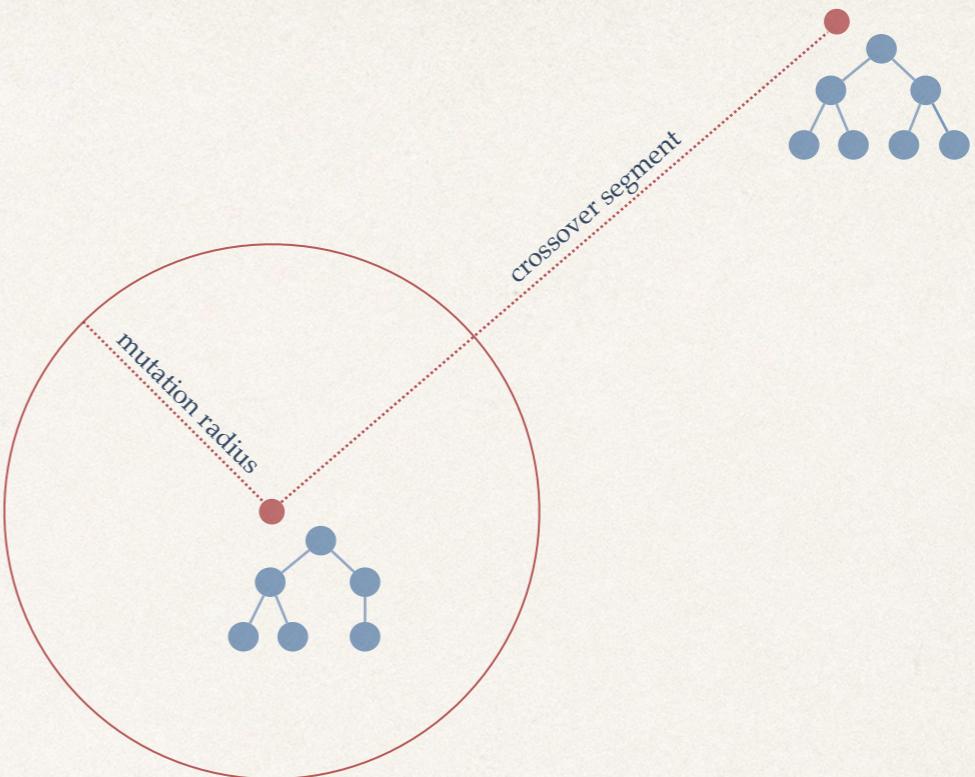


# Geometric Semantic Genetic Programming

Luca Manzoni

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# Geometric Operators

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Crossover and mutation  
as operations on a metric space

# Metric Spaces

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**Distance**

$$d : S \times S \rightarrow \mathbb{R}^+$$

Identity

$$d(x, y) = 0 \text{ iff } x = y$$

Symmetry

$$d(x, y) = d(y, x)$$

Triangular inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

**Segment**

$$S(x, y) = \{z \mid d(x, z) + d(z, y) = d(x, y)\}$$

**Ball of radius r**

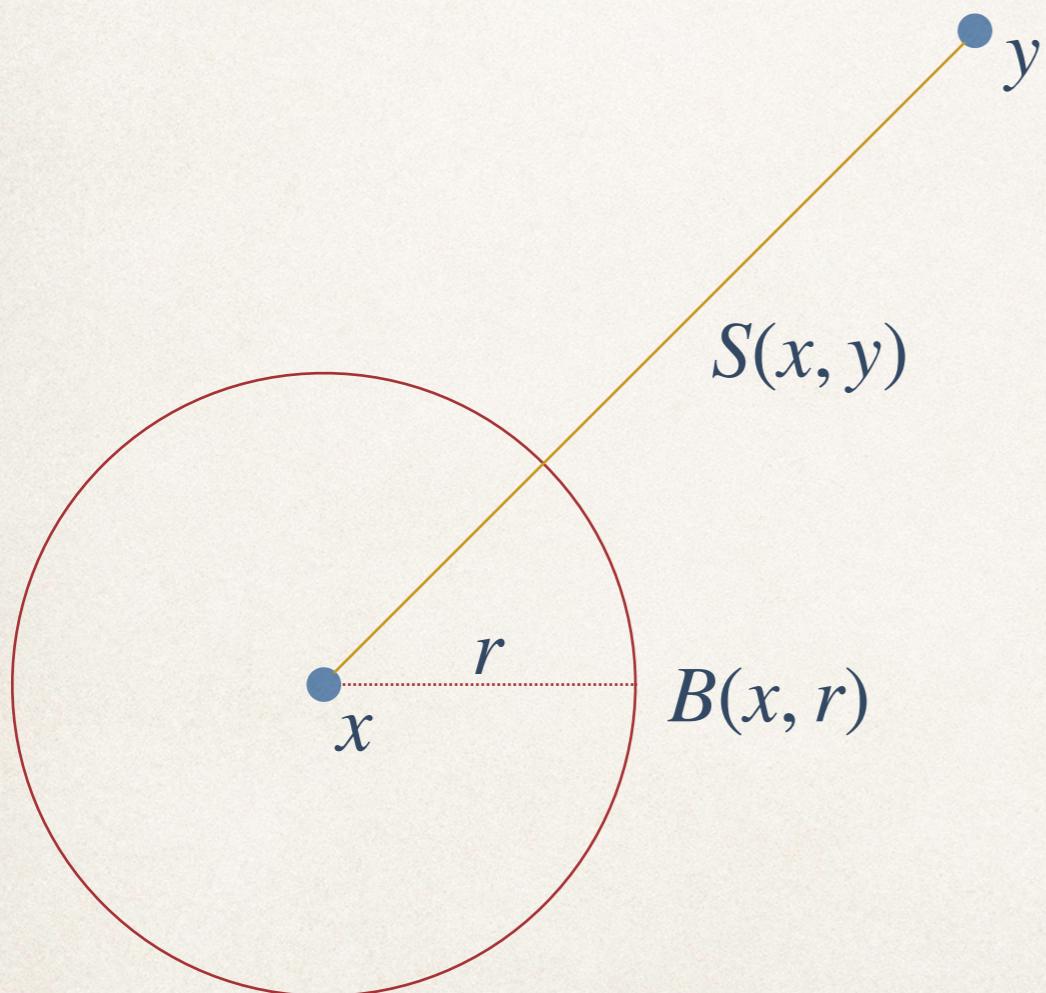
$$B(x, r) = \{y \mid d(x, y) \leq r\}$$

# Segments and Balls

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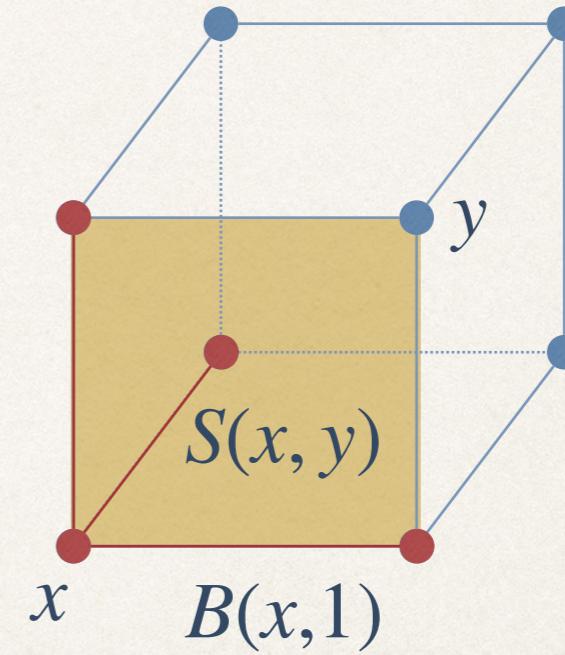
$\mathbb{R}^2$

Euclidean distance



$\{0,1\}^3$

Hamming distance



# Geometric Crossover

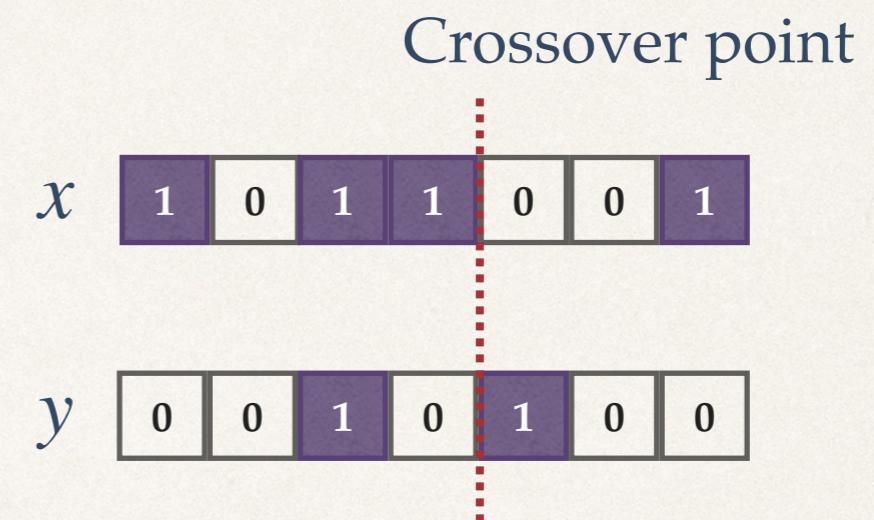
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## Geometric Crossover

For all  $x, y$

All possible results from crossover

Are inside  $S(x, y)$



# Geometric Crossover

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## Geometric Crossover

For all  $x, y$

All possible results from crossover

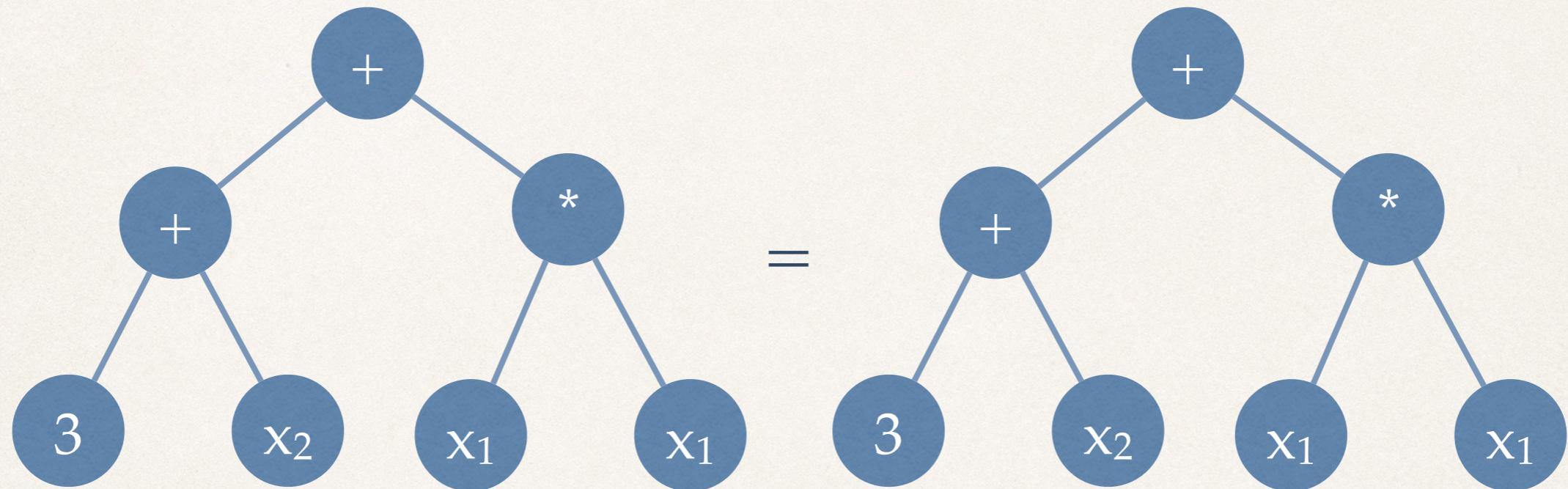
Are inside  $S(x, y)$

$$z_1 \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ \hline \end{array} \in S(x, y)$$

$$z_2 \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline \end{array} \in S(x, y)$$

# Are All Crossovers Geometric?

NO, NOT ALL CROSSOVER ARE GEOMETRIC

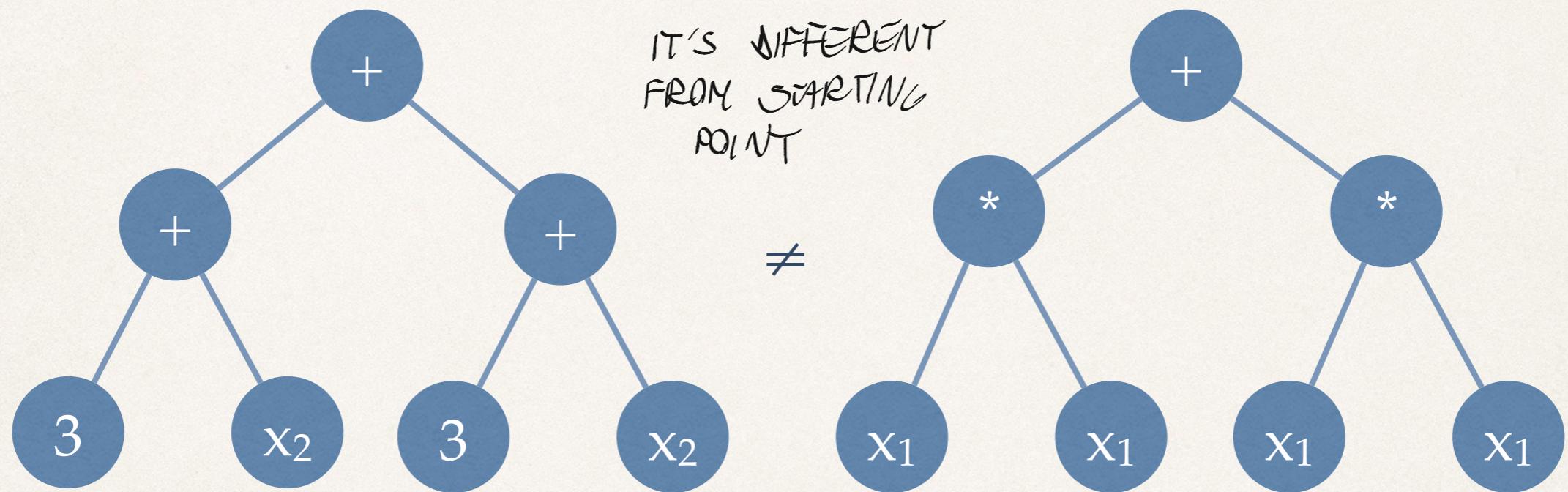


Crossover of a Tree with itself

$$S(x, x) = \{x\}$$

# Are All Crossovers Geometric?

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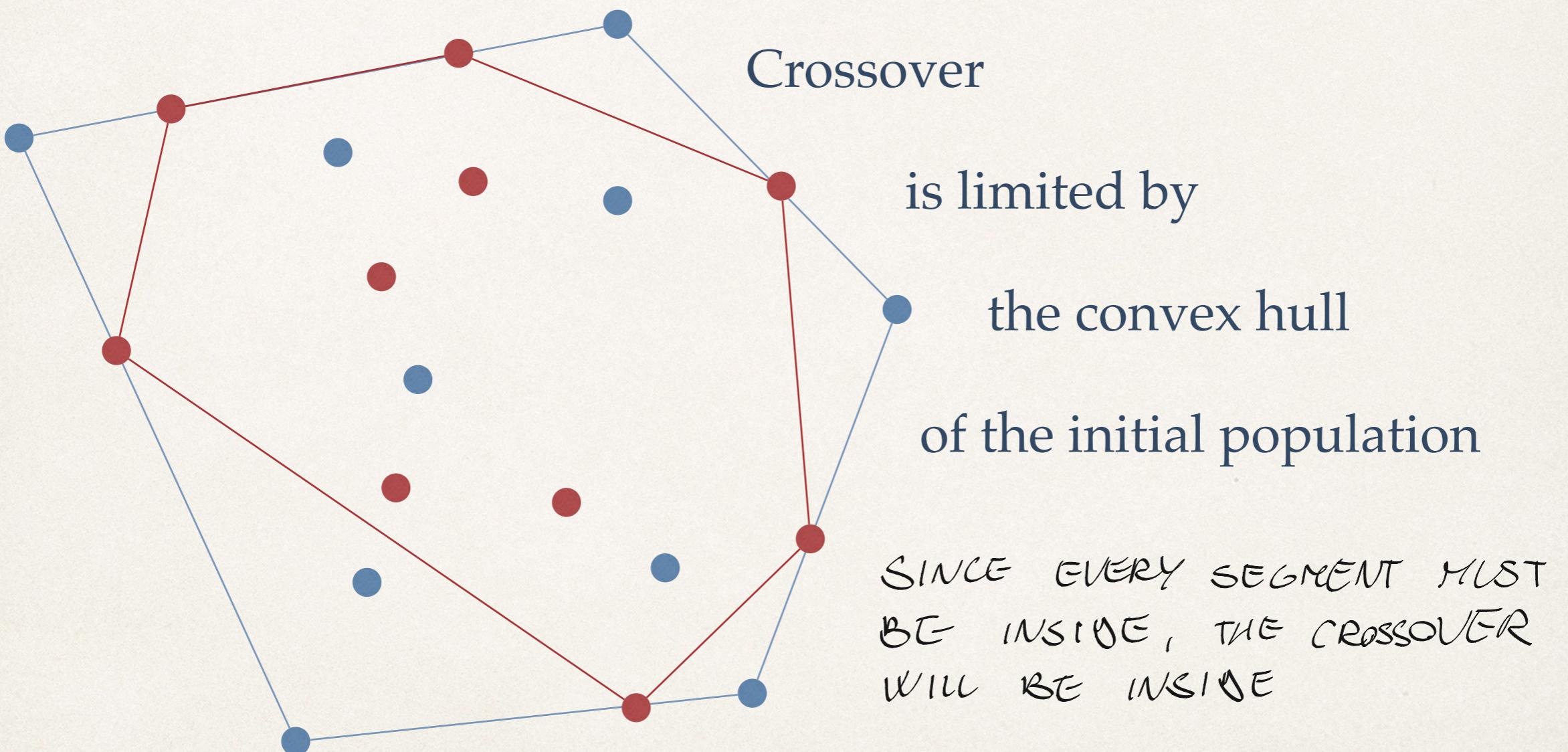
**Not a geometric crossover!**

IF IS NOT HOLOCAUS, IT CAN NOT

$$\hookrightarrow \text{crossover}(x/x) = x$$

# Consequences

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# Geometric Mutation

YOU NEED THIS ..

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Geometric Mutation  
(of radius r)

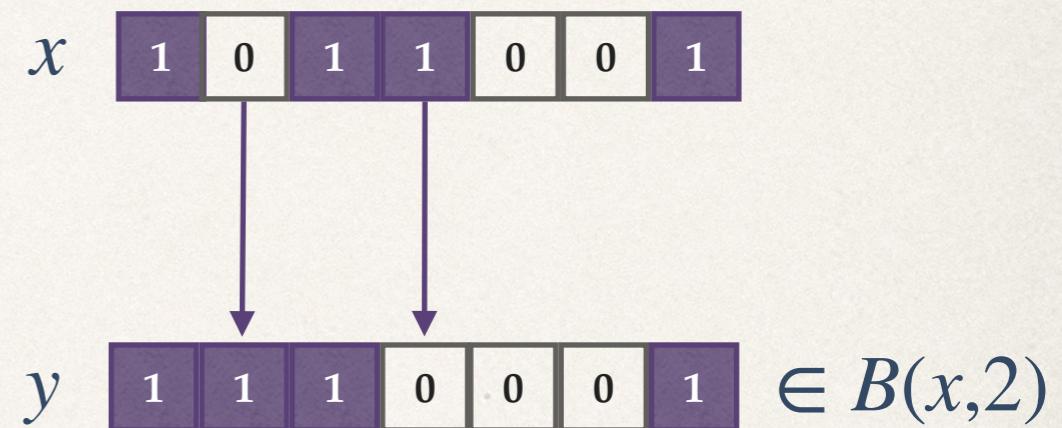
For all  $x$

All possible results from the mutation

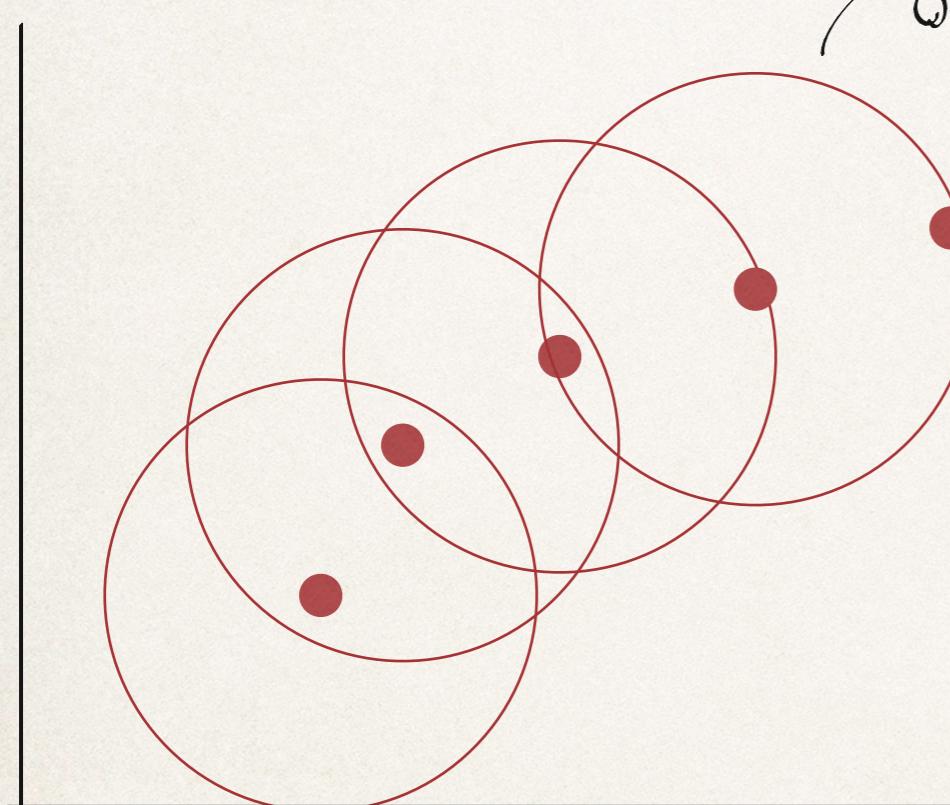
Are inside  $B(x, r)$

↪ BALL OF RADIUS  $r$

$$r = 2$$



# Mutation Can Take Us Everywhere\*



Exploration

Mutation enlarges the search space

Crossover restricts the search space

Exploitation

THE COMBINATION OF TWO ALLOW TO  
FIND THE OPTIMUM

\* Restrictions may apply for particulars spaces.

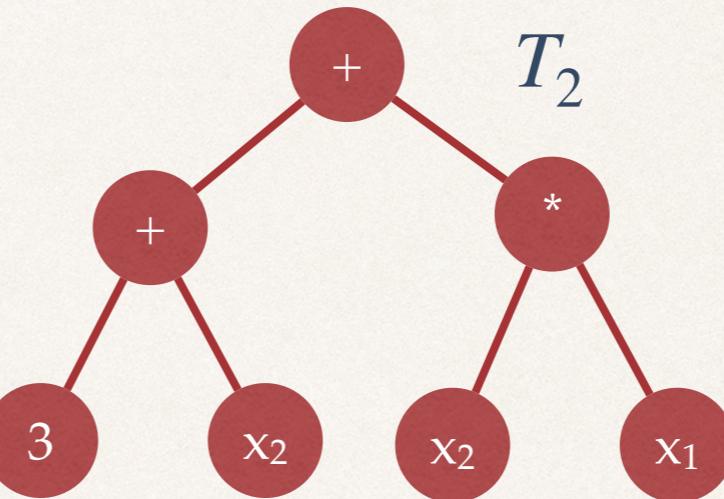
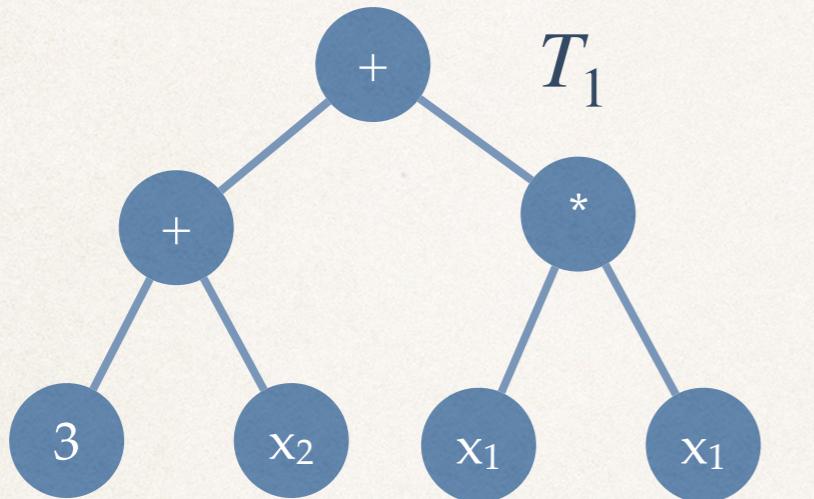
# Semantic Operators

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Controlling crossover and mutation effects  
on the *outputs* of an individual

# Syntax vs Semantics

BEFORE DEFINING WHAT SEMANTIC OPERATORS ARE :



Syntax

THIS IS JUST A  
REPRESENTATION

$x_1$	$x_2$	$T_1$	$T_2$
2	3	10	12
4	1	20	8
1	2	6	7

Semantic vectors

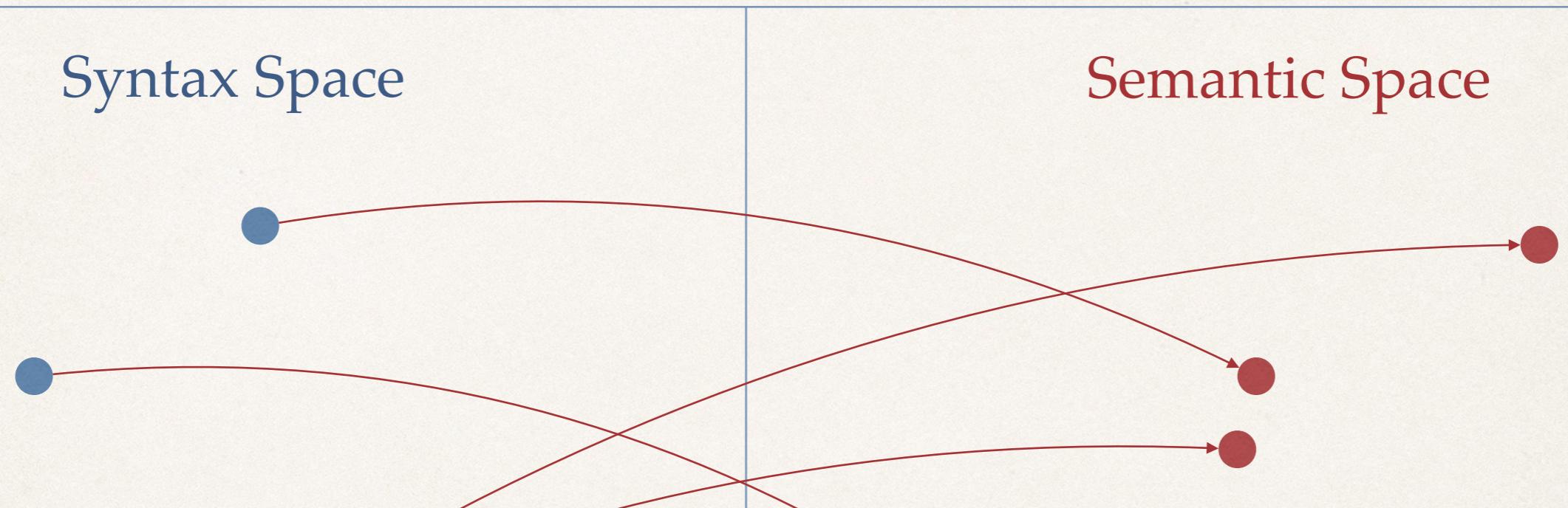
(10, 20, 6)

(12, 8, 7)

Semantics

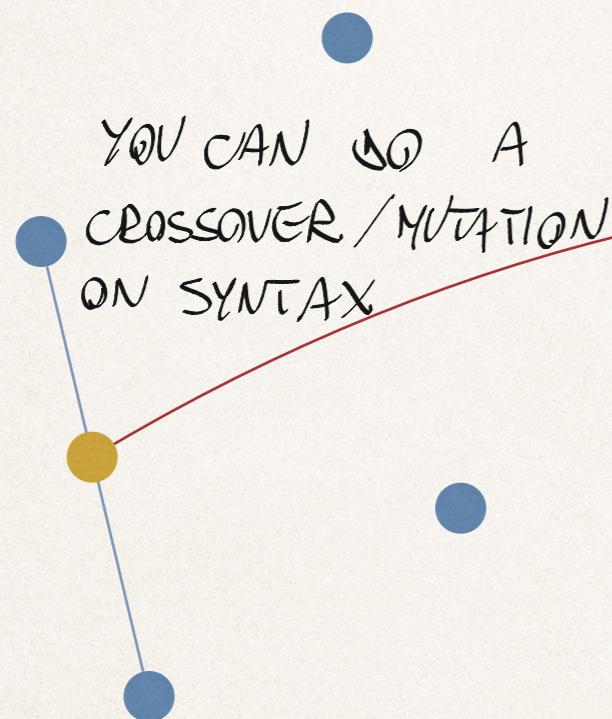
CAN I CHANGE IT

# Syntax vs Semantics



# Syntax vs Semantics

Syntax Space



Semantic Space

BUT YOU HAVE NO  
MFGA ON WHAT YOU  
GET

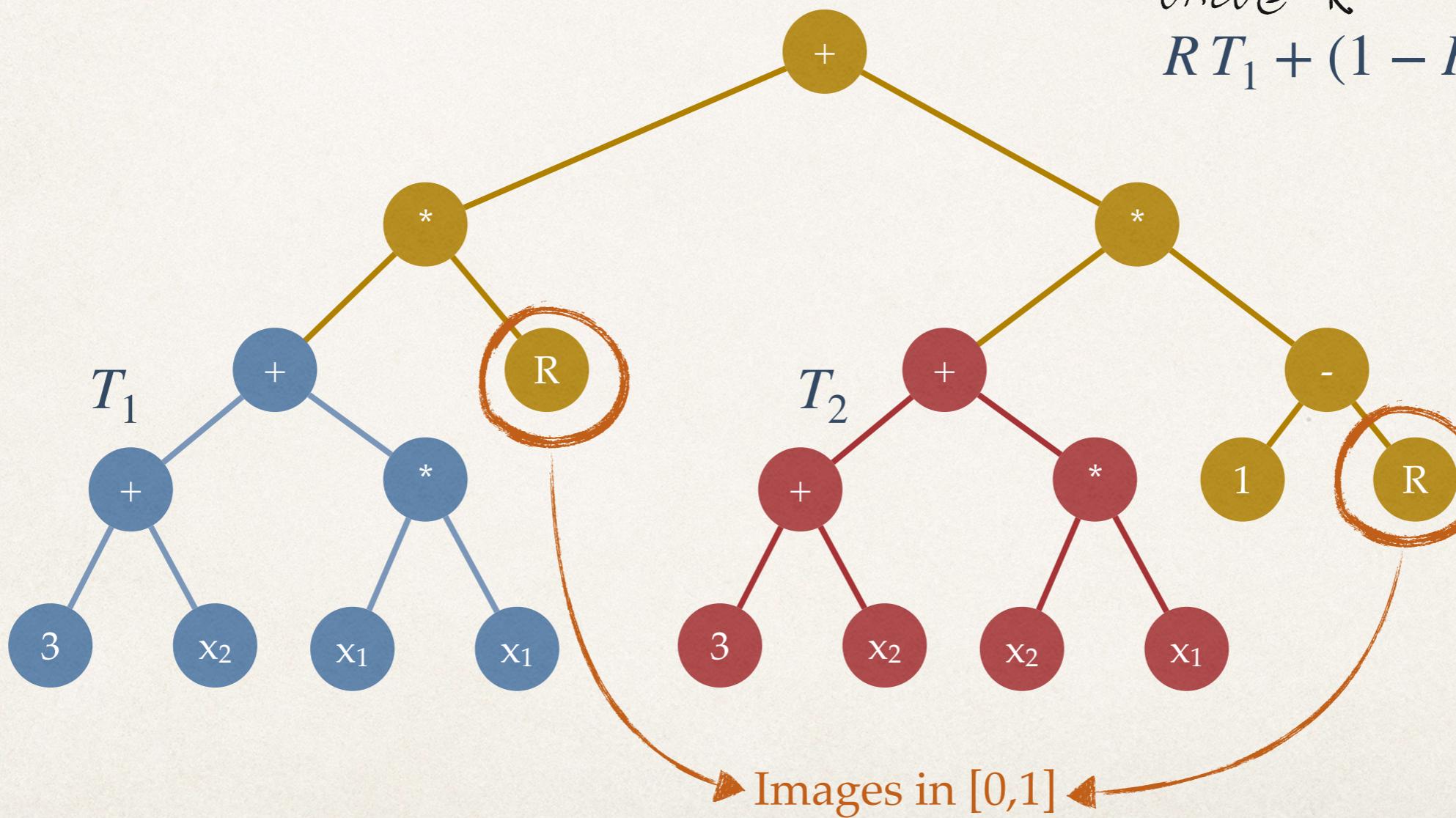


Can we make operators that are  
**geometric** in the **semantic** space?

YES, BUT IT'S A PROBLEM OF  
INEFFICIENCY

# Geometric Semantic Crossover

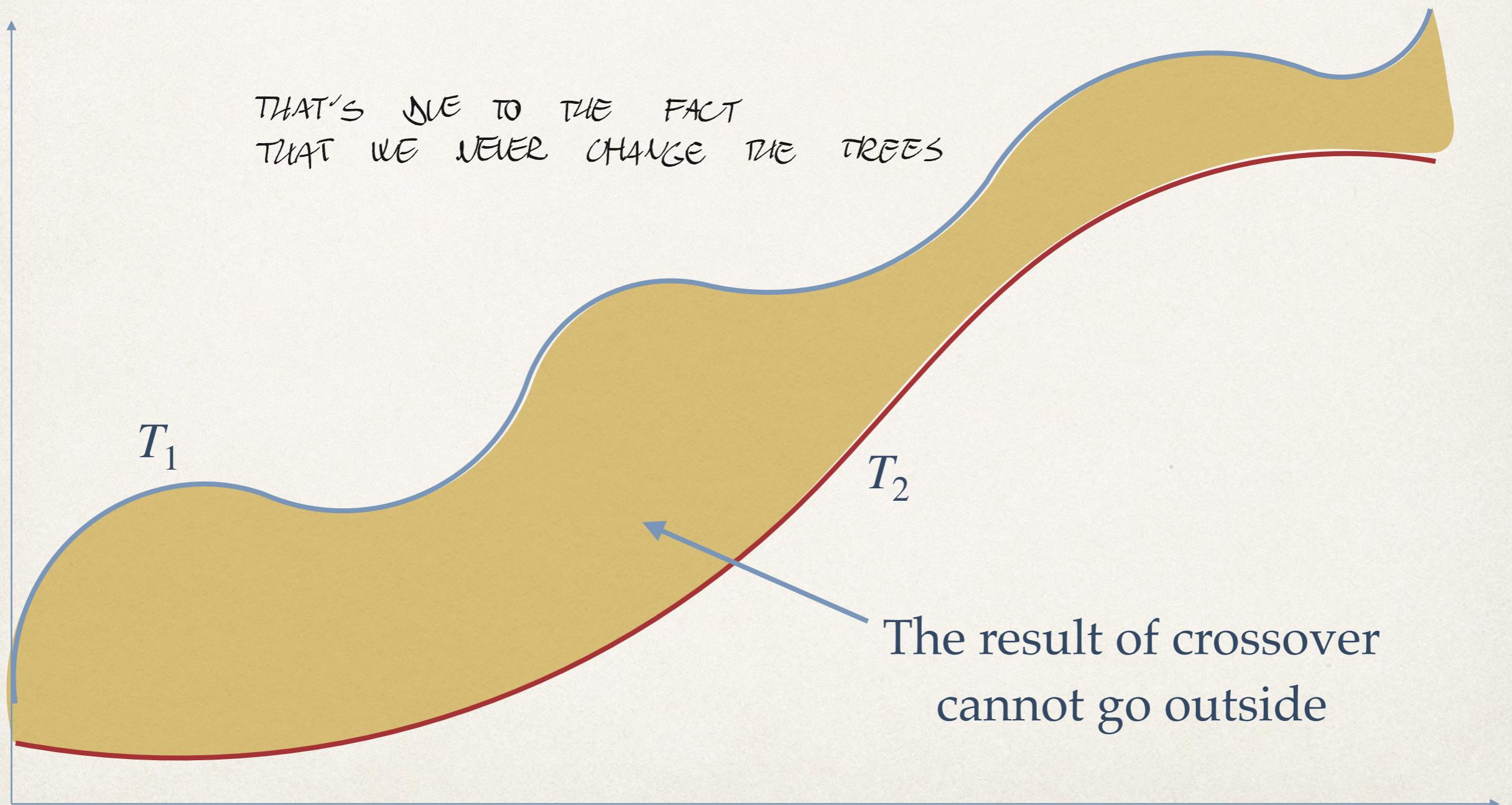
IT'S A WEAK COMBINATION OF THE TWO



I WEIGHT ONE TREE  
OR THE OTHER  
BASED ON SOME  
VALUE  $R$   
 $RT_1 + (1 - R)T_2$

# Effect on the Semantics

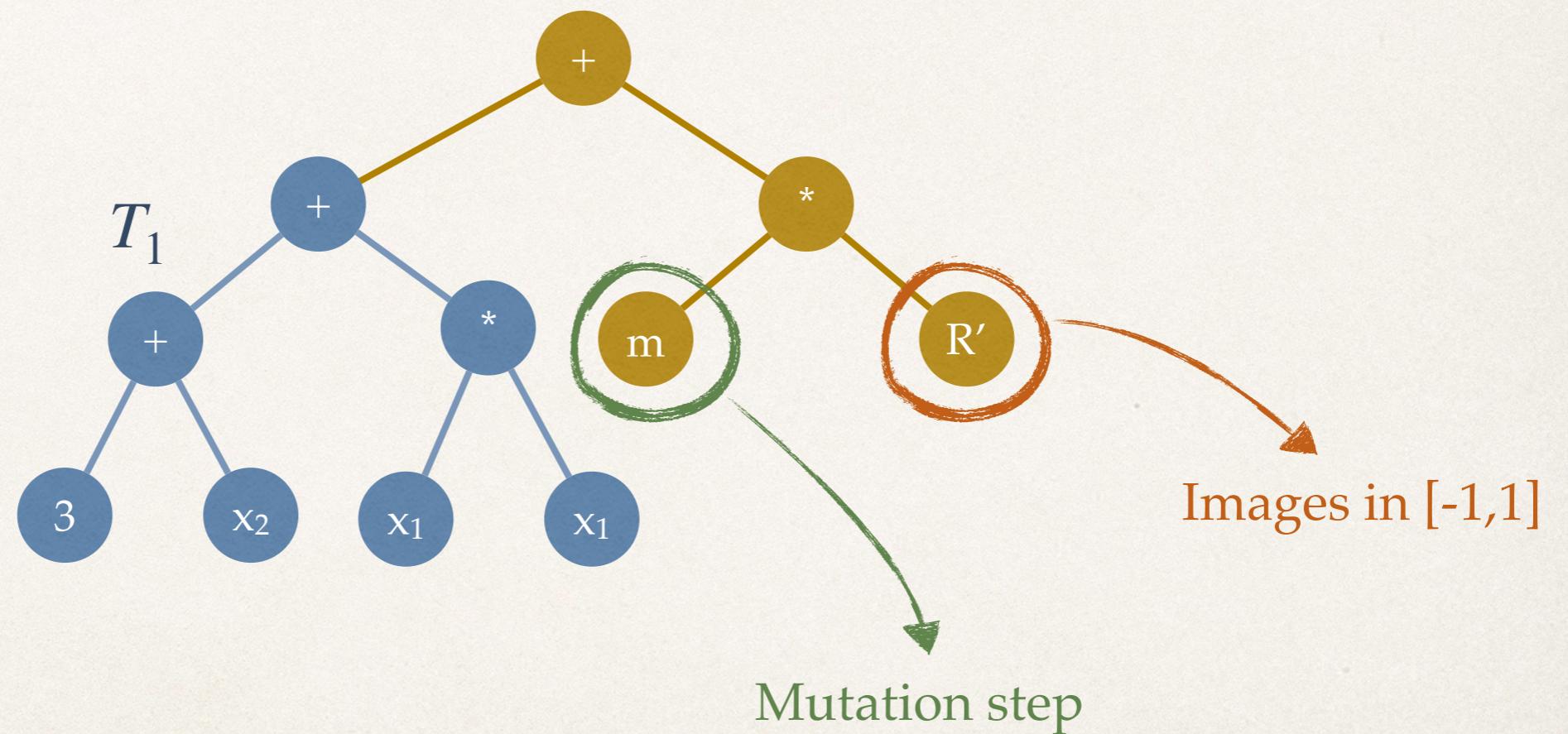
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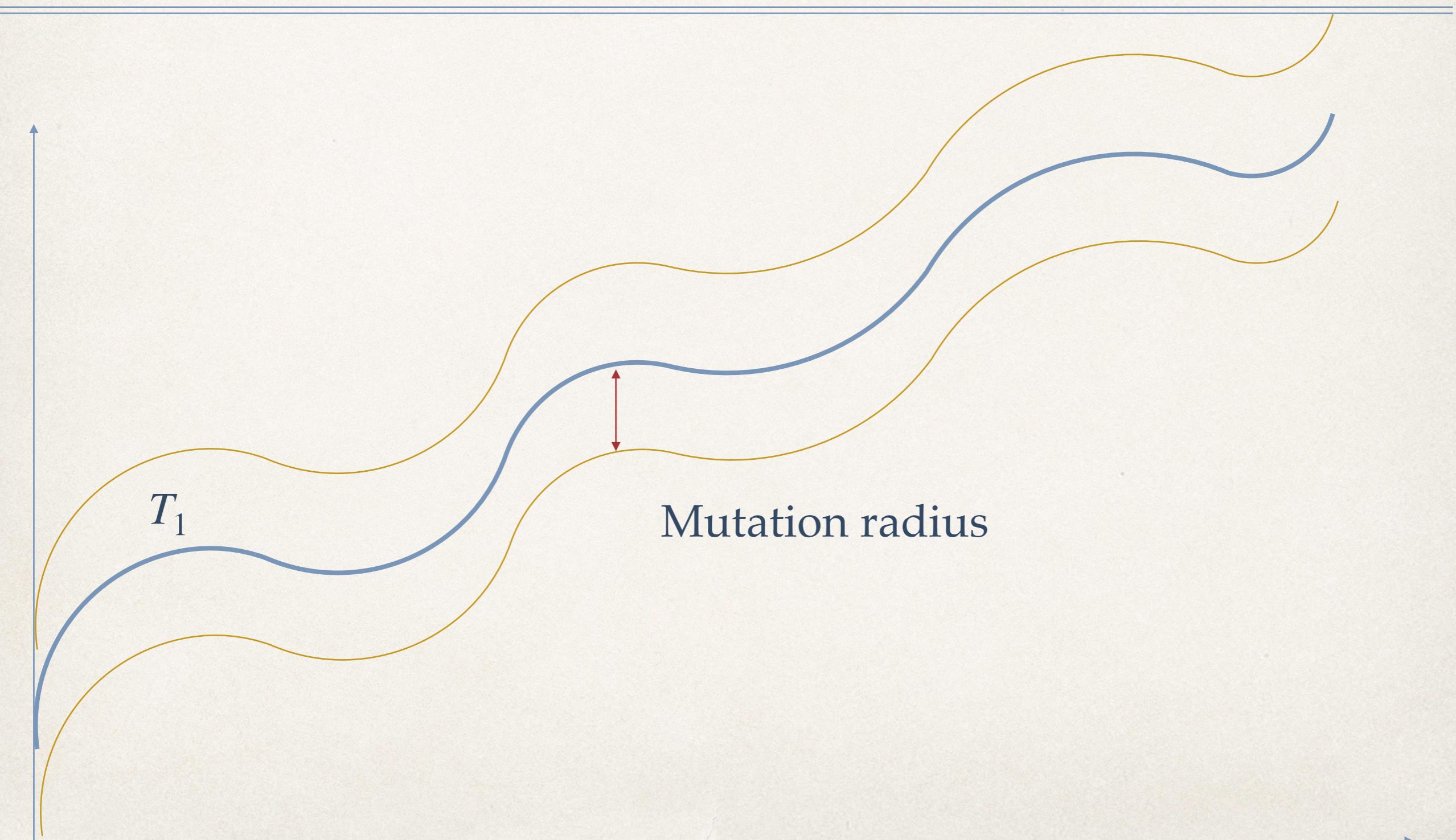
# Geometric Semantic Mutation

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$$T_1 + m R'$$



# Effect on the Semantics



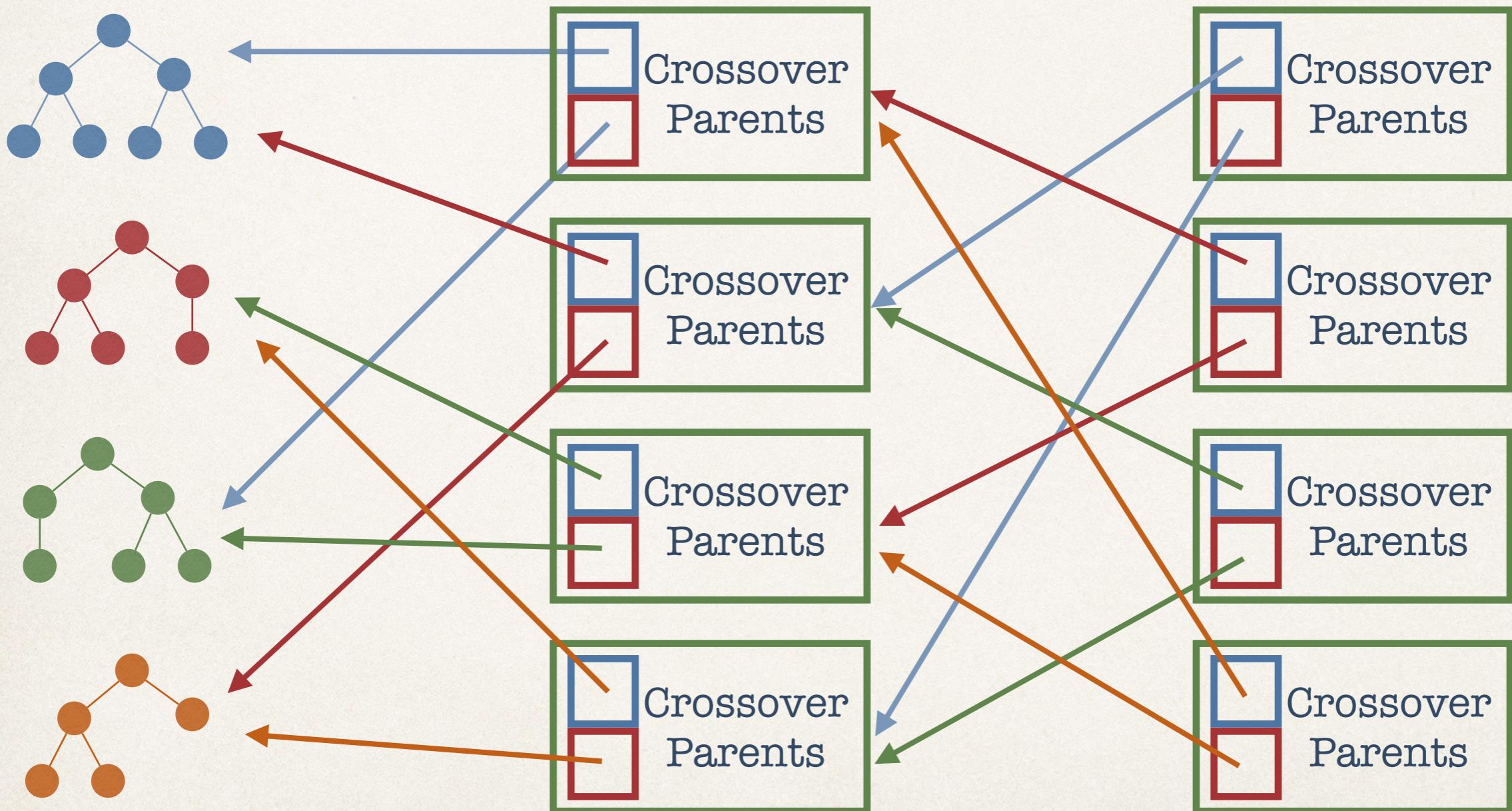
# Is this everything?

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- ❖ Different operators for different domains:  
Boolean, programs, etc.
- ❖ The tree-based representation is not the best one
  - ◆ Exponential growth wrt number of generations
  - ◆ Linear growth with subtree sharing.

# Fast GSGP

IS MORE OR LESS DYNAMIC PROGRAMMING



# An Example

