Sequential Vote Buying with Heterogeneous Agents

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Motivation

- Vote buying is a prevailing phenomenon, where political leaders offer side payments or other forms of benefits to voters or voting coalitions (like voting blocks) to gain support for a candidate or the passing of policies.
- The model that fits best with such scenario is sequential bargaining, where a principal negotiates with a group of n agents sequentially, and needs q agents' cooperation ($q \leq N$) to "win". For simplicity, we only consider the case with one political leader and multiple voters to avoid competition between various vote buyers.

Key issue

- The main problem to solve is in what order should the vote buyer approach a sequence of vote sellers (if applicable), and what offer should they make to each seller.
- Inspired by Cai (2003), the three papers, Xiao (2018), Iaryczower and Oliveros (2019), and Chen and Zapal (2021), provide different results under varied specifications.

Model Comparison

Table: Model Comparison

Model	Xiao(2018)	Iaryczower and Oliveros (2019)	Chen and Zapal(2021)
Endogenous Sequencing	Yes	No	Yes
Seller Bargaining power	Yes, via lots	Yes, via who initiates offer	No
Seller Heterogeneity	Yes	No	Yes, via different costs
Allow renegotiation	Yes	Yes	Yes

- The table identifies the key differences in the models applied by three existing papers, we'll then elaborate on each of them.

- Infinite-horizon bargaining game with complete-information.
- The game has N+1 players (1 buyer and N sellers). We denote the buyer by B and the set of sellers by 1, 2, ..., N. All players have the same discount factor $\delta \in (0, 1)$.
- The buyer gets a fixed positive payoff if she acquires at least q votes in the end
- The sellers get payoffs of unknown sign (will be specified in each model) if the seller acquires at least q votes in the end.
- In each period, the buyer approaches seller one-at-a-time. The buyer will reach out to another seller no matter an agreement is made or not with the previous seller (yet may re-approach depending on model).
- The game continues until sufficient number of votes are acquired by the buyer or no new purchases can be made in any future periods.

Set-up

- $q = n$. i.e. unanimity is required
- Before the policy passes, seller i gets profit v_i each period. If the policy passes, the buyer gets profit 1 at that period, and each seller i gets profit 0 since that period. If the policy does not pass, the buyer gets profit 0 at the terminal period, while the sellers get v_i till infinity. We assume that $v_1 > v_2 > ... > v_N$.

Set-up

- In each subgame between the seller and one buyer i, the seller makes an offer $p_{i,1}$. If the seller accepts the offer, the buyer makes the payment immediately. If the seller rejects the offer, the game goes to the second period, where the seller makes an offer $q_{i,1}$. If the buyer accepts, she makes the payment immediately. If she rejects, no agreements made.
- Seller can re-approach buyers.

Graphical Illustration

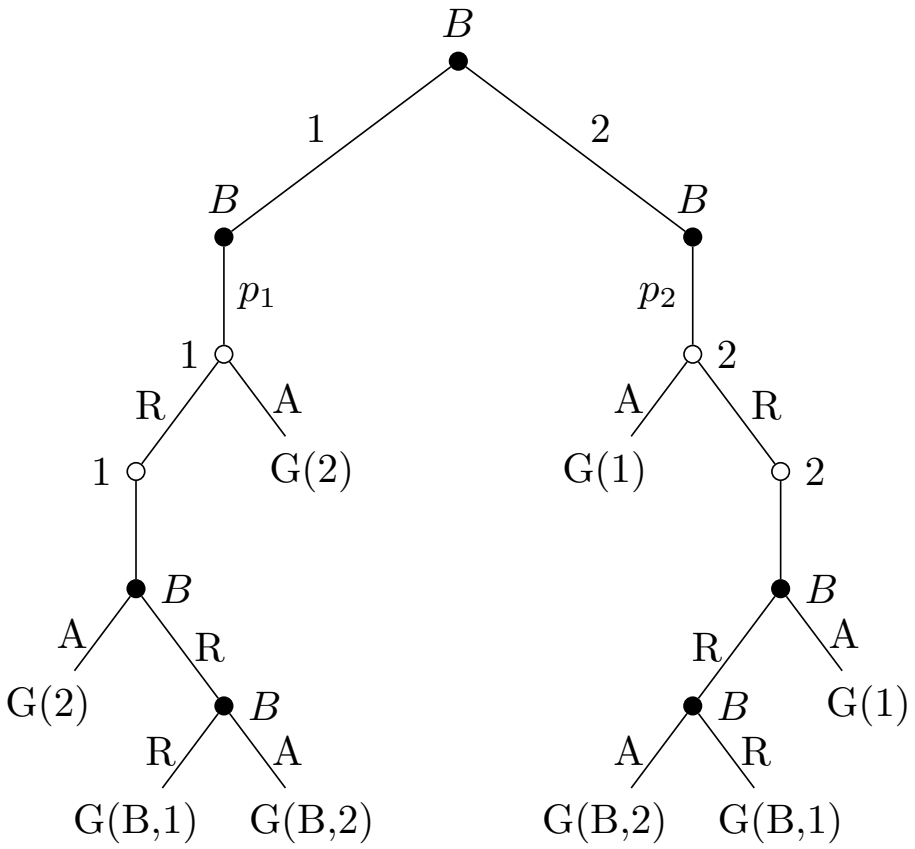

Figure: Xiao(2010) 2-seller game

Payoffs

An outcome is denoted as $(p_1, p_2, ..., p_N, t_1, t_2, ..., t_N)$ where seller i sells his lots of votes in period t_i at price p_i. If a seller i doesn't sell his lot of votes, we can assume t_i goes to infinity and $p_i = 0$ For a seller i, the present discounted payoff is denoted as

$$\pi_i = H_{i,t_i-1} + \delta^{t_i-1} p_i \tag{1}$$

$$= v_i(1-\delta)\sum_{s=1}^{t-1} \delta^{s-1} + \delta^{t_i-1} p_i \tag{2}$$

Here, H_{i,t_i-1} denotes the continuous payoff before the policy passes, and the later half denotes the payment by the buyer.

Payoffs

For the buyer, the present discounted payoff is denoted as

$$\pi_B = \delta^{max\{t_1,\ldots,t_N\}-1} - \sum_{i=1}^{N} \delta^{t_i-1} p_i \tag{3}$$

Results

Markov subgame perfect equilibrium has predicting power to the payoffs in this specific sequential bargaining setting with restrictions on parameters.

Setup

- Buyer only needs to secure $q < N$ votes to pass the policy.
- Each agent has one vote and identical payoffs if policy passes.
- In each period t, the buyer is randomly assigned to one of the k(t) sellers who hasn't sold his vote to the seller with equal probability 1/k(t).
- The buyer makes an offer to i with probability ϕ for some $\phi \in [0, 1]$, while with probability $1 - \phi$ the seller makes an offer to the buyer. The agent (buyer/seller) receiving the offer either accepts or rejects.

Results

"Collective Hold-up": As sellers' bargaining power rises (higher probability for sellers' to initiate an offer), coalitions may form where groups of sellers may cooperate to delay or breakdown the passing of the policy to maximize their payoffs.

Setup

- A policy requires $q \leq N$ votes to pass, each seller holds one vote, but have heterogeneous costs if the policy passes.
- Two types of payment methods: transfer promise (payment made if the policy passes) and up-front payment (payment made when the seller accept the offer).
- Before enough votes are collected, the payoff of all players (not including payments) are normalized to 0. As soon as q votes are purchased, the policy passes, which yields payoff $y > 0$ to the buyer, and heterogeneous cost $-x_i \leq 0$ to each seller $i \in 1, 2, ..., N$. Assume $x_1 < x_2 < ... < x_N$.
- In each bargaining game, the buyer makes an offer, which the seller can accept/reject. Buyer can't renegotiate with sellers who've rejected their offers.

Comparison of Two Payment Methods

- For patient enough agents, the transfer promise yields higher payoffs compared to the up-front payment. The intuition is that, in up-front payment, the seller can exploit sellers with higher costs at later periods to reduce present-discounted payments.
- Here, we visualize an example with the following setting: $x_1 = 1, x_2 \in (1, 10), x_3 = 10, y = 10, \delta = 0.2$.

Comparison of Two Payment Methods

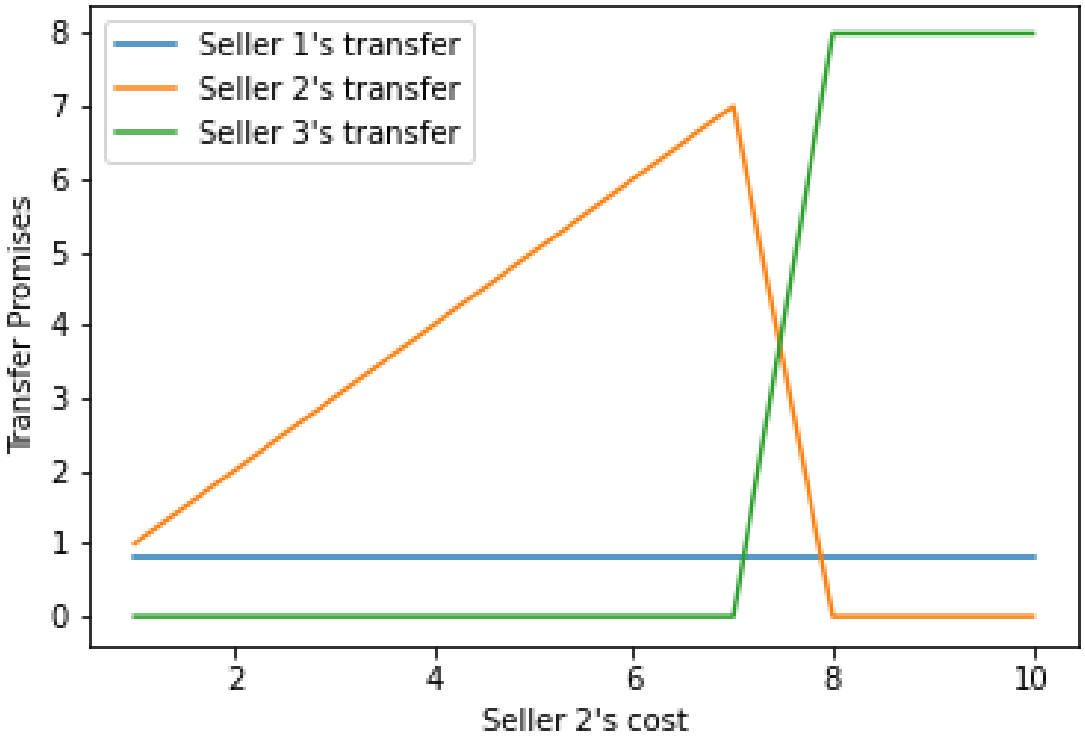

Figure: Sellers' payments under Transfer Promises

Comparison of Two Payment Methods

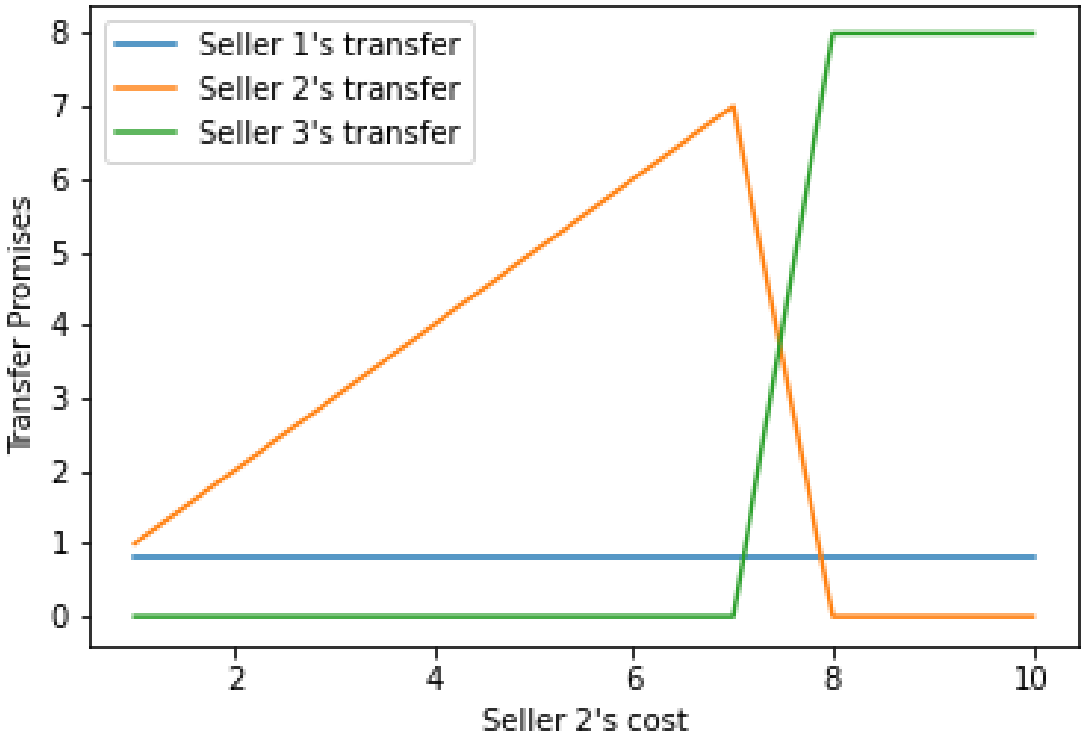

Figure: Sellers' payments under Transfer Promises

2nd-year Paper

In my second-year paper, I plan to extend the Chen-Zapal model by adding in the formation of coalition as an extra stage before the actual bargaining. The main focus, rather than considering the best sequencing, would be rationalize the coalition formation behavior.

Cai, Hongbin (2003): "Inefficient Markov perfect equilibria in multilateral bargaining," *Economic Theory*, 22(3), 583–606.

Chen, Ying, and Jan Zapal (2021): "Sequential Vote Buying," *Working Papers*.

Iaryczower, Matias, and Santiago Oliveros (2019): "Collective Hold-Up," *Working Papers*.

Xiao, Jun (2018): "Bargaining Orders in a Multi-Person Bargaining Game," *Game and Economic Behavior*, 107, 364–379.