M526: HW4

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May 8, 2023

1. First, we construct a linear Gaussian state–space model that can estimate the particle's trajectory $x_{1:N}$, $y_{1:N}$, $z_{1:N}$.

$$
\begin{aligned}
x_1 &\sim \text{Normal}(\mu_x, v) \\
y_1 &\sim \text{Normal}(\mu_y, v) \\
z_1 &\sim \text{Normal}(\mu_z, v) \\
x_n|x_{n-1} &\sim \text{Normal}(x_{n-1}, v), && n = 2, \cdots, N \\
y_n|y_{n-1} &\sim \text{Normal}(y_{n-1}, v), && n = 2, \cdots, N \\
z_n|z_{n-1} &\sim \text{Normal}(z_{n-1}, v), && n = 2, \cdots, N \\
X_n|x_n &\sim \text{Normal}(x_n, \sigma^2), && n = 1, \cdots, N \\
Y_n|y_n &\sim \text{Normal}(y_n, \sigma^2), && n = 1, \cdots, N \\
Z_n|z_n &\sim \text{Normal}(z_n, \sigma^2), && n = 1, \cdots, N
\end{aligned}
$$

   Where $\mu_x = X_1$, $\mu_y = Y_1$ and $\mu_z = Z_1$. $\sigma = 0.15$ μm and $v = 0.0054$ μm^2. For simplicity, we concatenate our parameters and data into column vectors and matrices as follows: $\tilde{r}_n = [x_n \, y_n \, z_n]^\top$ $\tilde{W}_n = [W_n \, Y_n \, Z_n]^\top$ for $n = 1, \cdots, N$. $\tilde{\mu} = [\mu_x \, \mu_y \, \mu_z]^\top$, $\Xi = \sigma^2 I$, and $\Lambda = vI$.

2. Next we give the graphical representation of this model. See Figure 1

3. Now we implement the Kalman Filtering Algorithm on our data with the specified parameters.

4. See Figure 2.

5. Now we implement the Kalman Smoothing Algorithm on our data with the specified parameters.

6. See Figure 2

Graphical Model

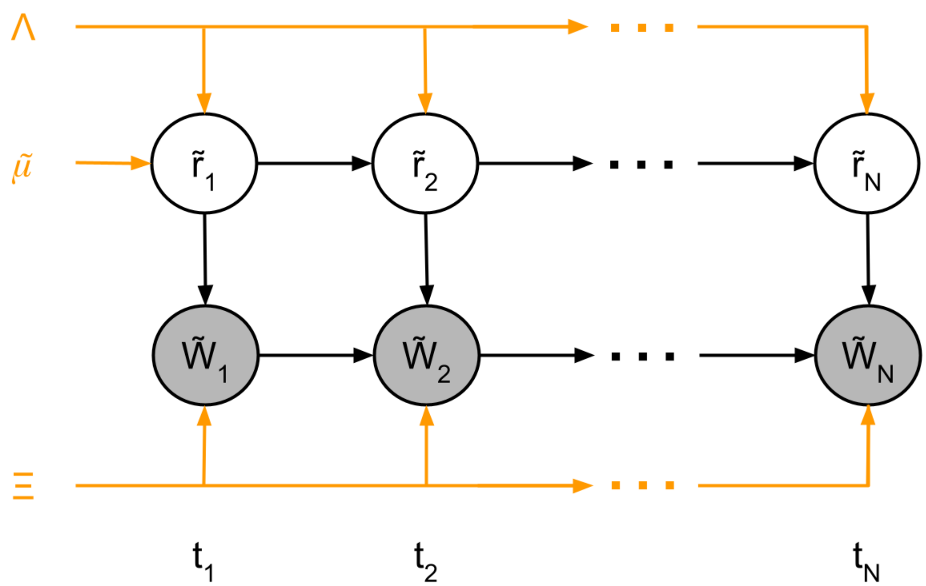

Figure 1: Graphical representation of our Kalman Filter. Note we use our vector notation for simplification of the number of nodes.

Kalman Filtering and Smoothing Results in X–Dimension

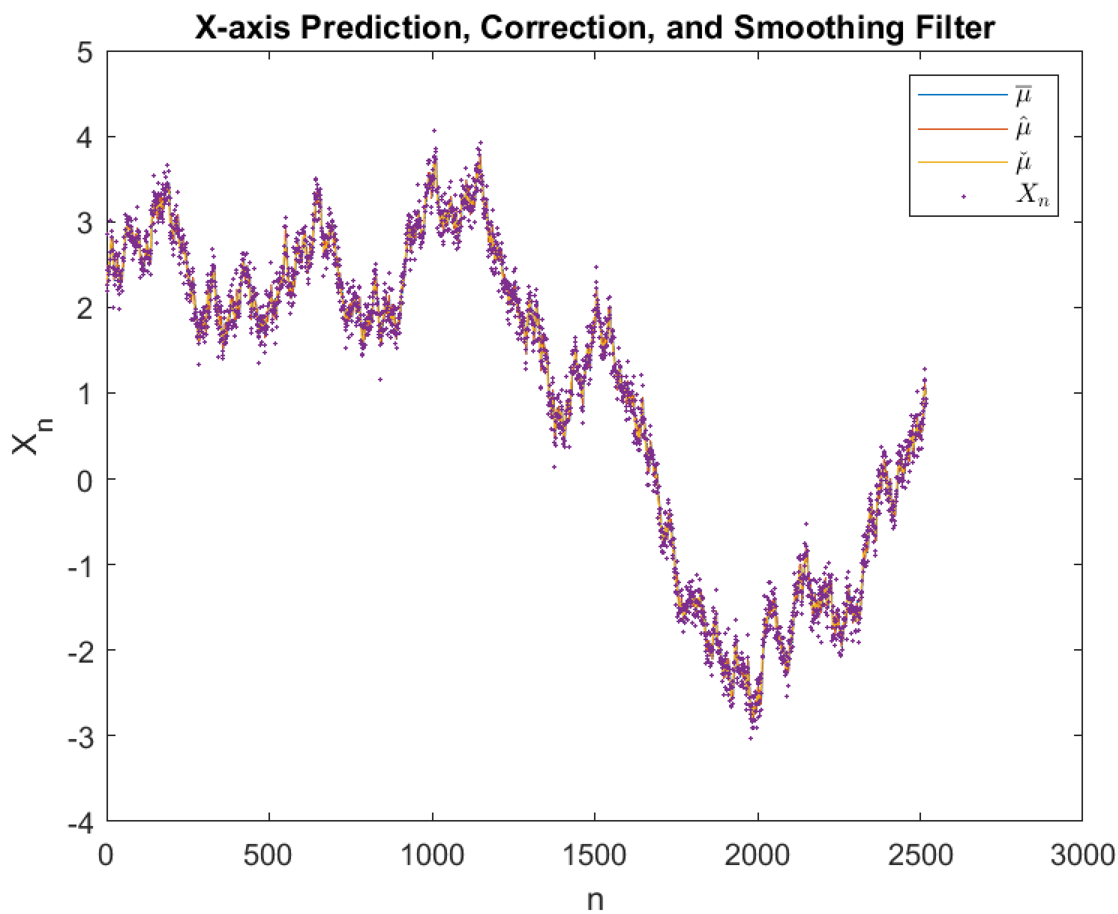

Figure 2: Kalman Filtering and Smoothing Results in X–Dimension. Note that all three estimates of the particle position are too close to differentiate. See Figure 3

Kalman Filtering and Smoothing Results in X–Dimension (Zoomed)

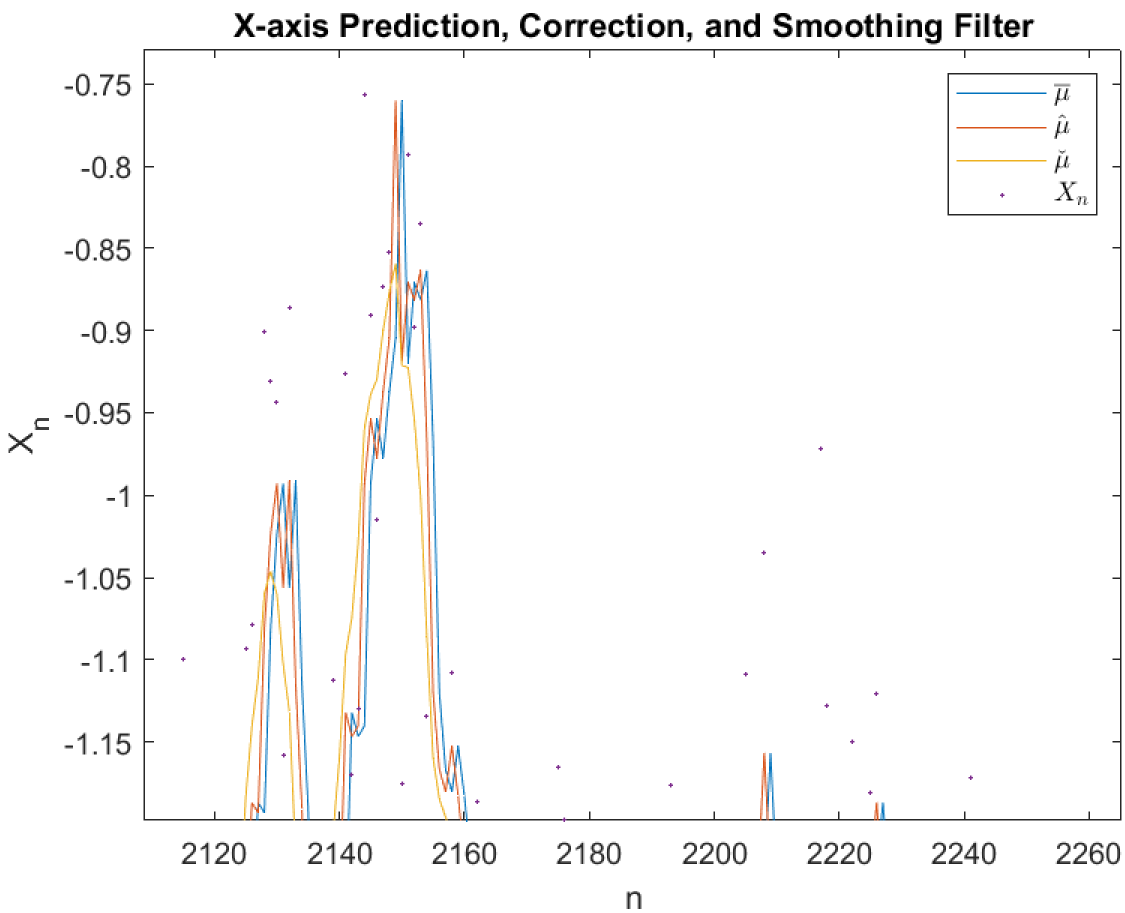

Figure 3: With this magnification, we can see our prediction mean $\overline{\mu}$, correction mean $\hat{\mu}$, and smoothed mean $\check{\mu}$ are all close to each other, but slightly different. We expect the accuracy of each of these to be in order of least accurate to most accurate estimate of the particle's true position simply by the data available when calculating each estimate.