COMP9334 ASS1

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Question 1.

(a).

Service demand of a job at the device j is the total service time required by that job. As we know the demand law:

$$\begin{cases} D(j) = V(j)S(j) \\ V(j) = \frac{X(j)}{X(0)} \end{cases}$$ in which $V(j)$ is the visit ratio.

$S(j)$ is the service time

$X(j)$ is the output of device j. $\Rightarrow D(j) = \frac{U(j)}{X(0)}$, $X(0)$ is the output of CPU

So we can get $\begin{cases} D(j) = \frac{U(j)}{X(0)} \\ U(j) = \frac{B(j)}{T} \\ X(0) = \frac{C(0)}{T} \end{cases} \Rightarrow D(j) = \frac{B(j)}{C(0)}$

$B(j) = 2565S$, $C(0)$ = Number of requests completed.

So $C(0) = 676$ jobs

$$D(cpu) = D(0) = \frac{B(0)}{C(0)} = \frac{4729}{676} \approx 6.996 \text{ seconds}$$

$$D(disk) = D(j) = \frac{B(j)}{C(0)} = \frac{2565}{676} \approx 3.794 \text{ seconds.}$$

(b) Yes, it is possible.

If cpu is bottleneck,	If disk is bottleneck,
$U(cpu) = S(CPU) \cdot X(0)$	$U(disk) = S(disk)X(0)$
$\mu = \frac{1}{S} = \frac{676}{4729}$	$S(disk) = \frac{676}{2565}$
$X(0) = \frac{1}{S}$	

So, it is possible to determine the bottleneck

(C) the bottleneck analysis is

$$X(0) \le \min\left[\frac{1}{\max D_i}; \frac{N}{\sum_{i=1}^{k} D_i + T}\right]$$

the $\frac{1}{\max D_i}$ is $\frac{1}{D(CPU)} = \frac{1}{6.996} \approx 0.143$ jobs/s

$$\frac{N}{\sum_{i=1}^{k} D_i + T} = \frac{30}{6.996 + 3.794 + 31} \approx 0.718 \text{ jobs/s}$$

So the asymptotic bound will be 0.143 jobs/s.

(d) minimum response time.

$\frac{M}{R}$ $M = X_0(R+Z)$

so $R = \frac{M}{X(0)} - Z = \frac{30}{0.143} - 31 \approx 178.8s$

Q(2)

(a) This is a system with two M/M/1

$\lambda = 20, \quad \mu_1 = 10, \quad \mu_2 = 15$

$U_1 = \frac{\lambda_1}{\mu_1} = \frac{\lambda p}{\mu_1}$

$U_2 = \frac{\lambda_2}{\mu_2} = \frac{\lambda(1-p)}{\mu_2} \Rightarrow U_1 = U_2$

$\frac{20p}{10} = \frac{20(1-p)}{15} \Rightarrow p = 0.4$

(b) $T = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu - \lambda}, \quad \rho = \frac{\lambda}{\mu}$

$T_1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{10 - 20\times 0.4} = \frac{1}{2}s$

$T_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{15 - 20\times 0.6} = \frac{1}{3}s$

$T_{mean} = 0.4T_1 + 0.6T_2 = 0.4s$

So the mean response time is $0.4s$

(c).

We can get that $R = p\cdot\frac{1}{\mu_1(1-\rho_1)} + (1-p)\cdot\frac{1}{\mu_2(1-\rho_2)}$

$= p\cdot\frac{1}{10(1-\frac{20p}{10})} + (1-p)\frac{1}{15(1-\frac{20(1-p)}{15})}$

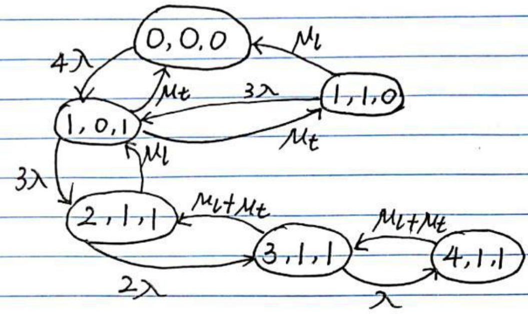
$$\begin{cases} 1 - \frac{20p}{10} > 0 \\ 1 - \frac{20(1-p)}{15} > 0 \end{cases} \Rightarrow 0.25 < p < 0.5.$$

So I write a python program q2c.py with a for loop increase by 0.001 each step. From the program I get that when $p = 0.388$, the smallest mean response time is $0.395s$.

Q3

(a) $\lambda = \frac{1}{600}$ for team leader $\mu_l = \frac{1}{60}$, for trainee $\mu_t = \frac{1}{90}$

So we can get the diagram as follow:



(b)

$$\frac{1}{150}\cdot P(0,0,0) - \frac{1}{90}\cdot P(1,0,1) + 0\times P(2,1,1) + 0\cdot P(3,1,1) + 0\cdot P(4,1,1) - \frac{1}{60}P(1,1,0) = 0$$

$$-\frac{1}{150}P(0,0,0) + \frac{29}{1800}P(1,0,1) - \frac{1}{60}P(2,1,1) + 0\cdot P(3,1,1) + 0\cdot P(4,1,1) + 0\cdot P(1,1,0) = 0$$

$$0\cdot P(0,0,0) - \frac{1}{200}P(1,0,1) + \frac{7}{225}P(2,1,1) - \frac{1}{36}P(3,1,1) + 0\cdot P(4,1,1) - \frac{1}{200}P(1,1,0) = 0$$

$$0\cdot P(0,0,0) + 0\cdot P(1,0,1) - \frac{1}{300}P(2,1,1) + \frac{7}{225}P(3,1,1) - \frac{1}{36}P(4,1,1) + 0\cdot P(1,1,0) = 0$$

$$0\cdot P(0,0,0) + 0\cdot P(1,0,1) + 0\cdot P(2,1,1) - \frac{1}{600}P(3,1,1) + \frac{1}{36}P(4,1,1) + 0\cdot P(1,1,0) = 0$$

$$0\cdot P(0,0,0) + 0\cdot P(1,0,1) - \frac{1}{90}P(2,1,1) + 0\cdot P(3,1,1) + 0\cdot P(4,1,1) + \frac{13}{600}P(1,1,0) = 0$$

(c) With the python ~~go~~ program q3c.py I get probability.

In the program. I set $P(0,0,0) = x$, $P(1,0,1) = y$, $P(2,1,1) = z$

$P(3,1,1) = m$, $P(4,1,1) = n$, $P(1,1,0) = o$

$\lambda = a$, $\mu_l = b$ and $\mu_t = c$

So $P(0,0,0) \approx 0.5918$ $\quad P(1,0,1) \approx 0.3081$ $\quad P(2,1,1) \approx 0.0611$

$P(3,1,1) \approx 0.0073$ $\quad P(4,1,1) \approx 0.0004$ $\quad P(1,1,0) \approx 0.0313$

(d) The probability of at least three machines are available are.

$P(0,0,0)$, $P(1,1,0)$ and $P(1,0,1)$

So $P(0,0,0) + P(1,1,0) + P(1,0,1) \approx 0.9312$

(e) $N_{avg} = 0\cdot P(0,0,0) + 1\times P(1,0,1) + P(1,1,0) + 2\cdot P(2,1,1) + 3P(3,1,1) + 4P(4,1,1) = 0.4851$

So mean number of failed machines is 0.4851

(f) $R_{avg} = \frac{N_{avg}}{X}$, we already know N_{avg}, so we just need X

$X = 4\lambda P(0,0,0) + 3\lambda P(1,0,1) + 3\lambda P(1,1,0) + 2\lambda P(2,1,1) + \lambda P(3,1,1) + 0P(4,1,1) \approx 0.0059$

So $R_{avg} = \frac{0.4851}{0.0059} \approx 82.2203$ s.