1 Lemmas and semi-proves from Cai (2003)

Lemma 1. If the vote sellers form a coalition and bargain with player B as in a standard Rubinstein game, player B gets $\delta/(1+\delta)$S and the seller coalition gets 1(1 + w)=h('). The outcome is efficient, and player B gets half of the total gain from trade when w)=h(' goes to 1.

The total share the vote sellers can get when bargaining separately will always be larger than that when bargaining as a coalition. Therefore, the sellers have no incentives to form such a coalition. Assume that the completion of the project is verifiable, then an agreement between player B and seller i specifies a share $f_i \in [0,1]$ from player B to seller i contingent on the completion of the project. With contingent contracts, the bargaining problem is the standard split-the-pie problem.

Lemma 2. In any Subgame Perfect Equilibrium of the game with N sellers, if player B reaches agreements with all the sellers except player i, player B will reach an agreement with player i without delay. Player i asks for a share of $(1-\sum_{j\neq i} f_i)/(1+\delta)$ and player B accepts so she gets $(1-\sum_{j\neq i} f_i)\delta/(1+\delta)$.

When there is only one seller left, the remaining subgame is a standard bilateral Rubinstein bargaining game.The joint surplus of this bargaining game between player B and the last vote seller is the total surplus (i.e., 1) minus the total shares player B has promised all the other sellers. Therefore, with contingent contracts, the sellers who reach agreements earlier have an inherent advantage over the other sellers, because their contracts with player B effectively constrain the joint surplus available in the later rounds of bargaining with the remaining other sellers. One corollary of Lemma 2 is that player B and the seller who signs the last contract will get the same payoff as w)=h(' goes to one. Consequently, the most player B can hope for is half of the total surplus when δ is close to one.