Math 526 – Statistics II, Spring 2023

## Assignment 2

Due: Monday, March 20, by 8:00PM

**Problem:** In this assignment, you will set up and solve a clustering problem via a Bayesian model.

The provided dataset consists of independent and identically acquired datapoints  $w_n = (x_n, y_n)$  of two scalar components each. The components of each datapoint w = (x, y) follow the statistics

$$x|s \sim \mathsf{Normal}\left(X_s, \frac{1}{ au}\right)$$
  $y|s \sim \mathsf{Normal}\left(Y_s, \frac{1}{ au}\right)$ 

where  $X_s, Y_s$  are cluster specific parameters.

- 1. Assuming that the measurements are generated by 3 clusters, formulate a Bayesian model for cluster analysis. Make your own choices for the necessary priors and briefly reason on your selection.
- 2. Represent graphically your model.
- 3. Describe a Markov chain Monte Carlo scheme to sample from your model's posterior.
- 4. Implement your Markov chain Monte Carlo scheme and generate samples to characterize the joint posterior distribution of all cluster specific location parameters. Summarize your results graphically.
- 5. Use your samples to approximate the posterior probability that the datapoints  $w_{11}$  and  $w_{982}$  belong to the same cluster.

Associated data: The dataset is provided in cluster\_data.mat.