Упражнение 12

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Объяснение

Проверено

Шаг 1

1 из 3

Given is the following function:

$$f(x) = \begin{cases} 2x - 3, & 0 \leq x \leq 2 \\ 6x - x^2 - 7, & 2 < x \leq 3 \end{cases}$$

We want to check whether f satisfies the hypotheses of the Mean Value Theorem. The conditions it has to satisfy are:

- f has to be continuous over a closed interval $[a, b]$
- f has to be differentiable over an open interval (a, b)

In this case, $[a, b] = [0, 3]$

Следующее упражнение

Шаг 2

Since both $f_1(x) = 2x - 3$ and $f_2(x) = 6x - x^2 - 7$ are continuous and differentiable in every point, we only need to check the continuity and differentiability of f at the point $x = 2$.

For f to be continuous at $x = 2$, equation $f_1(2) = f_2(2)$ has to be true:

$$f_1(2) = 2 \cdot 2 - 3 = 1 = 6 \cdot 2 - 2^2 - 7 = f_2(2) \quad \checkmark$$

For f to be differentiable at $x = 2$, equation $f_1'(2) = f_2'(2)$ has to be true:

$$f_1'(x) = 2$$
$$f_2'(x) = 6 - 2x$$

$$f_1'(2) = 2 = 6 - 2 \cdot 2 = f_2'(2) \quad \checkmark$$

So, $\text{\color{#blbadf} f does satisfy the hypotheses of the Mean Value Theorem.}$

Результат

f satisfies the hypotheses of the Mean Value Theorem

Следующее упражнение