COMP9101 ASS3

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1. Subproblem : find out the largest number of dams that can be bulit in x_i meters from head.

Recursion : $opt(i) = \max\{1 + opt(x_i - r_i), opt(x_i - 1)\}$

If we build a dam at x_i , so we need to find the larget number of dams within $x_i - r_i$ meters from head, the result should be $1 + opt(x_i - r_i)$

If there is no dam at x_i , we need to find out the larget number of dams within $x_i - 1$ meters, the result should be $opt(x_i - 1)$

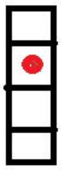
Edge case : $opt(1) = 1$

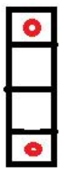
Time complexity: $O(n)$

2.

A) 8 legal patterns that can occur in any column.

The first situation:  , all the squares are empty.

The second situation:  ,  ,  ,  , in each case ,there is only one pebble placed in each column, there are four cases.

The third situation:  ,  ,  , in each case, there are two pebble placed in each column, there are three this kind of cases.

In conclusion, there are 8 legal patterns that can occur in any column.

B) Subproblem : get the max intergers in the square that are covered by pebbles in ith column which is compatible with the i-1th column·

We set the location of pebbles on one column using a four bit vector, $\vec{p} = (p_1, p_2, p_3, p_4)$, where $p_i = 1$ iff there is a pebble on row i in this column. Then there are eight feasible binary patterns. We will use the notation $p_i(k)$ to denote the ith bit of $\vec{p}(k)$ where

$0 < k \leq 8$.

Recursion: opt(comp(i),i)=max{opt(comp(i-1),i-1)+ $p_i(k)$ }, where $0 < k \leq 8$, maximum score for placing pebbles in the first i−1 columns, and placing pebbles in column i according to the pattern $\vec{p}(k)$, chose the max sum and comp(i) means the compatible patten in ith column with i-1th column.

Edge case : $\operatorname{opt}(comp(1),1) = \max(sum(p_1(k)))$

Time complexity: $O(n)$

3. Firstly, we sort team members based on their height in non-decreasing order and sort skis based on the length in non-decreasing order.

Case 1: n=m

Assign the member and skis one by one in order is the optimal.

Proof: If we exchange any skis or member' s order, the result won' t less than the optimal strategy.

If $l_i < l_{i+1} < h_i < h_{i+1}$:

$$|l_i - h_i| + |l_{i+1} - h_{i+1}| = |l_{i+1} - h_i| + |l_i - h_{i+1}|$$

If $h_i < h_{i+1} < l_i < l_{i+1}$:

$$|l_i - h_i| + |l_{i+1} - h_{i+1}| = |l_{i+1} - h_i| + |l_i - h_{i+1}|$$

If $l_i < h_i < l_{i+1} < h_{i+1}$:

$$|l_i - h_i| + |l_{i+1} - h_{i+1}| < |l_{i+1} - h_i| + |l_i - h_{i+1}|$$

If $h_i < l_i < h_{i+1} < l_{i+1}$:

$$|l_i - h_i| + |l_{i+1} - h_{i+1}| < |l_{i+1} - h_i| + |l_i - h_{i+1}|$$

If $l_i < h_i < h_{i+1} < l_{i+1}$:

$$|l_i - h_i| + |l_{i+1} - h_{i+1}| < |l_{i+1} - h_i| + |l_i - h_{i+1}|$$

Only when $l_i < l_{i+1} < h_i < h_{i+1}$ and $h_i < h_{i+1} < l_i < l_{i+1}$, the exchange makes no change. In other three cases, the change one becomes larger. So our method is the optimal one.

Case 2: n<m

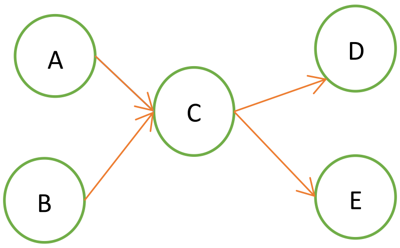
Subproblem: Assign the ith member with the jth skis and the sum of difference between height and length is the smallest.

If i =j, we need to $\sum_{i,\ j} | h_i - l_j |$ where i=j.

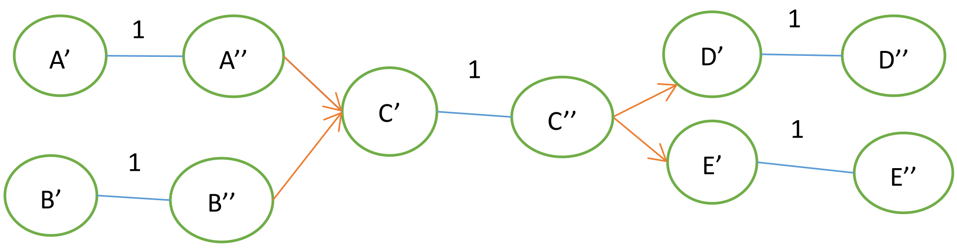
Recursion: opt(i,j)= $\{\min(opt(i-1, j-1)+|h(i)-l(j)|, opt(i, j-1))\}$ where j>i.

when the ith member chooses the jth skis, we get opt(i-1,j-1)+|h(i)-l(j)| and when the ith member does not choose the jth skis, we get opt(i,j-1).
Edge case : i=1 or j=1.

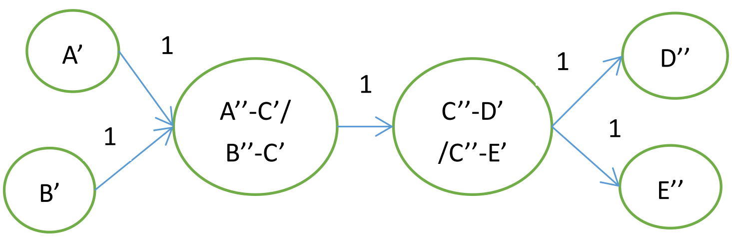
4.
A) We can use Max Flow-Mini Cut algorithm. We assume that each spy as a vertex and each channel as an edge. Then add a super source S and a super sink T, source S only has flow out and sink T only has flow in. The capacity of each edge is 1 so that min cut equals tonumber of edges crossing the cut.
B) Because we cannot compromise channel so that we assume that the capacity of each edge is infinite. For example: the red line mean the capacity is infinite



We can assume that each vertex as two vertexes and linked by an edge which capacity is 1, like the following figure:



So that we can get the figure with each edge capacity is 1:



So that we can use Max Flow-Mini Cut algorithm to get the number of spies that we need to bribe. However, when S can communicate with T directly by a channel, there is no solution.

5. We add the edge from s to u and v to t respectively and the capacity are both infinite. Then we

can use Max Flow-Mini Cut algorithm to find a minimum $s-t$ cut. Another option is to use a supersource connected to s and u and a supersink connected to t and v by edges of infinite capacity.