Week-01

Real-Number System, Complex Numbers

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Today Agenda

1. Real-Number System

2. Complex Numbers

Real-Number System

Real-Number System

As we call rational and irrational numbers.

"Every rational number is either a finite decimal or infinitely repeating decimal while irrational numbers infinitely not repeating decimal"

Examples, $0.\bar{2} = 0.222222... = \frac{2}{9}$, $0.25 = \frac{1}{4}, \sqrt{2}, \sqrt{3}$ respectively.

Definition

The union of the set of rational numbers and the set of irrational numbers is the set of real numbers.

Other words, a number can be represent in real number line. [1]

[1] Visual representation of the real numbers

Roots and Radicals

Definition

In mathematics, an nth root of a number x is a number r which, when raised to the power n, yields x:
$r^n = x$ where n is positive integer some time called degree

Definition

The radical symbol $\sqrt{.}$ used to indicate a root. It can be square root or a cubic root depending on index number.

Example

$\sqrt[3]{8}$, Where 3 denote Index 8 denote, radicand $\sqrt{.}$, and denote root symbol.

Roots

We know that $4 = 2.2$. Therefore, 4 is the square of 2, and 2 is a square root of 4. We also know that $4 = -2.(-2)$. Therefore, 4 is the square of 2, and 2 is a square root of 4. We write this as $4 = 2^2$ and $4 = (-2)^2$. Because there are two numbers whose square is 4, 4 has two square roots, written $\sqrt{4} = \pm 2$

- Square Root

Definition

A square root of k is one of the two equal factors whose product is k.

- Cubic Root

Definition

The cube root of k is one of the three equal factors whose product is k.

- The n^{th} Root of a Number

Definition

The nth root of k is one of the n equal factors whose product is k.

Simplifying Radicals

We know that $\sqrt{4.9} = \sqrt{36} = 6$ and that $\sqrt{4}\ .\ \sqrt{9} = 2\ .\ 3 = 6.$ therefore,

$$\sqrt{4.9} = \sqrt{4}\ .\ \sqrt{9}$$

Algebraic turns, for all non-negative real numbers a and b;

$$\sqrt{a.b} = \sqrt{a}\ .\ \sqrt{b}.$$

This notation can be used when $\sqrt{a}\ .\ \sqrt{b}$ are rational numbers. When $\sqrt{a.b}$ is an irrational number, we can often write it in simpler form.

Example

$\sqrt{450} = \sqrt{9}\ .\ \sqrt{50} = 3\sqrt{50}$
$\sqrt{450} = \sqrt{25}\ .\ \sqrt{18} = 5\sqrt{18}$
$\sqrt{450} = \sqrt{225}\ .\ \sqrt{2} = 15\sqrt{2}$

We say that $15\sqrt{2}$ is the simplest form of $\sqrt{450}$. As we known that $x^3.x^3 = x^6$. Then for x $\geq$ 0. $\sqrt{x^6} = x^3$. In general, $x^n.x^n = x^{2n}$. Therefore, for x $\geq$ 0:

$$\sqrt{x^{2n}} = x^n$$

Fractional Radicands

When the radicand of an irrational number is a fraction, the radical is in simplest form when it is written with an integer under the radical sign. For example. $\sqrt{\frac{2}{3}}$ does not have a perfect square factor in either the numerator or the denominator of the radicand. The radical is not in simplest form. To simplify this radical,write $\frac{2}{3}$ as an equivalent fraction with a denominator that is a perfect square.

$$\sqrt{\frac{2}{3}} = \sqrt{\frac{2}{3} \times \frac{3}{3}} = \sqrt{\frac{6}{9}}$$

Note

In algebraic turns, for any non-negative a and positive c:

$$\sqrt{\frac{a}{c}} = \frac{\sqrt{a}}{\sqrt{c}}$$

$$\sqrt{\frac{a}{c}} = \frac{\sqrt{a}}{\sqrt{c}} \times \frac{\sqrt{a}}{\sqrt{c}} = \frac{\sqrt{ac}}{c}$$

Hand on

Write About Mathematics Sarah said that in the set of real numbers, $\sqrt{a}$ is one of the two equal factors whose product is a. Therefore, $\sqrt{a}.\sqrt{a} = a$ for some values of a. Do you agree with Sarah? Explain why or why not.

Developing skill

Appling skill Find the length of the hypotenuse of a right triangle if the length of the longer leg is 20 feet and the length of the shorter leg is 12 feet.

Operation on Radicals

- Adding and Subtracting Radicals We can apply the same principles to the addition or subtraction of radical expressions. To express the sum or difference of two radicals as a single radical, the radicals must have the same index and the same radicand, that is, they must be like radicals. We can use the same procedure that we use to add or subtract like terms

$$1.\ 2\sqrt{5}+4\sqrt{5}=6\sqrt{5}$$
$$2.\ 4\sqrt{3b}-3\sqrt{3b}=\sqrt{3b}$$

Two radicals that do not have the same radicand or do not have the same index are unlike radicals. The radicals $\sqrt{8}$ and $\sqrt{50}$ have the same index but do not have the same radicand. We can add and subtract these radicals if they can be written in simplest form with a common radicand.

Operation on Radicals

Example

1. Write $\sqrt{12} + \sqrt{20}$ - $\sqrt{\frac{1}{3}}$ in simplest form.

Example

2. Simplify: $3x\sqrt{\frac{1}{3x}} + \sqrt{300x}$

Example

Solve and check: $4x - \sqrt{8} = \sqrt{72}$

Operation on Radicals

- Multipying and Dividing Radicals Recall that if a and b are non-negative numbers, $\sqrt{ab} = \sqrt{a} \,.\, \sqrt{b}$ Therefore, by the symmetric property of equality, we can say that $\sqrt{a} \,.\, \sqrt{b} = \sqrt{ab}$. Recall also that for any positive number a, $\sqrt{a} \,.\, \sqrt{b} = \sqrt{a^2} = a$. We can use these rules to multiply radicals.

Example

$\sqrt{4} \,.\, \sqrt{25} = \sqrt{100} = 10$

Note

$\sqrt{-2} \times \sqrt{-8} = \sqrt{16}$ because $\sqrt{-2}$ and $\sqrt{-8}$ are not real numbers

That is, since is true for non-negative real numbers, the following is also true by the symmetric property of equality:

Complex Numbers

Complex Numbers

Definition

A complex number is a number of the form $a + bi$, where a and b are real numbers

Thank you!