Algorithms: COMP3121/3821/9101/9801

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MORE DIVIDE AND CONQUER ALGORITHMS

Counting the number of inversions

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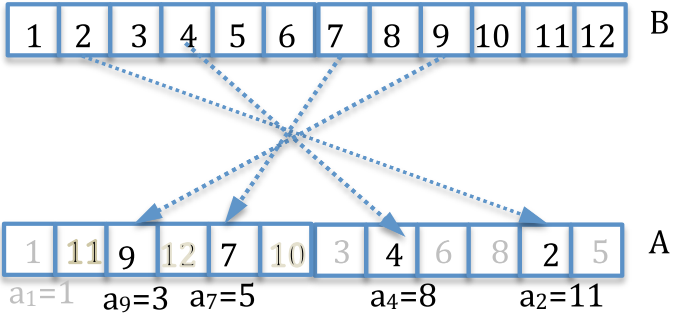
- In other words, we count the number of pairs of movies i, j such that $i < j$ (movie i precedes movie j on $B's$ list) but $a_i > a_j$ (movie i is in the position a_i on $A's$ list which is after the position a_j of movie j on $A's$ list.

Counting the number of inversions



For example 1 and 2 do not form an inversion because $a_1 < a_2$ ($a_1 = 1$ and $a_2 = 11$ because a_1 is on the first and a_2 is on the eleventh place in A); However, for example 4 and 7 do form an inversion because $a_7 < a_4$ ($a_7 = 5$ because a_7 is on the fifth place in A and $a_4 = 8$)

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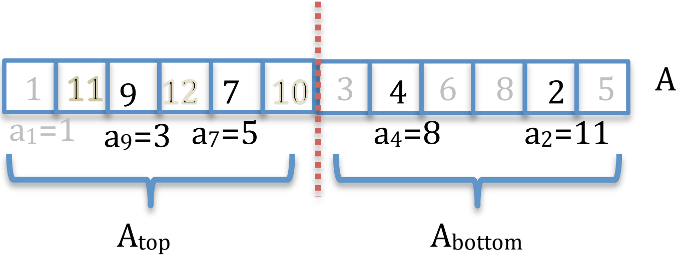
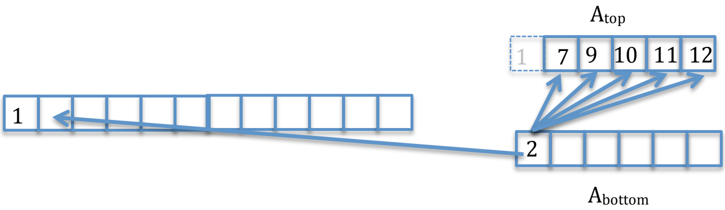
Counting the number of inversions

- We split the list $\langle a_1, a_2, \ldots, a_n \rangle$ into two (approximately) equal parts $A_{top} = \langle a_1, \ldots, a_{\lfloor 2/n \rfloor} \rangle$ and $A_{bottom} = \langle a_{\lfloor 2/n \rfloor + 1}, \ldots, a_n \rangle$.
- Note that the total number of inversions in array A is equal to the sum of the number of inversions $I(A_{top})$ in A_{top} (such as 9 and 7) plus the number of inversions $I(A_{bottom})$ in A_{bottom} (such as 8 and 11) plus the number of inversions $I(A_{top}, A_{bottom})$ across the two halves (such as 7 and 4).



Counting the number of inversions

- We now recursively sort arrays A_{top} and A_{bottom} and obtain the number of inversions $I(A_{top})$ in the sub-array A_{top} and the number of inversions $I(A_{bottom})$ in the sub-array A_{bottom};
- We now merge the two sorted arrays A_{top} and A_{bottom} while counting the number of inversions which are across the two sub-arrays, i.e., such that $i < j$ but such that $a_i \in A_{bottom}$ and $a_j \in A_{top}$:



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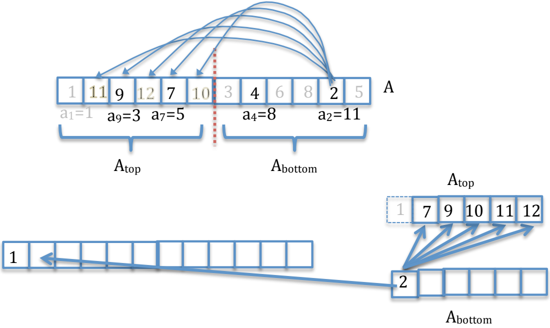

- We now merge the two sorted arrays A_{top} and A_{bottom} while counting the number of inversions $I(A_{top}, A_{bottom})$ which are across the two sub-arrays.
- When the next smallest element among all elements in both arrays is an element in A_{bottom}, such an element clearly is in an inversion with all the remaining elements in A_{top} and we add the total number of elements remaining in A_{top} to the current value of the number of inversions across A_{top} and A_{bottom}.

Counting the number of inversions

- Whenever the next smallest element among all elements in both arrays is an element in A_{top}, such an element clearly is not involved in any inversions across the two arrays (such as 1, for example).
- After the merging operation is completed, we obtain the total number of inversions $I(A_{top}, A_{bottom})$ across A_{top} and A_{bottom}.
- The total number of inversions $I(A)$ in array A is finally obtained as:

$$I(A) = I(A_{top}) + I(A_{bottom}) + I(A_{top}, A_{bottom})$$