Tutorial - 1

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S 2 - G

Question1 what do you understand by Asymptotic notation, define different asymptotic notation with Example?

(i) Big $O(n)$

$f(n) \Rightarrow O(g(n))$

if $f(n) \leq g(n) \times c \; \forall \; n \geq n_0$

for some constant, $c > 0$

$g(n)$ is 'tight' upper bound of $f(n)$

Example: $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c * n^3$

$n^2 + n = O(n^3)$

(2) Big omega (Ω)

when $f(n) = \Omega(g(n))$

means $g(n)$ is tight lowerbound of $f(n)$ i.e $f(n)$ can go beyound $g(n)$

i.e $f(n) = \Omega\, g(n)$

if and only if

$f(n) \geq c \cdot g(n)$

$\forall \; n_2 > n_0$ and c = constant > 0

Example:

$f(n) \Rightarrow n^3 + 4n^2$

$g(n) \Rightarrow n^2$

i.e $f(n) \geq c \times g(n)$

$n^3 + 4n^2 = \Omega(n^2)$

(3) Big theta (Θ)

$f(n) = \Theta\, g(n)$ gives the tight upper bound and lower bound both

i.e $f(n) = \Theta(g(n))$

if and only if

$c_1 * g(n_1) \leq f(n) \leq c_2 * g(n_2)$

for all $n \geq \max(n_1, n_2)$, some constant $c_1 > 0$ & $c_2 > 0$

Example:

$3n+2 = \Theta(n)$ as $3n+2 \geq 3n$ if

$3n+2 \leq 4n$ for n, $c_1 = 3$, $c_2 = 4$ & $n_0 = 2$

④ Small o(θ)

When $f(n) = o\,g(n)$ gives the upper bound i.e $f(n) = o\,g(n)$ if and only if

$f(n) < c * g(n)$

$\forall\ n > n_0$ & $n > 0$

Example $f(n) = n^2$; $g(n) = n^3$

$f(n) < c * g(n)$

$n^2 = o(n^3)$

Question 2

for $i \Rightarrow 1, 2, 4, 6, 8 \ldots n$ times

i.e series is a G.P

$a = 1$, $r = 2/1$ or $4/2$ or...

Kth value of G.P:

$t_K = ar^{K-1}$

$t_K = 1(2)^{K-1}$

$2n = 2^K$

$\log_2(2n) = K\log 2$

$\log_2 2 + \log_2 n = K$

$\log_2 n + 1 = K \Rightarrow$ neglecting 1

Time complexity $O(\log n)$ Ans

Question 3

$$T_n = \begin{cases} 3T(n-1) & \text{if } n > 0 \\ \text{otherwise } 1 \end{cases}$$

$T(n) \Rightarrow 3T(n-1)$ — ①

$T(n) \Rightarrow 1$

putting $(n = n-1)$ in ①

(3)

$T(n-1) = 3T(n-2)$ —(2)

putting (2) in (1)

$T(n) = 3 \times 3T(n-2)$

$T(n) = 9T(n-2)$

Generalising series

$T(K) = 3^K T(n-K)$ —(*)

for Kth term, let $n-K=1$

$K = n-1$

putting K in (*) we get

$T(n) = 3^{n-1} T(1)$

$T(n) = 3^{n-1}$

$T(n) = O(3^n)$

Question 4

$T(n) = 2T(n-1) - 1$ —(1)

put $n = n-1$

$T(n-1) = 2T(n-2) - 1$ —(2)

put in equ(1)

$T(n) = 2(2T(n-2) - 1) - 1$

$= 4T(n-2) - 2 - 1$ —(3)

put $n = n-2$ in (1)

$T(n-2) = 2T(n-3) - 1$

put in equ(1)

$T(n) = 8T(n-3) - 4 - 2 - 1$

Series.

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \ldots 2^0$

$k = n-1$

$T(n) = 2^{n-1} - 2^{n-1}\left(1/2 + 1/2^2 + \ldots + 1/2^{n-1}\right)$

$a = 1/2$ and $r = 1/2$

$$T(n) = 2^{n-1}\left(1-\left(\tfrac{1}{2}\,\frac{1-(1/2)^{n-1}}{1-1/2}\right)\right)$$

$$= 2^{n-1}\left(1-1+(1/2)^{n-1}\right)$$

$$= \frac{2^{n-1}}{2^{n-1}}$$

$$\boxed{T(n) = O(1)}$$

Anss. $i = 1\ 2\ 3\ 4\ 5\ 6 \ldots$

$S = 1+3+6+10+15+\ldots n$ — 1

$S = 1+3+6+10+\ldots T_{n-1}+T_n$ — 2

$0 = 1+2+3+4+\ldots n - T_n$

$T_k = 1+2+3+4+\ldots+k$

$T_k = \frac{1}{2}k(k+1)$

$1+2+3+\ldots+k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k^2) <= n$

$k = O\sqrt{n}$

$T(n) = O(\sqrt{n})$

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Ans6. $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}$

$\sum_{i=1} 1+2+3+4+\ldots+\sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$T(n) = \frac{n\sqrt{n}}{2}$

$\boxed{T(n) = O(n)}$

Ans7. $a = 1, r = 2$

$\frac{a(r^n - 1)}{r-1}$

$= \frac{1(2^k - 1)}{1}$

$n = 2^k - 1$

$n+1 = 2^k$

$\log_2(n) = k$

i	j	k
1	$\log n$	$\log n \times \log n$
2	$\log n$	$\log n \times \log n$
$\vdots$	$\vdots$	$\vdots$
n	$\log n$	$\log n \times \log n$

$T(n) = O(n \times \log n \times \log n)$

$\boxed{T(n) = O(n \log 2 (n))}$

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Ans

$$T(n) = n^2 + T(n-3)$$

$$T(n-3) = (n^2.3)^2 + T(n-6)$$

$$T(n-6) = (n^3.6)^2 + T(n-9)$$

and $T(1) = 1$

$$\Rightarrow T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$$

$$k = \frac{(n-1)}{3}$$

$$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$$

$$T(n) \cong (k-1)/3 - n^2$$

So,

$$\boxed{T(n) = O(n^3)}$$

Ans

$$n^k = O(c^n)$$

$$n^k \leq a(c^x)$$

$\forall n \geq n_0$ and constant, $a > 0$

for $n_0 = 1$, $c = 2$

$$1^k < a2$$

$$\boxed{n_0 = 1 \text{ and } c = 2}$$