1. A packet contains 10 distribution firecrackers out of which 4 are defective. If three firecrackers are drawn at random (without replacements) from the packet then the probability that all three firecrackers are defective equals

A. $\frac{1}{10}$ B. $\frac{1}{20}$

C. $\frac{1}{30}$ D. $\frac{1}{40}$

2. Let E and F be any two independent events with $0<P(E)<1$ & $0<P(F)<1$. Which one of the following statements is NOT true?

A. P (Neither E nor F occurs) = $(P(E)-1)(P(F)-1)$

B. P (Exactly one of E and F occurs) = $P(E)+P(F)-P(E)P(F)$

C. P (E occurs but F does not occur) = $P(E)-P(E\cap F)$

D. P (E occurs given that F does not occur) = $P(E)$

3. Let E, F and G be any three events with $P(E)=0.3$, $P(F|E)=0.2$, $P(G|E)=0.1$ and $P(F\cap G|E)=0.05$. Then $P(E-(F\cup G))$ equals

A. 0.155 B. 0.175

C. 0.225 D. 0.255

4. Two biased coins C_1 & C_2 have probability of getting heads $\frac{2}{3}$ & $\frac{3}{4}$, respectively, when tossed. If both coins are tossed independent two times each then the probability of getting exactly two heads out of these four tosses is

A. $\frac{1}{4}$ B. $\frac{37}{144}$

C. $\frac{41}{144}$ D. $\frac{49}{144}$

5. Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega=\{1,2,3,4\}$. Consider the events $E=\{1,2,\}$, $F=\{1,3\}$ & $G=\{3,4\}$. Then which of the following statements(s) is (are) true?

A. E and F are independent

B. E and G are independent

C. F and G are independent

D. E, F and G are independent

6. Consider four coins labelled as 1,2,3 and 4. Suppose that the probability of obtaining a 'head' in a single toss of the i^{th} coins is $\frac{i}{4}$, $i=1,2,3,4$. A coin is chosen uniformly at random and flipped. Given that the flip resulted in a 'head', the conditional probability that the coin was labelled either 1 or 2 equals

A. $\frac{1}{10}$ B. $\frac{2}{10}$

C. $\frac{3}{10}$ D. $\frac{4}{10}$

7. Let E and F be two events with $0<P(E)<1,\ 0<P(F)<1$ & $P(E)+P(F)\geq 1$. which of the following statements is are TREU?

A. $P(E^c)\leq P(F)$ B. $P(E\cup F)<P(E^c\cup F^c)$

C. $P(E|F^c)\geq P(F^c|E)$ D. $P(E^c|F)\leq P(F|E^c)$

8. Player P_1 tosses 4 fair coins and player P_2 tosses a fair die independently of P_1. The probability that the numbers of heads observed is more that the numbers on the upper face of the die equals

A. $\frac{7}{16}$ B. $\frac{5}{32}$

C. $\frac{17}{96}$ D. $\frac{21}{64}$

9. Let E and F be two events with $0 < P(E) < 1,\ 0 < P(F) < 1$ & $P(E|F) > P(E)$. Which of the following statements is (are) TRUE?

A. $P(F|E) > P(F)$

B. $P(E|F^c) > P(E)$

C. $P(F|E^c) < P(F)$

D. E and F are independent

10. An institute purchases laptops from either vendor V_1 or vendor V_2 with equal probability. The lifetimes (in years) of laptop from vendor V_1 have a $U(0,4)$ distribution and the lifetime (in years) of laptop from vendor V_2 have an $Exp(1/2)$ distribution. If a randomly selected laptop in the institute has lifetime more than two years then the probability that it was supplied by vendor V_2 is

A. $\frac{2}{2+e}$

B. $\frac{1}{1+e}$

C. $\frac{1}{1+e^{-1}}$

D. $\frac{2}{2+e^{-1}}$

11. Let E and F be two independent events with

$$P(E|F)+P(F|E)=1,\ P(E\cap F)=\frac{2}{9} \ \&\ P(F)<P(E).$$

Then $P(E)$ equals

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

12. 2000 cashew nuts are mixed thoroughly in flour. The entire mixture is divided into 1000 equals parts and each part is used to make one biscuit. Assume that on cashews are broken in the process. A biscuit is picked at random. The probability that it contains no cashew nuts is

A. between 0 and 0.1

B. between 0.1 and 0.2

C. between 0.2 and 0.3

D. between 0.3 and 0.4

13. There are two boxes each containing two components. Each component is defective with probability 1/4 independent of all components. The probability that exactly one box contains exactly one defective component equals

A. 3/8

B. 5/8

C. 15/32

D. 17/32

14. There are two urns U_1 & U_2. U_1 contains four black balls and four white balls and U_2 is empty. Four balls are drawn at random from U_1 and transferred to U_2. Then a ball is drawn at random from U_2. The probability that the ball drawn from U_2 is white is

A. $\frac{1}{3}$ B. $\frac{1}{2}$

C. $\frac{2}{3}$ D. $\frac{3}{4}$

15. Four persons P, Q, R and S take turns (in the sequence P, Q, R, S, P, Q, R, S, P…) in rolling a fair die.

The first person to get six wins. Then the probability that S wins is

A. $\frac{125}{671}$ B. $\frac{125}{571}$

C. $\frac{125}{471}$ D. $\frac{125}{371}$

16. A system consisting of n components function if and only if at least one of n components function. Suppose that all the n components of the system function independently, each with probability $\frac{3}{4}$. If the probability of functioning of the system is $\frac{63}{64}$, then the value of n is

A. 2 B. 4

C. 3 D. 5

17. Let E and F be two events with $P(E)=0.7,\ P(F)=0.4\ \&\ P(E\cap F^c)=0.4$. Then $P(F\,|\,E\cup F^c)$ is equal to

A. $\frac{1}{2}$ B. $\frac{1}{3}$

C. $\frac{1}{4}$ D. $\frac{1}{5}$

18. A four-digit number is chosen at random. The probability that there are exactly two zeros in that numbers is

A. 0.73 B. 0.973

C. 0.027 D. 0.27

19. A person makes repeated attempts to destroy a target. Attempts are made independent of each other. The probability of destroying the target in any attempts is 0.8. Given that he fails to destroy the target in the first five attempts. The probability that the target is destroyed in the 8^{th} attempt is

A. 0.128
B. 0.032
C. 0.160
D. 0.064

20. A fair die is rolled 3 times. The conditional probability of 6 appearing exactly once given that it appeared at least once equals

A. $\dfrac{3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)}{1-\left(\frac{5}{6}\right)^3}$
B. $\dfrac{\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2}{1-\left(\frac{5}{6}\right)^3}$
C. $\dfrac{3\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2}{1-\left(\frac{5}{6}\right)^3}$
D. $\dfrac{\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)}{1-\left(\frac{5}{6}\right)^3}$

21. Two coins with probability of heads $u \, and \, v$ respectively are tossed independently. If P (both coins show up tails) = P (both coins show up heads). Then $u+v$ equals

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. $\frac{3}{4}$
D. 1

22. Let E and F be two events with $P(E)>0, P(F|E)=0.3$ & $P(E\cap F^c)=0.2.$ Then P(E) equals

A. $\frac{1}{7}$
B. $\frac{2}{7}$
C. $\frac{4}{7}$
D. $\frac{5}{7}$

23. For detecting a disease, a test given correct diagnosis with probability 0.99. If is known that 1% of a population suffers this disease. If a randomly selected individual from this population tests positive then the probability that the selected individual actually has the disease is

A. 0.01
B. 0.05
C. 0.5
D. 0.99

24. A nonempty subset P is formed by selection elements at random and without replacements form a set B consisting of $n(>1)$ distinct elements. Another nonempty subset Q is formed in a similar fashion from the original set B consisting of the same n elements. Then the probability that P and Q do not have any common elements is

A. $\dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n-i}\binom{n}{i}\binom{n-i}{j}}{\sum_{i=1}^{n}\sum_{j=1}^{n}\binom{n}{i}\binom{n}{j}}$

B. $\dfrac{\sum_{i=0}^{n}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n-i}{j}}{\sum_{i=0}^{n}\sum_{j=0}^{n}\binom{n}{i}\binom{n}{j}}$

C. $\dfrac{\sum_{i=1}^{n-1}\sum_{j=1}^{n-i}\binom{n}{i}\binom{n-i}{j}}{\sum_{i=1}^{n}\sum_{j=1}^{n}\binom{n}{i}\binom{n}{j}}$

D. $\dfrac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n-i}{j}}{\sum_{i=0}^{n}\sum_{j=0}^{n}\binom{n}{i}\binom{n}{j}}$

25. Let E and F be two events such that $0<P(E)<1$ & $P(E|F)+P(E|F^c)=1$. Then

A. E and F are mutually exclusive

B. $P(E^c|F)+P(E^c|F^c)=1.$

C. E and F are independent

D. $P(E|F)+P(E^c|F^c)=1.$

26. Independent trials consisting of rolling a fair die are performed. The probability that 2 appears

before 3 or 5 is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

27. An archer makes 10 independents attempts at a target and his probability of hitting the target at each attempt is $\frac{5}{6}$ Then the conditional probability that his last two attempts are successful given that he has a total of 7 successful attempts is

A. $\frac{1}{5^5}$

B. $\frac{7}{15}$

C. $\frac{25}{36}$

D. $\frac{8!}{3!5!}\left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)^3$

28. Let E and F be two mutually disjoint events. Further, Let E and F be independent of G. If $p = P(E) + P(F)$ & $q = P(G)$, then $P(E \cup F \cup G)$ is

A. $1 - pq$
B. $q + p^2$
C. $p + q^2$
D. $p + q - pq$

29. Let E, F and G be three events such that the events E and F are mutually exclusive, $P(E \cup F) = 1$, $P(E \cap G) = \frac{1}{4}$ & $P(G) = \frac{7}{12}$. Then $P(F \cap G)$ equals

A. $\frac{1}{12}$
B. $\frac{1}{4}$
C. $\frac{5}{12}$
D. $\frac{1}{3}$

30. There are three urns, labeled Urn 1, Urn 2 and Urn 3. Urn 1 contains 2 white balls and 3 black balls Urn 2 contains 1 white ball and 3 back balls and Urn 3 contains 3 white balls and 1 black ball. Consider two coins with probability head in their single trials as 0.2 and 0.3. The two coins are tossed independently once and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained; Urn 3 is selected if 2 tails are obtained otherwise Urn 2 is selected. A ball is then drawn at random from the selected urn. Then

P (Urn 1 is selected | the ball drawn is white)

Is equal to

A. $\frac{6}{109}$
B. $\frac{12}{109}$
C. $\frac{1}{18}$
D. $\frac{1}{9}$

31. Consider three coins having probability of obtaining head in a single trail as $\frac{1}{4}, \frac{1}{2}$ & $\frac{3}{4}$ respectively. A player selects one these coins at random (each coin is equally likely to be selected). If the player tosses the selected coin five times independently then the probability of obtaining two tails in five tosses is equals to

A. $\frac{85}{384}$
B. $\frac{255}{384}$
C. $\frac{125}{384}$
D. $\frac{64}{384}$

32. Let E_1, E_2, E_3 & E_4 be four events such that

$P(E_i \mid E_4)=\frac{2}{3},\ i=1,2,3;\ P(E_i \cap E_j^c \mid E_4)=\frac{1}{6},\ i,j=1,2,3;\ i\neq j\ \&\ P(E_1 \cap E_2 \cap E_3^c \mid E_4)=\frac{1}{6}$

Then, $P(E_1 \cup E_2 \cup E_3 \mid E_4)$ is equals to

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{5}{6}$

D. $\frac{7}{12}$

33. Let E_1, E_2 & E_3 be three events such that $P(E_1)=\frac{4}{5}, P(E_2)=\frac{1}{2}$ & $P(E_3)=\frac{9}{10}$

Then which of the following statements is False?

A. $P(E_1 \cup E_2 \cup E_3)\geq \frac{9}{10}$

B. $P(E_2 \cap E_3)\leq \frac{1}{2}$

C. $P(E_1 \cup E_2 \cup E_3)\leq \frac{1}{6}$

D. $P(E_1 \cup E_2)\geq \frac{4}{5}$

34. A year is chosen at random form the set of years {2012, 2013….,2021}. Four the chosen year a month is chosen at random and from the chosen month a day is chosen at random Given that chosen day is the 29^{th} of a month the conditional that the chosen month is February equals

A. $\frac{279}{9965}$

B. $\frac{289}{9965}$

C. $\frac{269}{9965}$

D. $\frac{259}{9965}$

35. Suppose that four persons enter a lift on the ground floor of a building. There are seven floors above the ground floor and each person independently chooses her exit floor as one of these seven floors, If each of them chooses the topmost floor with probability $\frac{1}{3}$ and each of the remaining floors with an equal probability, Then the probability that on two of them exit at the same floor equals

A. $\frac{200}{729}$

B. $\frac{220}{729}$

C. $\frac{240}{729}$

D. $\frac{180}{729}$

36. Let A and B be two events such that $0<P(A)<1$ & $0<P(B)<1$

Then which one of the following statements is NOT true?

A. If $P(A|B) > P(A)$, then $P(B|A) > P(B)$

B. If $P(A \cup B) = 1$, then A and B cannot be independent

C. If $P(A|B) > P(A)$, then $P(A^c|B) < P(A^c)$

D. If $P(A|B) > P(A)$, then $P(A^c|B^c) < P(A^c)$

37. A computer lab has two printers handling certain types of printing jobs. Printer-I and Printer-II handle 40% and 60% of the jobs respectively for a typical printing job, printing tome (in minutes) of Printer-I follows $N(10,4)$ distribution and that of Printer-II follows $U(1,21)$ distribution. If a randomly selected printing job is found to have been completed in less than 10 minutes then the conditional probability that it was handled by the Printer-II equals _________(round off to two decimal places)

38. In an ethic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), That a randomly selected adult male form this population does not have disease given that the test indicates high cholesterol level equals_____________

39. In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and apprise the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected at least two are approved for shipment equals

40. Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals_______

41. Let A_1, A_2 & A_3 be three events such that

$$P(A_i)=\frac{1}{3}, i=1,2,3; P(A_i \cap A_j)=\frac{1}{6}, 1 \le i \ne j \le 3 \text{ and } P(A_1 \cap A_2 \cap A_3)=\frac{1}{6}.$$

Then the probability that none of the events A_1, A_2, A_3 occurs equals __________

42. A fair die is rolled times independently. Given that 6 appeared at least once, the conditional probability that 6 appeared exactly twice equals ____________

43. Let E.F and G b three events such that

$$P(E \cap F \cap G)=0.1, P(G | F)=0.3 \text{ and } P(E | F \cap G)=P(E | F).$$

Then $P(G | E \cap F)$ equals_______

44. In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k; k=1,2,.....$ A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is__________

45. Two points are chosen at random on a line segments of length 9 cm. The probability that the distance between two points is lees that 3 cm is___________

46. A system comprising of n identical components works if at least one of the components works. Each of the components works probability 0.8, independent of all other components. The minimum value of n fro which the system works with probability at least 0.97 is _________

47. Two cars A and B are travelling independently in the same direction. The speed of car A is normally distribution with the mean 100 kilometers per hour standard deviation $\sqrt{5}$ kilometers per hour and the speed of car B is normally distributed with the mean 100 kilometers per hour a standard deviation 2 kilometer per hour. Initially (at time t = 0) the car A is 3 kilometers ahead of car B. Find the probability that after 3 hours these two vars will be within of 3 kilometers ___________

48. Suppose that 5 distinct balls are distribution at random into 3 distinct boxes in such a way that each of the 5 balls can get into any are of the 3 boxes. Find the probability that exactly one box is empty. Also find the probability that all boxed are occupied.

49. Let X_1 & X_2 denote the lifetimes (in months) o bulls produced at factories F_1 & F_2 respectively. The random variables X_1 & X_2 are $Exp(1/8)$ & $Exp(1/2)$ respectively. A shop procures 80% of its supply of bulbs form factory F_1 and 20% form factory F_2. A randomly selected bulb from the shop is put on test and is found to be working after 4 months. What is the probability that it was procured from factory F_2?

50. Ram rolls a pair of dice. If the sum of the numbers shown on upper faces is 5,6,10,11 or 12 then Ram wins a gold coin. Otherwise, he rolls the pair of dice once again and wins a silver cions if the sum of the numbers show on the upper faces in the second throw is the same as the sum of the numbers in the first throw, what is the probability that he wins a gold or a silver coin?

51. Ten percent of bolts produced in a factory are defective. They are randomly packed in boxes such that each box contains 3 bolts. Four of these boxes are bought by a customer. The probability that the boxed that this customer bought have no defective bolt in them is equal to

52. Bulbs produced by a factory F_i have lifetimes (in months) distribution as $Exp\left(\frac{1}{3^i}\right)$ for $i=1,2,3.$ A firm randomly procures bulb from the firm is found to be working after 27 months. The probability that it was produced by the factory F_3 is

53. Let $U_1,U_2,.....U_n$ be n urus such that urn U_k contains k white and k^2 black balls $k=1,...n.$ Consider the random experiments of selecting an urn and drawing a ball out of it at random. If the probability of selection urn U_k is proportional to $(k+1)$, then

(a) find the probability that the ball drawn is black.

(b) find the probability that urn U_n was selected given that the ball drawn is white

54. (a) Student population of a university has 30% Asian students 50% of all American student, 60% of all European student and 20% of all African student are girls. Find the probability that a girl chosen at random from the university is an Asian.

(b) Let A_1, A_2 & A_3 be pairwise independent events with $P(A_i)=\frac{1}{2}$. $i=1,2,3.$ Suppose that A_3 & $A_1 \cup A_2$ are independent. Find the value of $P(A_1 \cap A_2 \cap A_3)$.

55. (a) One coin s selected at random form two coins. The probability of obtaining head for one of them is $\frac{1}{3}$ and for the other it is $\frac{1}{2}$. If the selected coin is tossed and the head shows up. What is the probability that it is the fair coin?

(b) Let *p* denote the probability that the weather (either wet or dry) tomorrow will be the same as that of today. If the weather is dry today, show that P_n, the probability that it will be dry *n* days later satisfies

$$P_n=(2p-1)P_{n-1}+(1-p),\ n\geq 1.$$

Hence or otherwise determine the value of P_{50} for $P=\frac{3}{4}$.

56. Let E_1, E_2, E_3 & E_4 be four independent events such that

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{4} \ \& \ P(E_4)=\frac{1}{5}.$$

Let *p* be the probability that at most two events among E_1, E_2, E_3 & E_4 occur. Then 240*p* is equal to________

57. Five men go to a restaurant together an each of them orders a dish that is different from the dishes ordered by the other members of the group. However, the waiter serves the dishes randomly. Then the probability that exactly one of them gets the dish he ordered equals________(round off to 2 decimal places)

58. There are four uruses labeled U_1, U_2, U_3 &U_4 each containing 3 blue and 5 real balls. The fifth urn, labelled U_5, contains 4 and 4 red balls. An is selected at random from these five urns a ball is drawn at random at from it. Given that selected ball is red, find the probability that it came from the urn U_5,

Answer Key

1. C	2. B	3. C	4. B

5. A, C	6. C	7. A, C, D	8. C
9. A, C	10. A	11. C	12. B
13. C	14. B	15. A	16. C
17. B	18. C	19. B	20. C
21. D	22. B	23. C	24. C
25. B	26. B	27. B	28. D
29. D	30. A	31. A	32. C
33. C	34. A	35. A	36. D

37. 0.55 to 0.60   38. 0.20 to 0.25

39. 0.88 to 0.95   40. 0.30 to 0.35

41. $\frac{1}{3}$ 42. $\frac{15}{91}$ 43. 0.3   44. 0.33

45. 0.56   46. 3   47. 0.2454   48. $\frac{50}{81}$

49. $\frac{1}{4e^{3/2}+1}$ 50. 0.537   51. 0.28

52. $\left(\frac{4}{3}e^{-8}+e^{-2}+1\right)^{-1}$ 53. (a). $\frac{n+1}{n+3}$ (b). $\frac{1}{n}$

54. (a). $6/23$ (b). $\frac{1}{8}$

55. (a). $\frac{3}{5}$ (b). $\left(\frac{1+2^{50}}{2^{51}}\right)$ 56. 218

57. 0.35 to .040   58. $\frac{1}{6}$