2.0 Electric field and forces, Electric field lines, Electric Dipoles, Charged particles in an electric field.

We follow up with the last lectures looking at the followings:

- Fields due to static and moving charges
- Lines of force and electric field lines
- Objects such as molecules, that are oppositely charged at two points or poles, DIPOLES
- Behavior of charged particles in an electric field

Field: A Physical Phenomenon such as a force, potential, that pervades a region. Examples are Electric field, gravitational field, magnetic field, etc.

2.1 Static Charges

Static = Non-moving, so that Static Charges are non-moving charges.

Any matter or particle is characterized by two independent but fundamental properties, namely mass and charge.

Electric Charge: The state of electrification of a body is defined in terms of its electrical charge, q. Two types of electrical charge: positive and negative. The net charge in the body is given by the algebraic sum of all the charges, i.e. $Q_t = \sum q_i$. A body with equal numbers of positive and negative charges is said to be electrically neutral.

2.2 Charge Distributions

A point charge is defined as the amount of charge covering a small region in space such that its dimensions are negligible compared to the point of observation. For a collection of charges occupying a finite space, it is more desirable to consider the charge density of the distribution. The density is defined in three ways:

(i) ρ the volume charge density $\rho = dq/dV$ C/m^3

(ii) σ the area charge density $\sigma = dq/dA$ C/m^2

(iii) μ the linear charge density $\mu = dq/dl$ C/m

2.3 Electric Field

We define the electric field ***E*** as any region where an electric charge experiences a force. The force is due to the presence of other charges in the region. For example, a charge q placed in a region where other charges q_1, q_2, q_3 experiences a force $F = F_1 + F_2 + F_3$. The electric field is a vector quantity which gives at every point the force per unit charge on q_1 due to q_2 (or in general other charges). Recall Coulomb's law equation and dividing by q_0, we obtain

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i \quad \text{N/C or Volts/meter}$$

Note that if q is positive the force ***F*** acting on the charge has the same direction as the electric field ***E***, but if q is negative the force F has a direction opposite to ***E***

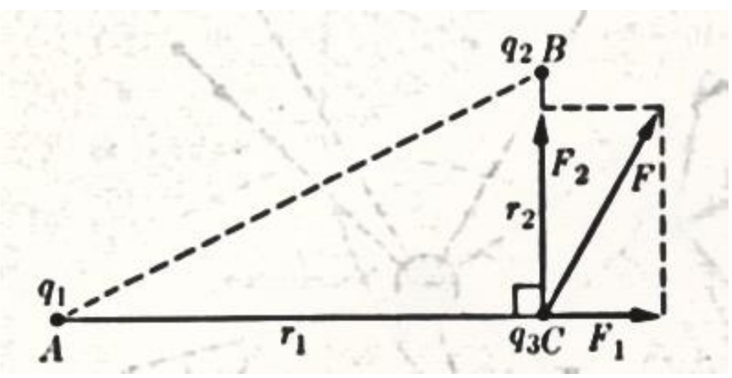
also for a continuous charge distribution a similar expression follows from equation

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}_i$$

The expression shows that it is the charges in the vicinity of the electric force that create the electric field at the given point.

Example

Consider Fig. 2.1 below, given that $q_1 = 1.5\ x\ 10^{-3}C;\ \ q_2 = -0.5\ x\ 10^{-3}C;\ q_3 = 0.2\ x\ 10^{-3}C; AC = 1.2m\ and\ BC = 0.5m$. Calculate the electric field, E produced by charges q_1 and q_2 at C.



Two approaches are possible.

1. Calculate the resultant force on C due to the charges q_1 and q_2 and thereafter calculate E

$$F_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_1^2} = 1.875\ x\ 10^3 N$$

$$F_2 = \frac{q_2 q_3}{4\pi\varepsilon_0 r_2^2} = -3.5\ x\ 10^3 N$$

The resultant force on q_3 is obtained by vector addition

$$F = \sqrt{F_1^2 + F_2^2} = 4.06\ x\ 10^3 N\ and\ \ E = \frac{F}{q_3} = 2.03\ x\ 10^7 N/C$$

2. Calculate E_1 and E_2 due to q_1 and q_2 on q_3, and find the resultant

$$E_1 = \frac{q_1}{4\pi\varepsilon_0 r_1^2} = 9.37\ x\ 10^6 N/C$$

$$E_2 = \frac{q_2}{4\pi\varepsilon_0 r_2^2} = 1.8\ x\ 10^7 N/C$$

$$E = \sqrt{E_1^2 + E_2^2} = 2.03\ x\ 10^7\ N/C$$

Furthermore, we can calculate the direction of the force or electric field lines (both are in the same direction) as

$$\tan\theta = \frac{E_2}{E_1} \text{or } \theta = \ tan^{-1}\left(\frac{E_2}{E_1}\right) = 62.5°$$

The angle is calculated using the vertical component divided by the horizontal component.

2.4 Electric field lines

An electric field can be represented by lines of force, which are lines that, are tangent to the direction of the electric field at that point. Fig. 1. 2a shows the electric field of a positive charge and those of Fig. 1.2b of that of a negative charge. They are straight lines through the charge.

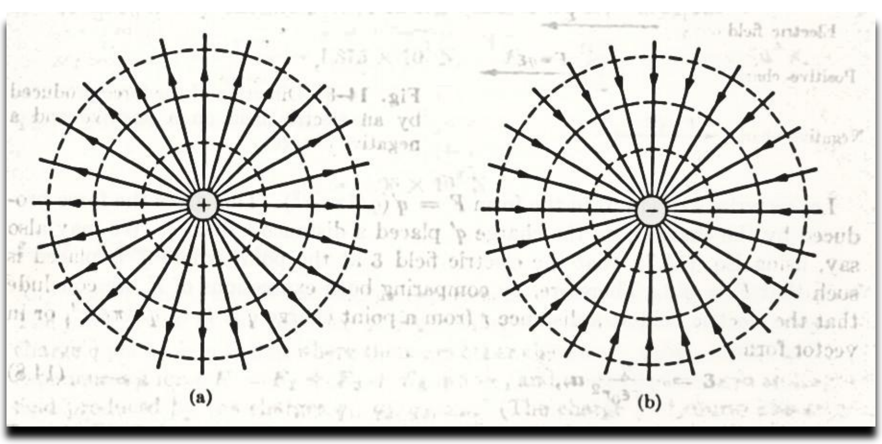


Fig. 2.2: Lines of force and equipotential surfaces of the electric field of positive and negative charges

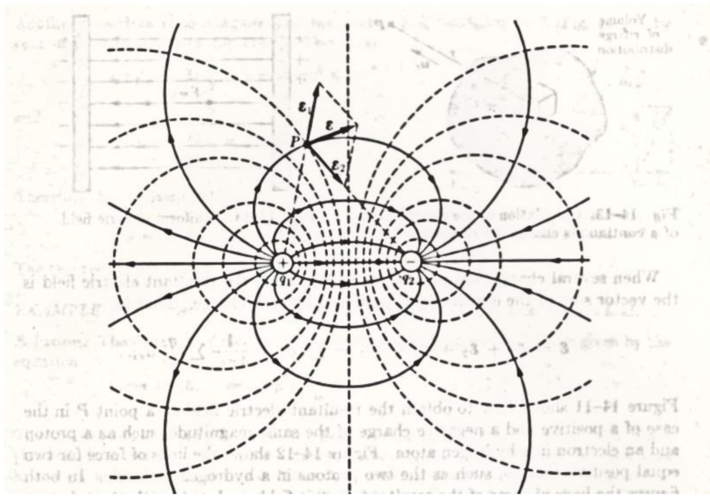


Fig. 2. 3: Lines of force and equipotential surfaces (dashed lines) of the electric field of two equal but opposite charges

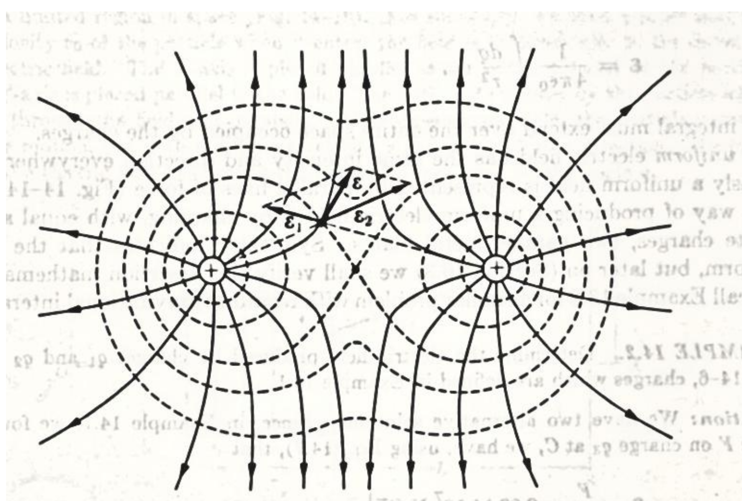


Fig. 2. 4: Lines of force and equipotential surfaces (dashed lines) of the electric field of two equal identical charges

The equipotential surface is that in which the electric potential is the same everywhere. Noting that potential, $V = \frac{kq}{r}$, thus all surfaces defined by the radius r are equipotential. They have the same potential and the work done in moving a charge on an equipotential surface is zero

2.4.1 Characteristics of Electric field lines

- Electric field lines radiate from the charges in 3 dimensions, hence an infinite number of lines could be drawn.
- They provide information about the strength of the field. Note that near the charges the field lines are close and the electric field strongest. As distances increase from the test charge the filed strength is weaker.*The number density (number of lines per unit area)*

passing perpendicular through an area is proportional to the magnitude of the electric field

- Electric field lines are not always straight. Most often they are curved
- Electric field lines always begin on a positive charge and end on the negative charge

2.5 Dipoles

An electric dipole consists of two separated charges that have the same magnitude but opposite signs. The electric field of the dipole is proportional to the product of the magnitude of one of the charges and their separation. The product is called ***dipole moment***.

2.6 The Electric Dipole

A dipole consists of a pair of equal and opposite charges separated by a distance r. The distance r is drawn from the negative to the charge and it is along the axis of the dipole. The dipole is a very important problem in electrostatics of the common occurrence of dipoles in every day life. The electric field calculation is also easy as only point charges are involved in the calculations.

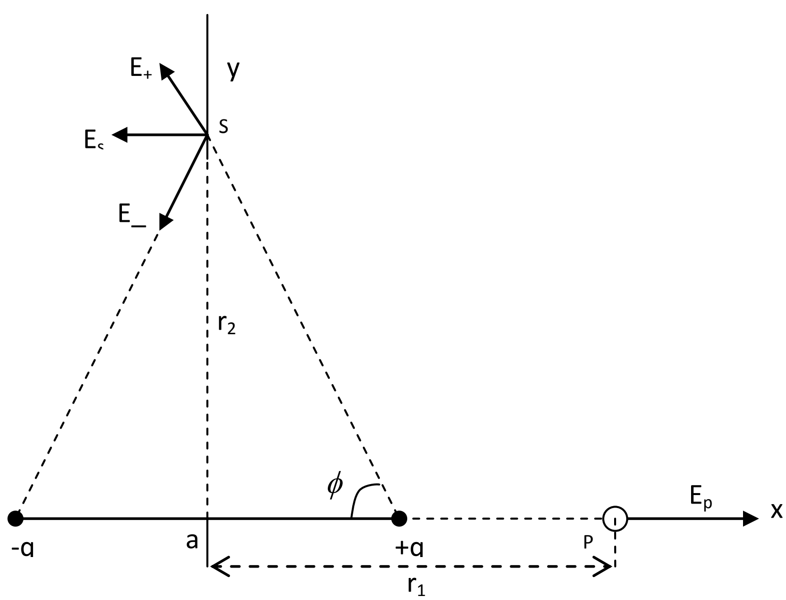


Fig. 2.5: The field due to the Electric Dipole

The field of the dipole:

(i) Along the axis of the dipole:

by superposition principle

$$E_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r_1 - a/2)^2} - \frac{q}{(r_1 + a/2)^2}\right]$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{2r_1 a}{(r_1^2 - a^2/4)^2}$$

(ii) Perpendicular to the dipole:

$$E_S = |E_+|\cos\phi + |E_-|\cos\phi$$

by the geometry of the dipole, the y components of the electric field cancel out.

$$E_S = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_2^2 + (a/2)^2} + \frac{q}{r_2^2 + (a/2)^2}\right]\frac{a/2}{\left[r_2^2 + (a/2)^2\right]^{1/2}}$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{a}{\left[r_2^2 + (a/2)^2\right]^{3/2}}$$

at points of observation such that $r^2 >> a^2/4$ (far fields)

$$E_P \cong \frac{1}{4\pi\varepsilon_0}\frac{2p}{r_1^3} \quad \text{N/C}$$

$$E_S \cong \frac{1}{4\pi\varepsilon_0}\frac{p}{r_2^3} \quad \text{N/C}$$

where ***p = qa*** is the dipole moment

Important deductions from this are

(i) the field of large charges close together is the same as the field of small charges at large distances as the field depends only on the product qa.
(ii) a single isolated charge has its electric field fall as $1/r^2$, a dipole with two charges has its field falls as $1/r^3$

2.6.1 The field of a dipole in any direction.

The field of the electric dipole in any other direction can be easily calculated by resolving the dipole for both the horizontal and vertical components. On the horizontal component, the dipole in the direction of the radius vector E_r is given by

$$E_r \cong \frac{1}{4\pi\varepsilon_0}\frac{2p_1}{r^3}$$

where $p_1 = p \cos \theta$. The above follows directly from equation.

At any other direction θ relative to the horizontal, the component is given

$$E_\theta \cong \frac{1}{4\pi\varepsilon_0}\frac{p_2}{r^3}$$

where $p_2 = p \sin \theta$.

For the Dipole the field lines are as shown below

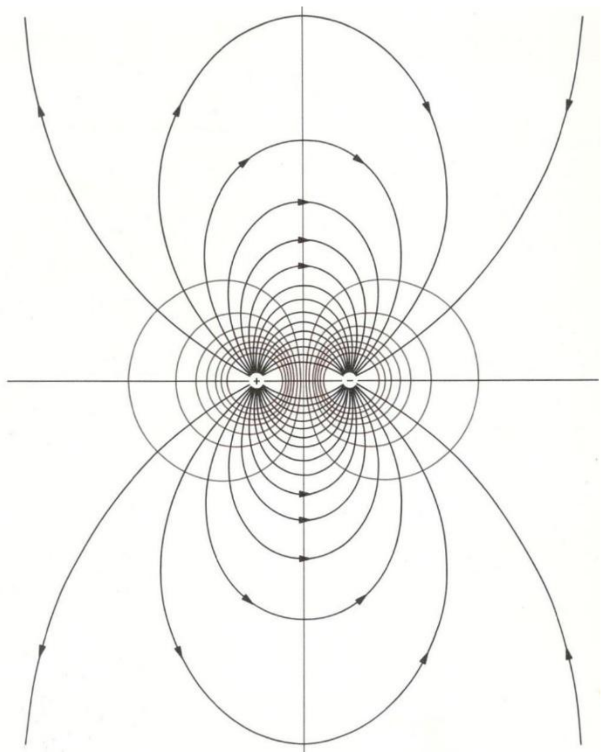


Fig. 2.6: Electric field lines and equipotential surfaces of a dipole. The surfaces are drawn so that at every point the surfaces are perpendicular to the field lines.

2.7 Charged Particles in an Electric field.

A charged particle in an electric field experience a force which causes it to move in the field. The equation of motion in the electric field is given by

$$ma = qE; \ \ or \ \ a = \frac{q}{m}E$$

Thus, the motion (***acceleration)*** in the field depends on the ratio q/m, this ratio is generally different for different charged particles or ions. This is unlike the gravitational field where the acceleration is constant, g. If the electric field is uniform the acceleration is constant and the path of the charged particle is a parabola.

Consider a charged particle passing through an electric field occupying a limited region in space as shown in Fig. 2.7. If the initial velocity is v_0 when it enters the field is perpendicular to the direction of the electric field. If v_0 is in the X -direction and the field in the Y-direction, the path followed by the particle is a parabola after the field the particle continues in a straight line but with a different velocity and direction. The electric field has produced a deflection by an angle α.

Inside the field the particle is described by

$$x = v_0 t, \qquad y = \frac{1}{2}\frac{q}{m}Et^2$$

From Newton’s laws ***(x =ut +1/2 at^2)***

Eliminating *t* we obtain

$$y = \frac{1}{2}\frac{q}{m}\frac{E}{v_0^2}x^2$$

Which is the equation of a parabola. The angle α is given by

$$\tan\alpha = \left(\frac{dy}{dx}\right)_{x=a} = \frac{qEa}{mv_0^2} \approx \frac{d}{L} \quad for\ BD \ll d$$

A device built upon the above expression will be useful in separating particles of different masses since q is the same.

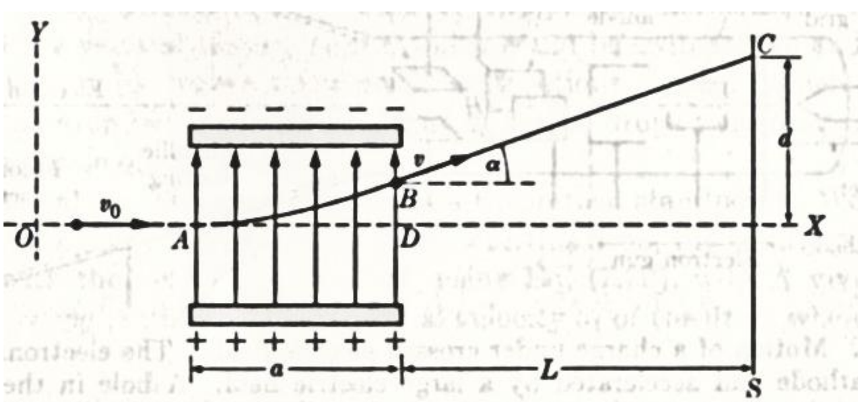


Fig. 2.7: Motion of Charged particle in a uniform Electric field

2.8 Assignment

1. A charge of 2.5×10^{-8} C is placed in an upward-directed uniform field whose intensity is 5×10^{4} N/C. What is the work of the electrical force on the charge when the charge moves (i) 45cm to the right (ii) 80cm downwards (iii) 200cm at angle 45 deg upward
2. Two point charges 5μC and -10μC are spaced 1m apart. Calculate (i) the magnitude and direction of the electric field at a point 0.6m from the first charge and 0.8m from the second charge. Obtain the point where the electric field is zero due to these two charges.