Math 526 – Statistics II, Spring 2023

Assignment 4

Due: Monday, May 8, by 8:00PM

Problem: In this assignment, you will perform time series analysis under a state-space model.

The provided dataset contains successive measurements $X_{1:N}, Y_{1:N}, Z_{1:N}$ of the position of a freely moving pollen particle. Each measurement in the dataset is contaminated with additive noise that is accurately represented by

$$X_n | x_n \sim \text{Normal}\left(x_n, \sigma^2\right)$$
$$Y_n | y_n \sim \text{Normal}\left(y_n, \sigma^2\right)$$
$$Z_n | z_n \sim \text{Normal}\left(z_n, \sigma^2\right)$$

where x_n, y_n, z_n is the exact position of the particle. In turn, the motion of the particle is accurately represented by

$$x_n | x_{n-1} \sim \text{Normal}\left(x_{n-1}, v\right)$$
$$y_n | y_{n-1} \sim \text{Normal}\left(y_{n-1}, v\right)$$
$$z_n | z_{n-1} \sim \text{Normal}\left(z_{n-1}, v\right)$$

which link the exact particle's positions $x_{1:N}, y_{1:N}, z_{1:N}$ across time. The standard deviation of the measurement noise has been calibrated separately and has the value $\sigma = 0.15$ μm. The variance of the transition noise has also been calibrated separately and has the value $v = 0.0054$ μm^2.

1. Set up a linear Gaussian state-space model that can estimate the particle's trajectory $x_{1:N}, y_{1:N}, z_{1:N}$.
2. Represent your model graphically.
3. Implement the Kalman filtering algorithm and estimate the particle's position.
4. Summarize your results graphically.
5. Implement the Kalman smoothing algorithm and estimate the particle's position.
6. Summarize your results graphically.

Associated data: The provided dataset is `pollen_motion.mat`. The dataset contains measurements $X_{1:N}, Y_{1:N}, Z_{1:N}$ which are reported in μm.