

# 1 Lemmas and semi-proves from Cai (2003)

Lemma 1. If the vote sellers form a coalition and bargain with player B as in a standard Rubinstein game, player B gets  $\delta/(1 + \delta)S$  and the seller coalition gets  $1/(1 + w) = h'$ . The outcome is efficient, and player B gets half of the total gain from trade when  $w = h'$  goes to 1.

The total share the vote sellers can get when bargaining separately will always be larger than that when bargaining as a coalition. Therefore, the sellers have no incentives to form such a coalition. Assume that the completion of the project is verifiable, then an agreement between player B and seller  $i$  specifies a share  $f_i \in [0, 1]$  from player B to seller  $i$  contingent on the completion of the project. With contingent contracts, the bargaining problem is the standard split-the-pie problem.

Lemma 2. In any Subgame Perfect Equilibrium of the game with  $N$  sellers, if player B reaches agreements with all the sellers except player  $i$ , player B will reach an agreement with player  $i$  without delay. Player  $i$  asks for a share of  $(1 - \sum_{j \neq i} f_j)/(1 + \delta)$  and player B accepts so she gets  $(1 - \sum_{j \neq i} f_j)\delta/(1 + \delta)$ .

When there is only one seller left, the remaining subgame is a standard bilateral Rubinstein bargaining game. The joint surplus of this bargaining game between player B and the last vote seller is the total surplus (i.e., 1) minus the total shares player B has promised all the other sellers. Therefore, with contingent contracts, the sellers who reach agreements earlier have an inherent advantage over the other sellers, because their contracts with player B effectively constrain the joint surplus available in the later rounds of bargaining with the remaining other sellers. One corollary of Lemma 2 is that player B and the seller who signs the last contract will get the same payoff as  $w = h'$  goes to one. Consequently, the most player B can hope for is half of the total surplus when  $\delta$  is close to one.