M525: Final

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1. Here, we implement a Monte Carlo method for estimating a multidimensional Gaussian integral.

   (i) We, consider J random variables distributed as

   $$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}_i \sim \text{Normal}_4\left(\begin{bmatrix} -2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 & \frac{4}{5} & 0 & 0 \\ \frac{4}{5} & 5 & -1 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}\right) \quad i = 1, \ldots, J$$

   To sample points from $p(x, y, z, w)$, we first find the Cholesky decomposition of our covariance matrix S; this gives us a lower triangular matrix L such that $LL^\top = S$. Next, we generate J four dimensional points with components following the standard normal distribution. Next we multiply these points by our matrix L do give them the proper covariance. Lastly, we add our mean m to the points to properly translate them in space.

   (ii) We take $J = 10000$ and generate the data. See Figure 1.

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   (iv) Finally, we estimate the following integral using the method of Monte–Carlo integration:

   $$I = \int\int\int_A (x-y)^2 p(x, y, z)\, dx\, dy\, dz,$$

   where A is the interior of the ellipsoid defined by

   $$\frac{(x+3)^2}{2} + \frac{y^2}{2} + \frac{z^2}{9} \leqslant 1.$$

   To accomplish this, we take all the points generated in our Monte–Carlo algorithm that fall within A; assume K points satisfy this condition. Note that our covariance matrix S gives us that $p(x, y, z|w) = p(x, y, z)$ since the only entry for w is on the diagonal. Then we find

   $$I \approx \frac{1}{J}\sum_{k=1}^{K}(x_k - y_k)^2 = 1.3492.$$

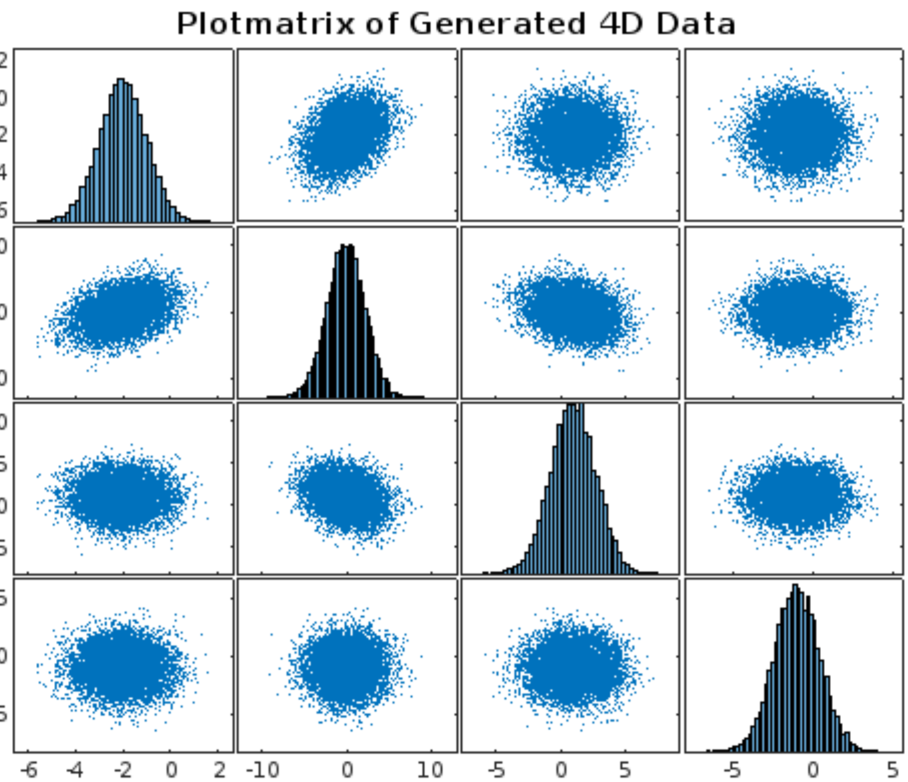

Figure 1: A visualization of 4 dimensional random data with distribution described in 1 (i).