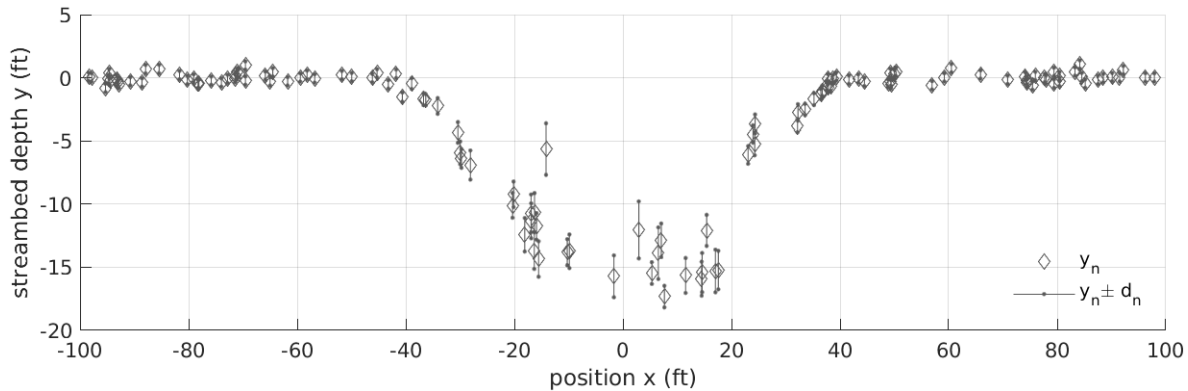# Assignment 1

Due: Monday, February 27, by 8:00PM

Problem: In this assignment, you will set up and solve a regression problem under a Gaussian process.

The dataset shown above provides measurements of the depth, y_n, of a water channel obtained at arbitrarily chosen positions, x_n, along its cross-section. Each depth measurement, y_n, is contaminated with additive normal noise of zero mean and standard deviation, d_n, which is shown by the error-bars.

1. Formulate a Bayesian regression model employing a Gaussian process to estimate the depth profile along the channel's cross-section. In the Gaussian processes prior make your own choices for the mean and co-variance and reason briefly on your selection.

2. Use Monte Carlo sampling to characterize your prior depth profiles and summarize the results graphically.

3. Use Monte Carlo sampling to characterize your posterior depth profiles and summarize the results graphically.

4. At each of the positions

$$x_A = -20, \qquad x_B = -10, \qquad x_C = 0, \qquad x_D = +10, \qquad x_E = +20 \text{ ft}$$

along the channel's cross-section, compute the posterior probability that the depth is less than 10 ft. Derive your results analytically and also using Monte Carlo sampling.

5. Compute the posterior probability that the depth is less than 10 ft in all positions x_A, x_B, x_C, x_D, x_E simultaneously. Derive your results analytically and also using Monte Carlo sampling.

Associated data: The dataset shown above is provided in `streambed_data.mat`. This dataset contains measurements of the depth, y, of a water channel at position, x, across its cross-section. In this dataset, d contains the standard deviation of each depth measurement. All quantities are reported in feet.