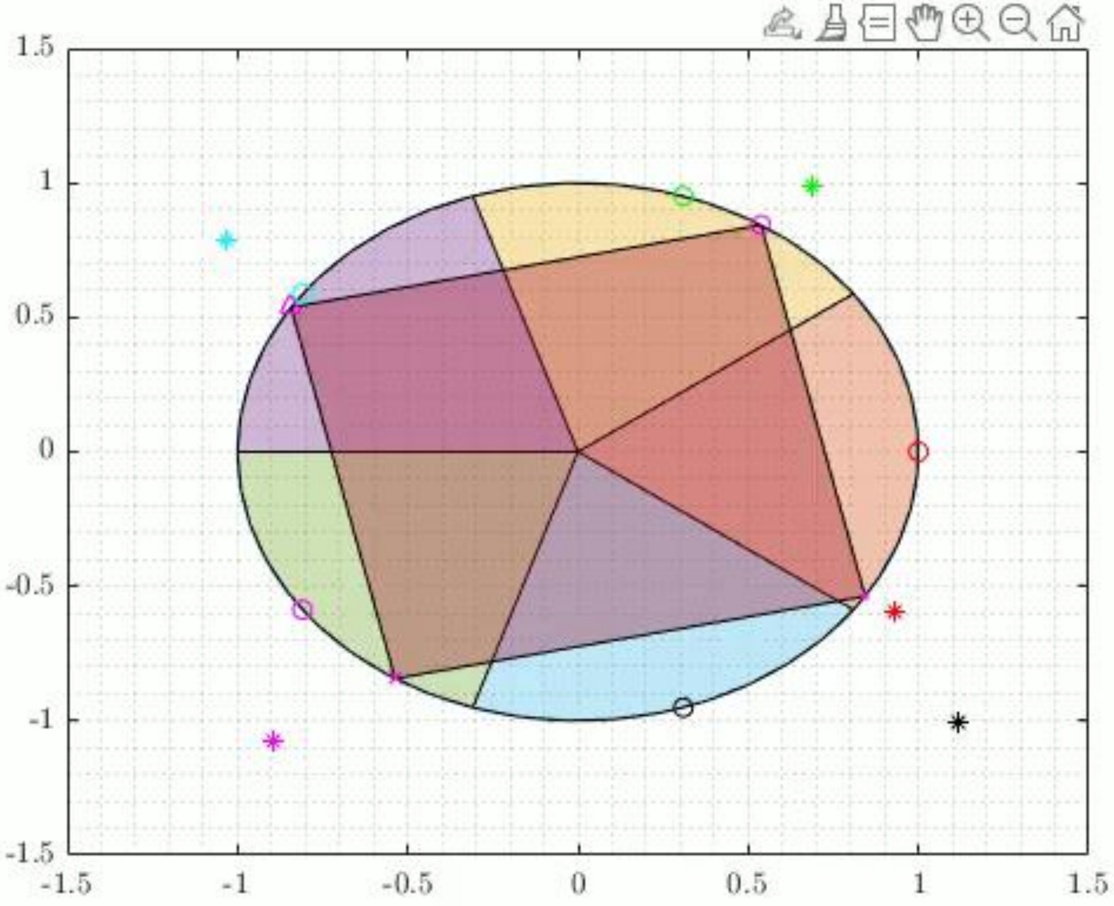
Top view



Gait generator doc

4 finger support

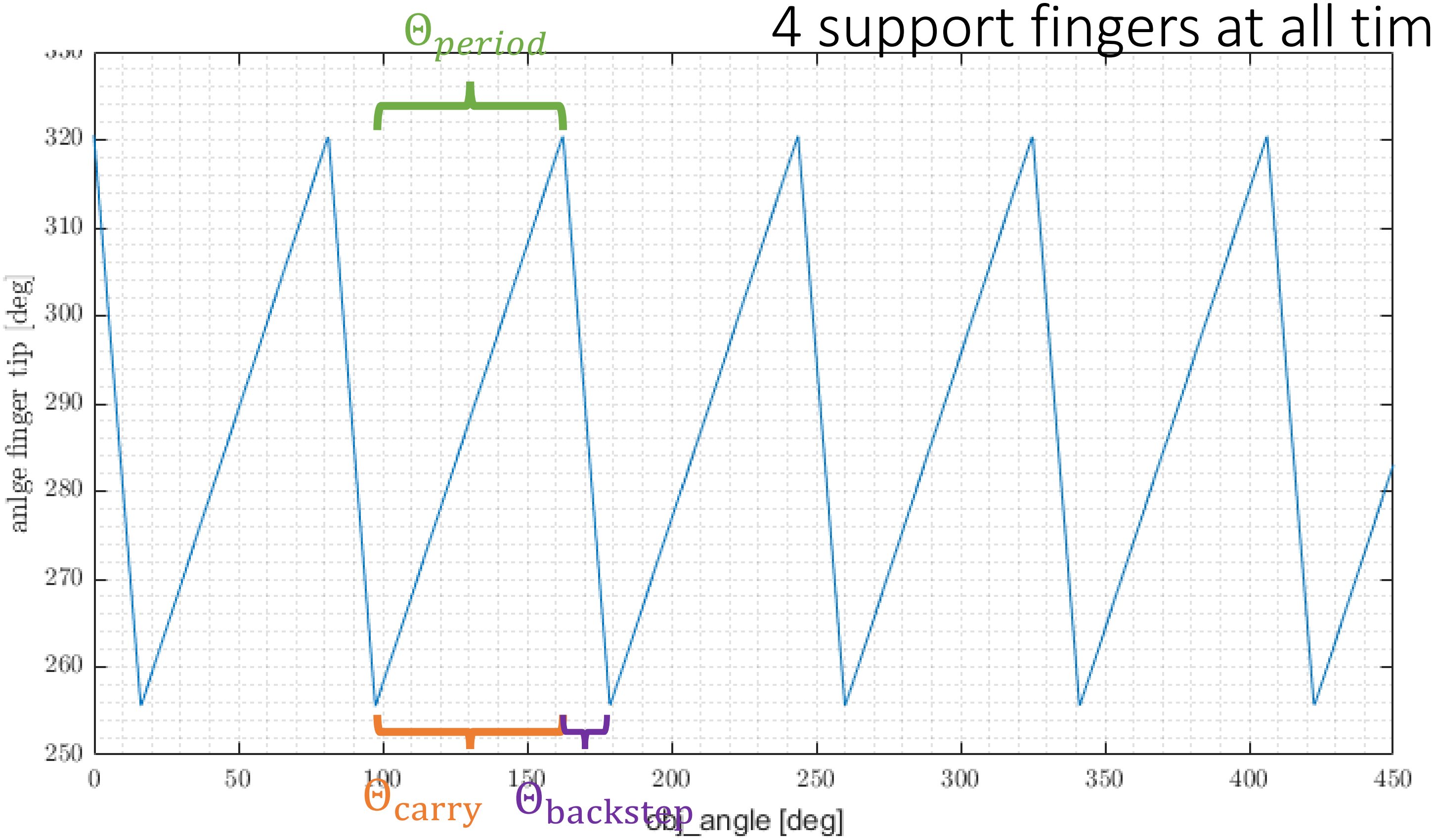
Goal: describe this mathematically



https://youtu.be/6gGeVKzd5BY

Gait generator doc

4 support fingers at all times



Observations

- Gait must be periodic
- Slope during the carry phase = 1
- Each finger has the same gait (with a different phase offset)
- The retraction phase of different fingers can not overlap

Analysis

Because retraction phases for fingers can not overlap (always want 4 support points)

$$5\Theta_{backstep} < \Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

Thus

$$4\Theta_{backstep} \leq \Theta_{carry}$$

$$\Theta_{backstep} = f_{backstep}\frac{1}{4}\Theta_{carry}$$

$f_{backstep} \in [0, 1]$ (design choise)

$$\Theta_{surplus} = (\Theta_{carry} - 4\Theta_{backstep})$$

(Time (in terms of obj rotation) when all fingers are touching)

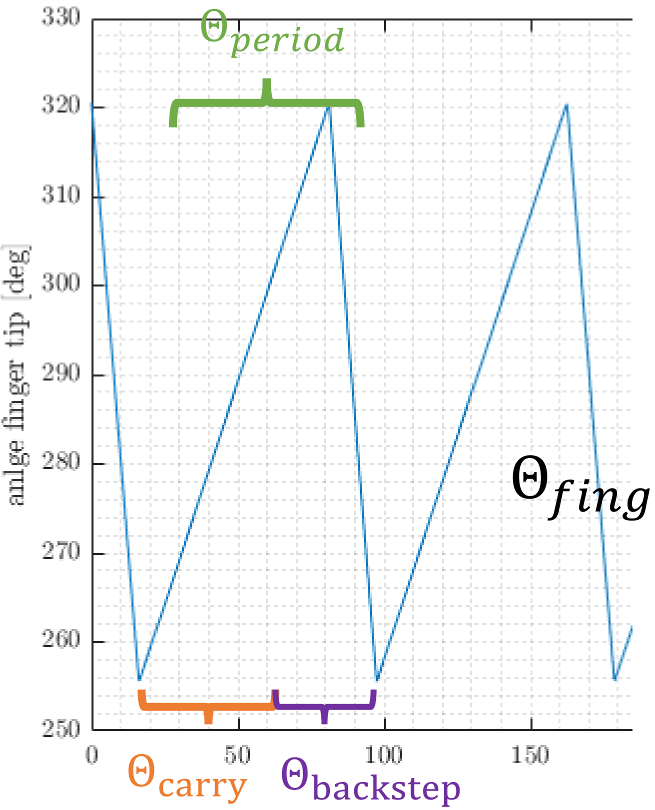
Can add a phase offset $\Theta_{phase\,offset} = f_{surplus}\Theta_{surplus}/5$ between fingers

$f_{surplus} \in [0, 1]$ (design choice = 1)

Need $\Theta_{carry} < \frac{360}{5}[deg]$

else finger tips will come to close to each other

Radial angle equation



$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

$$\Theta_{\mathrm{m}} = mod(\Theta_{obj}, \Theta_{period})$$

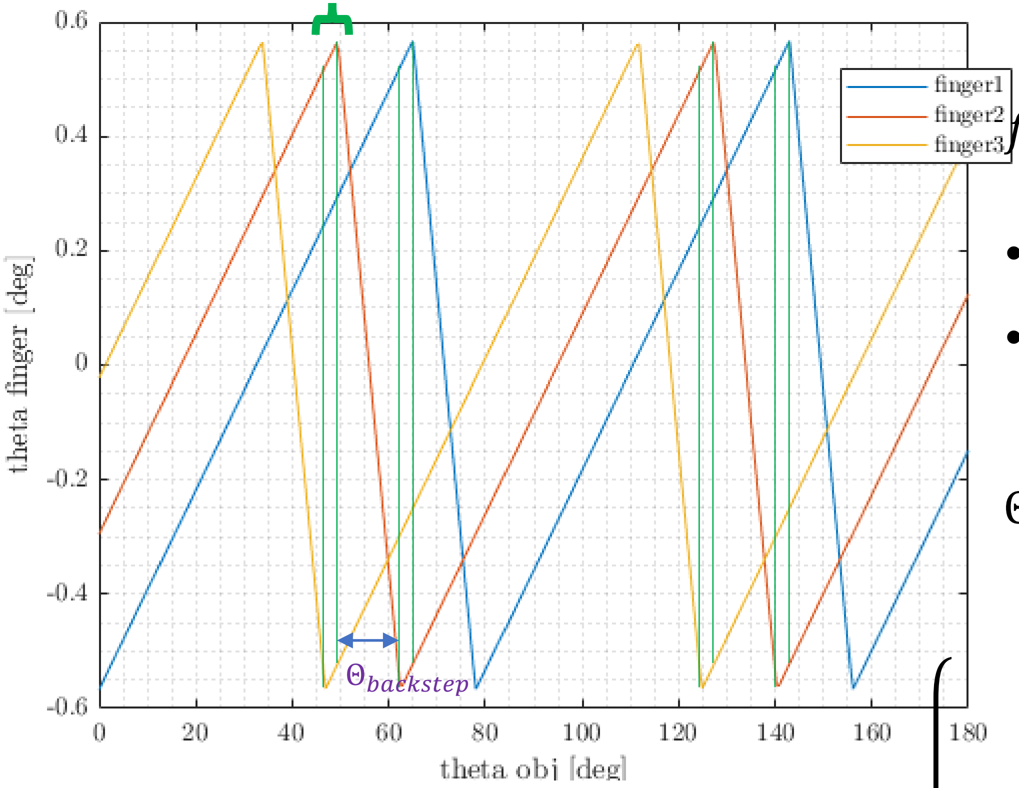
$$\Theta_{finger}(\Theta_{obj}) = \begin{cases} -\dfrac{\Theta_{carry}}{2} + \Theta_m & if\ \Theta_m \leq \Theta_{carry} \\ \dfrac{\Theta_{carry}}{2} - \dfrac{\Theta_{carry}}{\Theta_{backstep}}(\Theta_m - \Theta_{carry}) & if\ \Theta_m > \Theta_{carry} \end{cases}$$

$$\Theta_{finger_i}(\Theta_{obj}) = \Theta_{finger}(\Theta_{obj} + \Theta_{backstep} + (i-1)\Theta_{phase\,offset})$$

Handover (parallel)

- When 1 finger takes over a carry point from another finger, one should raise in height the other should lower

$\Theta_{handover}$



$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{4}\Theta_{carry}$$

$f_{backstep} \in [0,1]$ (1 is fast raise/lower, 0 means slow raise lower)

- This scheme moves a finger up and down at same rate
- Take over is not at the object height…

$$\Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

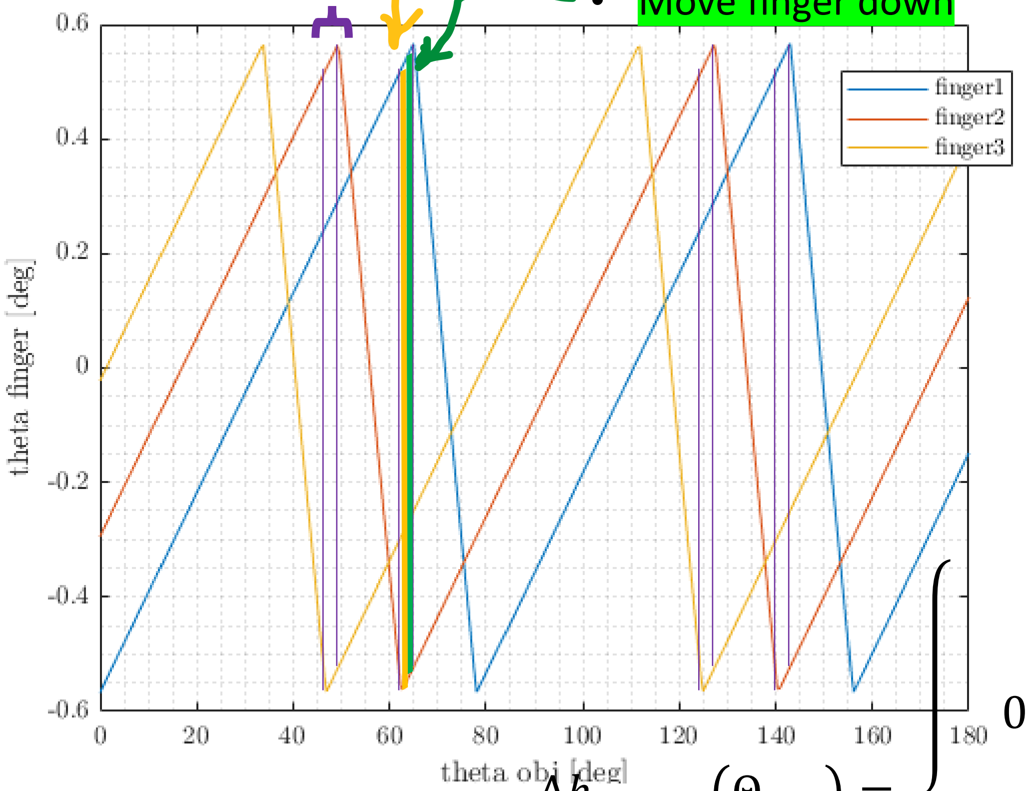
$$\Theta_{\mathrm{m}} = mod(\Theta_{obj}, \Theta_{period})$$

$$\Delta h_{finger}(\Theta_{obj}) = \begin{cases} -\Delta h \dfrac{\Theta_{handover} - \Theta_m}{\Theta_{handover}} & if\ \Theta_m \leq \Theta_{handover} \\ 0 & if\ \Theta_{handover} < \Theta_m \leq \Theta_{carry} - \Theta_{handover} \\ -\Delta h \dfrac{\Theta_m - (\Theta_{carry} - \Theta_{handover})}{\Theta_{handover}} & if\ \Theta_{carry} - \Theta_{handover} < \Theta_m \leq \Theta_{carry} \\ -\Delta h & if\ \Theta_{carry} < \Theta_m \end{cases}$$

Handover (sequential)

- Want handover to happen at the object height
  - Move one finger totally up before retracting the other…
  - Move finger up
  - Move finger down

$\Theta_{handover}$



$$\Theta_{handover} = (1 - f_{backstep})\frac{1}{4}\Theta_{carry}$$

$f_{backstep} \in [0, 1]$ (design choice)

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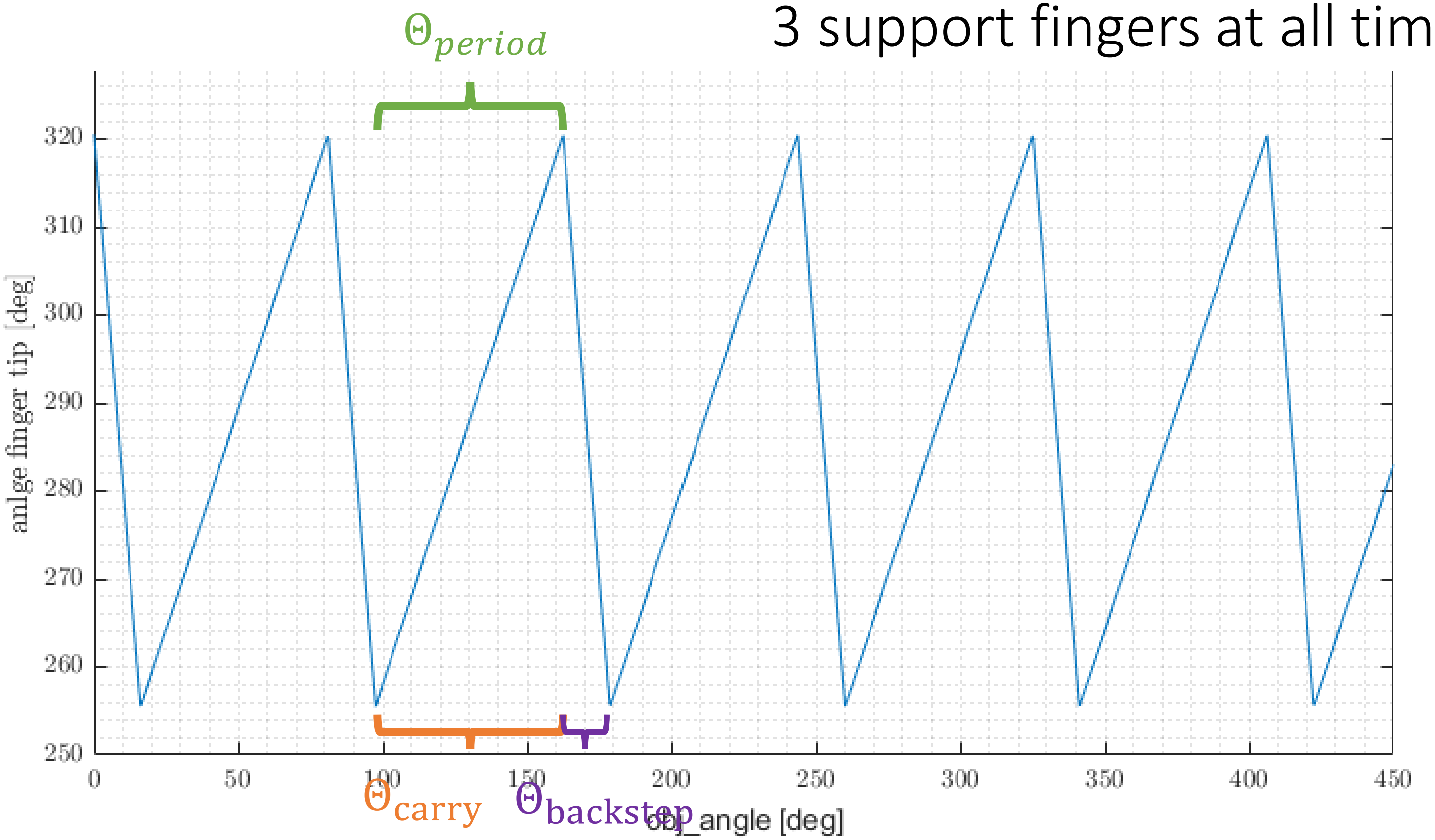
Linear height profile

Gait generator doc

3 support fingers at all times



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Because retraction phases for fingers can not overlap (always want 4 support points)

$$4\Theta_{backstep} < \Theta_{period} = \Theta_{carry} + \Theta_{backstep}$$

Thus

$$3\Theta_{backstep} \leq \Theta_{carry}$$

$$\Theta_{backstep} = f_{backstep}\frac{1}{3}\Theta_{carry}$$

$$f_{backstep} \in [0, 1]$$

By choosing $f_{backstep} < 1$, there are times where obj is supported by more than 3 points

$$\Theta_{phase\,offset} = (\Theta_{carry} - 3\Theta_{backstep})/5$$

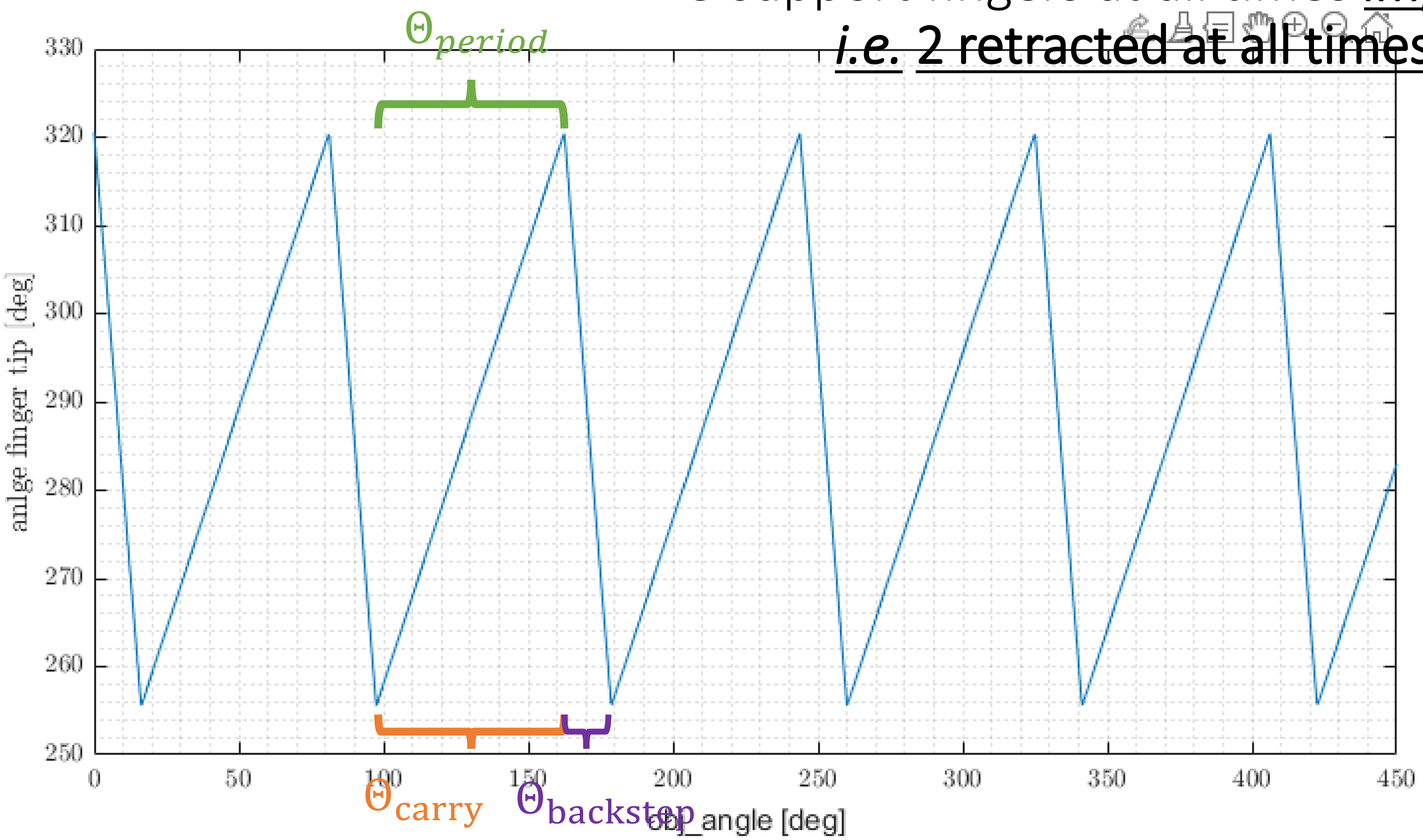
Need $\Theta_{carry} < \frac{360}{5}[deg]$

else finger tips will come to close to each other

Gait generator doc

3 support fingers at all times ***improved***

i.e. 2 retracted at all times



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$$\Theta_{period} = \Theta_{carry} + \Theta_{\mathrm{backstep}}$$

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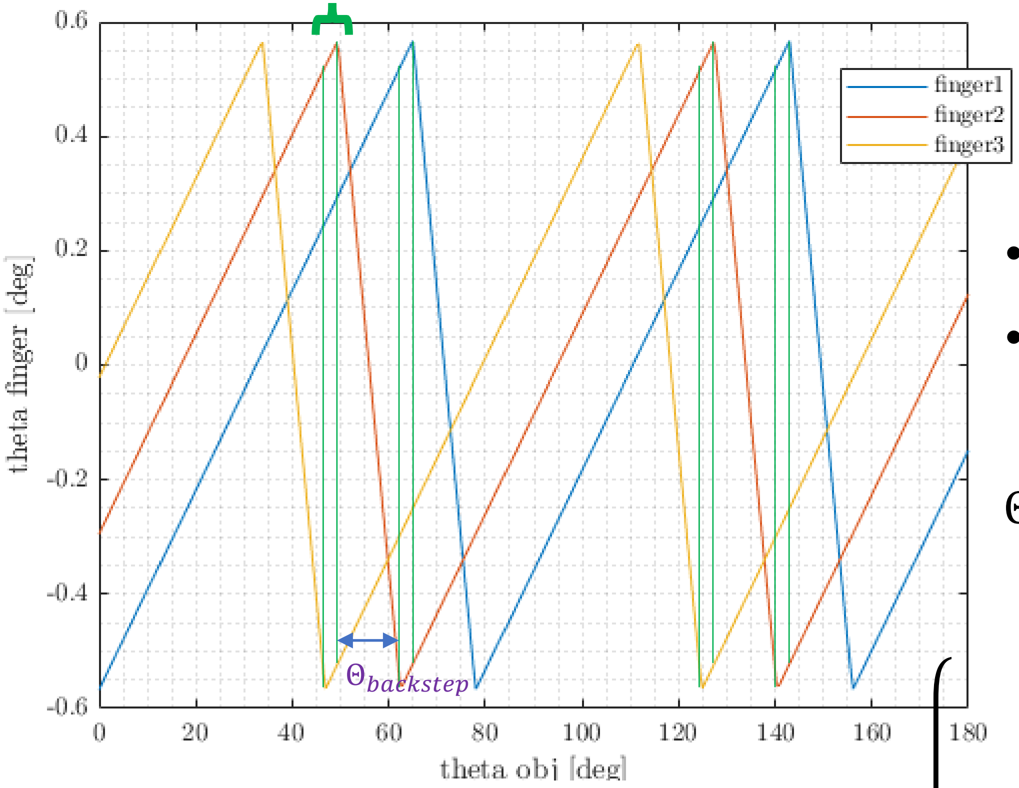

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$$f_{backstep} \in [0, 1]$$

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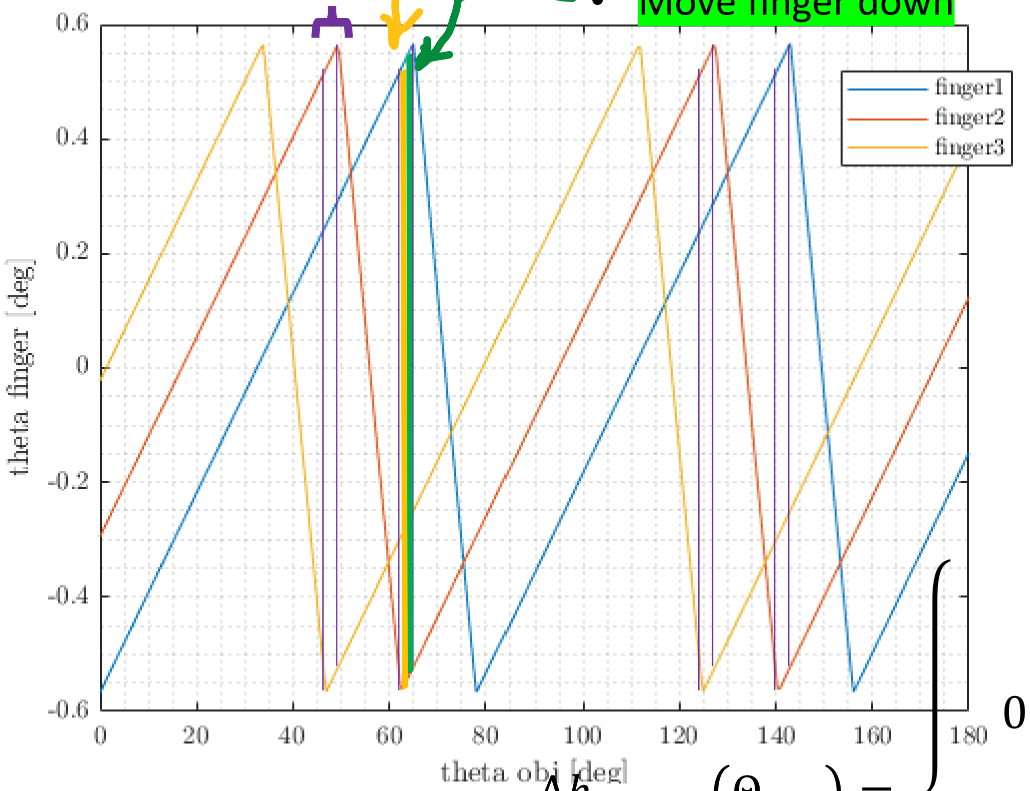

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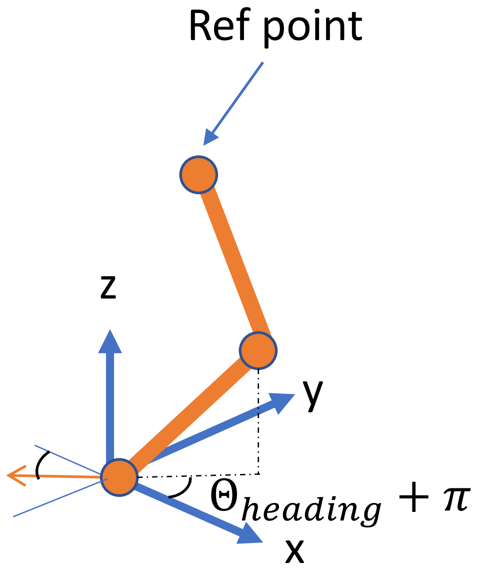
Linear height profile

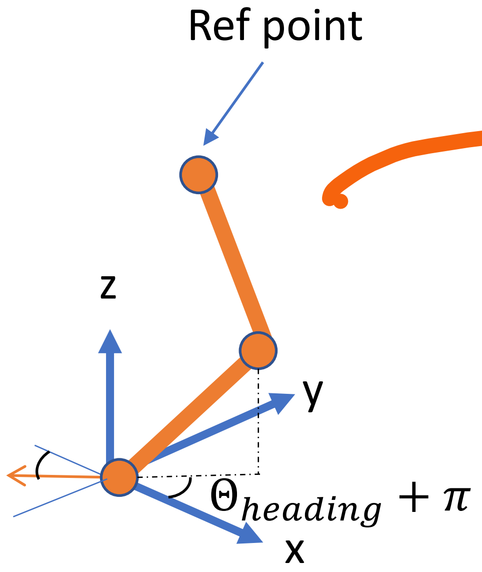
Inverse kinematics



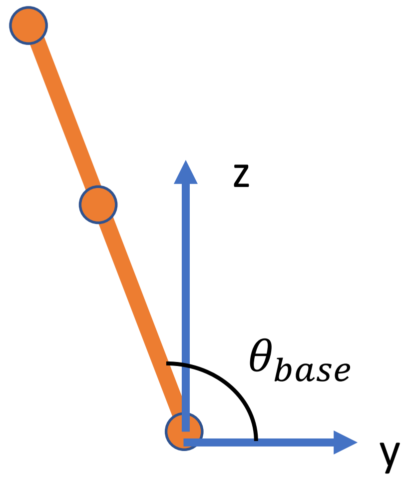
Goal

Given a reference point (generated by the gait generator) find the joint angles such that the finger reaches this point

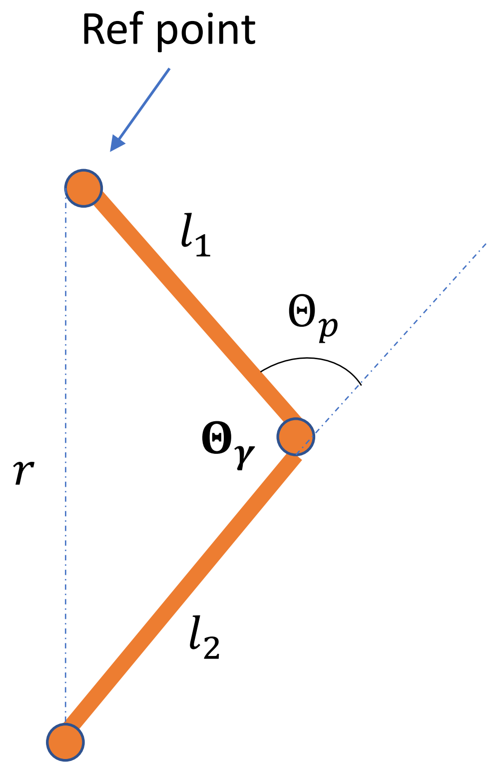
Inverse kinematics base angle



- 1. Rotate (ref point-base coordinate) by the heading
- 2. Project into 2D (ignore x coordinate)
- Apply sine/cosine rule
- $\Theta_{base} = \mathrm{asin}\left(\frac{z}{\sqrt{(z^2+y^2)}}\right)$
- $\Theta_{base} = \mathrm{acos}\left(\frac{y}{\sqrt{(z^2+y^2)}}\right)$



Inverse kinematics phalange joint

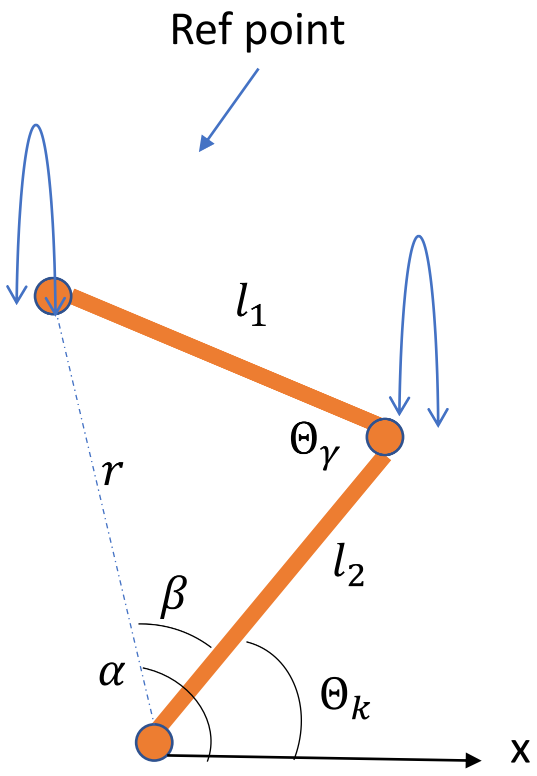


- $\Theta_p = \pi - \boldsymbol{\theta_\gamma}$
  - (Θ_p can be set by a servo in the finger)

- Law of cosines
- $r^2 = l_1^2 + l_2^2 - 2l_1 l_2 \cos(\theta_\gamma)$

- $\boldsymbol{\theta_\gamma} = \text{acos}(\frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2})$

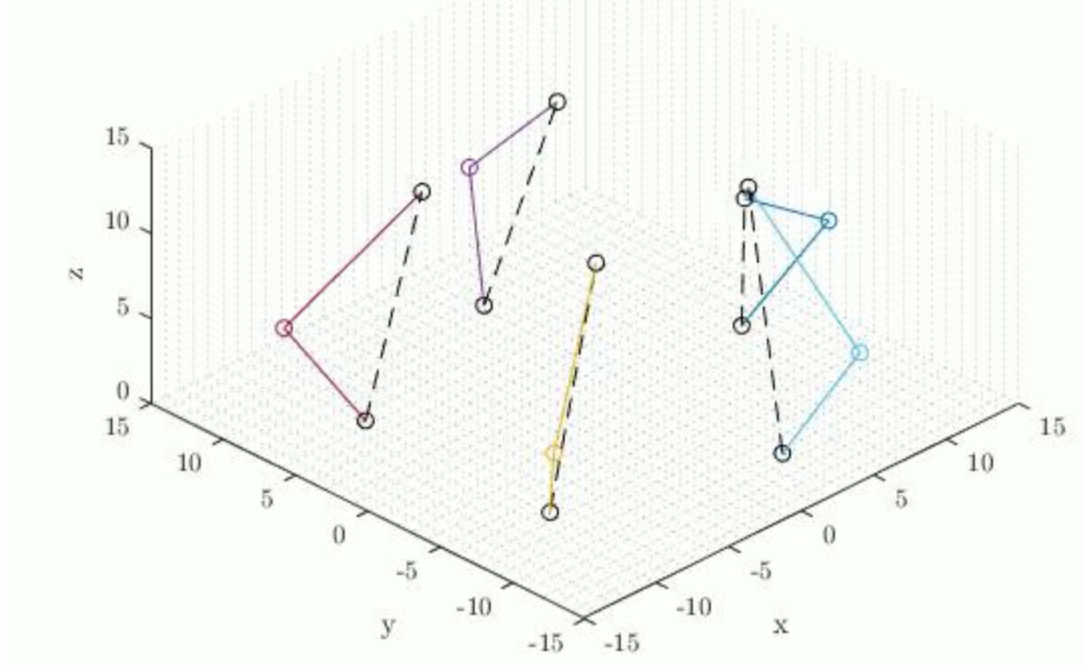
Inverse kinematics knuckle



- Observation
  - Rotation of the base angle does not change the x coordinates
- $\Theta_k = \alpha - \beta$
- (Θ_k can be set by a servo in the finger)
- Trig + law of cosines
- $\alpha = \text{acos}(\frac{x}{r})$
- $\beta = \text{acos}(\frac{r^2 + l_2^2 - l_1^2}{2rl_2})$

Putting it together

- Can use the simulation to tune (to look good)
  - Finger lengths
  - Radius around which the fingers are distributed
  - The radius around which the object is carried
    - In reality the controlled joint angles are not perfect and the finger has some play. This causes the finger to not reach the reference point. In practice a large object carry radius was necessary such that the object does not fall/slide off.



https://youtu.be/Ng2OiVzpYz4