Math 526 – Statistics II, Spring 2023

Assignment 3

Due: Monday, April 24, by 8:00PM

Problem: In this assignment, you will perform signal analysis under a hidden Markov model.

The provided dataset contains successive measurements $w_{1:N}$ of a highly corrupted signal with 3 dynamical states $\sigma_{1:3}$. The dataset is accurately represented by the hidden Markov model

$$\begin{aligned} s_1 &\sim \text{Categorical}_{\sigma_1,\sigma_2,\sigma_3}\left(\rho_{\sigma_1}, \rho_{\sigma_2}, \rho_{\sigma_3}\right) \\ s_n | s_{n-1} &\sim \text{Categorical}_{\sigma_1,\sigma_2,\sigma_3}\left(\pi_{s_{n-1}\to\sigma_1}, \pi_{s_{n-1}\to\sigma_2}, \pi_{s_{n-1}\to\sigma_3}\right), && n = 2, \ldots, N \\ w_n | s_n &\sim \text{Normal}\left(\phi_{s_n}, 1\right), && n = 1, \ldots, N \end{aligned}$$

1. Implement the (normalized) forward filtering algorithm and evaluate the (marginal) likelihood under the following parameter choices

$$\begin{aligned} \rho_{\sigma_1} &= 1, & \rho_{\sigma_2} &= 0, & \rho_{\sigma_3} &= 0 \\ \pi_{\sigma_1\to\sigma_1} &= 1/2, & \pi_{\sigma_1\to\sigma_2} &= 1/4, & \pi_{\sigma_1\to\sigma_3} &= 1/4 \\ \pi_{\sigma_2\to\sigma_1} &= 1/4, & \pi_{\sigma_2\to\sigma_2} &= 1/2, & \pi_{\sigma_2\to\sigma_3} &= 1/4 \\ \pi_{\sigma_3\to\sigma_1} &= 1/4, & \pi_{\sigma_3\to\sigma_2} &= 1/4, & \pi_{\sigma_3\to\sigma_3} &= 1/2 \\ \phi_{\sigma_1} &= -1, & \phi_{\sigma_2} &= +3, & \phi_{\sigma_3} &= +6 \end{aligned}$$

2. Implement the Viterbi algorithm and compute the optimal hidden state sequence under the same parameter choices.

3. Set up a Bayesian model to estimate all model parameters.

4. Describe a Markov chain Monte Carlo sampler to sample the model's posterior probability distribution. You do not need to implement your sampler.

Associated data: The dataset shown above is provided in `corrupted_data.mat`.