

$$T(n) = 1.9n \log n$$

tutorial-1

question 1

$$T(n) = 3T(n/2) + n^2$$

$$T(n) = aT(n/b) + f(n)$$

$$a \geq 1, b \geq 1$$

on comparing

$$a=3, b=2, f(n)=n^2$$

$$\text{Now } c = \log_b a = \log_2 3 = 1.5849$$

$$n^c = n^{1.5849} < n^2$$

$$f(n) > n^c$$

$$T(n) = O(n^2)$$

question 3

$$T(n) = 2T(n/2) + 2^n$$

$$a=2$$

$$b=2$$

$$f(n) = 2^n$$

$$c = \log_b a = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$f(n) > n^c$$

$$T(n) = O(2^n)$$

ques 5 $T(n) = 16T(n/4) + n$

$$a=16$$

$$b=4$$

$$f(n) = n$$

$$c = \log_4 16 = \log_4 (4^2) = 2 \log_4 4 = 2$$

$$n^c = n^2$$

$$f(n) < n^c$$

$$T(n) = O(n^2)$$

question 2

$$T(n) = 4T(n/2) + n^2$$

$$a=4, b=2, f(n)=n^2$$

$$c = \log_2 4 = 2$$

$$n^c = n^2 = f(n) = n^2$$

$$\therefore T(n) = \Theta(n^2 \log n)$$

ques 4

$$T(n) = 2^n T(n/2) + n^n$$

$$a=2^n$$

$$b=2$$

$$f(n) = n^n$$

$$c = \log_b a$$

$$= \log_2 2^n = n$$

$$n^c \Rightarrow n^n$$

$$f(n) = n^n$$

$$T(n) = \Theta(n^n \log n)$$

ques 6

$$T(n) = 2T(n/2) + n \log n$$

$$a=2, b=2$$

$$f(n) = n \log n$$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$n \log n > n$$

$$f(n) > n^c$$

$$T(n) = \Theta(n \log n)$$

ques 8

$$T(n) = 2T(n/4) + n^{0.5}$$

$$a=2, b=4$$

$$f(n) = n^{0.5}$$

$$n^c = n^{0.5}$$

$$n^{0.5} < n^{0.5}$$

$$f(n) > n^c$$

$$T(n) = \Theta(n^{0.5})$$

ques 10

$$T(n) = 16T(n/4) + n!$$

$$a=16, b=4, f(n)=n!$$

$$c = \log_b a = \log_4 16 = 2$$

$$n^c = n^2$$

$$A \times n! > n^2$$

$$T(n) = \Theta(n!)$$

ques 7

$$4T(n/2) + \log n$$

$$a=4, b=2$$

$$f(n) = \log n$$

$$c = \log_b a \cdot \log_2 h = 2$$

$$f(n) = \log n$$

$$\log n < n^2$$

$$f(n) < n^c$$

$$T(n) < n^c$$

$$T(n) = \Theta(n^2)$$

ques 9

$$T(n) = 0.5T(n/2) + 1/n$$

$$a=0.5, b=2$$

$$a > 1 \text{ but here is } 0.5$$

So we can't apply master theorem

ques 11

$$4T(n/2) + \log n$$

$$a=4, b=2$$

$$f(n) = \log n$$

$$c = \log_b a \cdot \log_2 h = 2$$

$$f(n) = \log n$$

$$\log n < n^2$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^2)$$

$$= \Theta(n^2)$$

ques 12

4.3

$$T(n) = \text{sqrt}(n) T(n/2) + \log n$$

$$a = \sqrt{n}, b = 2$$

$$c = \log_b a = \log_2 \sqrt{n} = 1/2 \log_2 n$$

$$f(n) \leq n^c$$

$$T(n) = \Theta(f(n))$$

$$= \Theta(\log(n))$$

ques 13

$$T(n) = 3T(n/2) + n$$

$$a = 3, b = 2, f(n) = n$$

$$c = \log_b a = \log_2 3 = 1.58496$$

$$n^c = n^{1.58496}$$

$$n < n^{1.58496}$$

$$T(n) = \Theta(n^{1.58496})$$

ques 14 $T(n) = 3T(n/3) + \text{sqrt}(n)$

$$a = 3, b = 3$$

$$c = \log_3 3 = 1$$

$$n^c = n^1 = n$$

As $\text{sqrt}(n) < n$ $T(n) = \Theta(n)$

$$f(n) < n^c$$

$$T(n) = \Theta(\text{sqrt}(n))$$

ques 15 $T(n) = 4T(n/2) + n$

$$a = 4, b = 2$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$n < n^2$ (for any constant)

$$f(n) < n^c$$

$$f(n) = \Theta(n^2)$$

Ques 22 $T(n) = T(n/2) + n(2 - \cos n)$

$a=1, b=2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$

Ques 19

$T(n) = 4T(n/2) + n \log n$

$a=4, b=2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n / \log n < n^2$

$T(n) = \Theta(n^2)$

Ques 21

$T(n) = 7T(n/3) + n^2$

$a=7, b=3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

Ques 17

$T(n) = 3T(n/3) + n/2$

$a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

$n/2 < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

Ques 18

$T(n) = 6T(n/3) + n^2 \log n$

$a=6, b=3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^{1.6309} < n^2 \log n$

$T(n) = \Theta(n^2 \log n)$