EXERCISE 1

$X_i \sim N(\mu, 4)$

$n = 50$

$\hat{x} = 105.2\ cm$

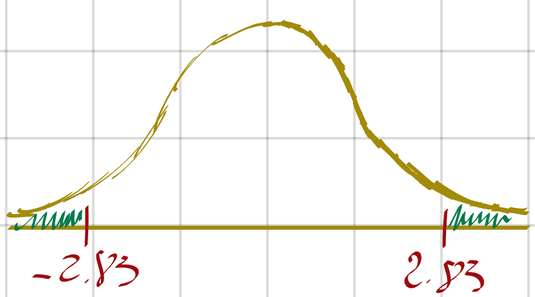
WE SET THE HYPOTESIS BEFORE LOOKING AT THE OBTAINED SAMPLE MEAN:

$\begin{cases} H_0: \mu = 106 \\ H_1: \mu \neq 106 \end{cases}$

REQUIRES A $\mu = 106\ cm$

WE SET $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ AND USE THE STD VERSION

WE COMPUTE P-VALUE : $Z_{|H_0} = \frac{\hat{x} - 106}{\sqrt{4/50}} = -2.83$



THE NEXT STEP FOR P-VALUE:

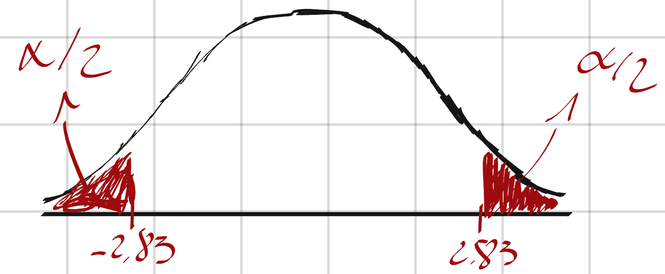
$2(1 - \Phi(2.83)) = 0.005$

2(1 - QNORM(2.83))

WE COMPUTE NOW THE REJECTION REGION FOR TWO SIDED HYPOTESIS :

$$R_\alpha = \{x : |Z| > Z_{1-\alpha/2}\}$$

$$= \{x : Z < -Z_{1-\alpha/2} \cup Z > Z_{1-\alpha/2}\}$$



WE SET $\alpha = 0.001 \Rightarrow Z_{0.9995} = 3.29$

$$= \{x : Z < -3.29 \cup Z > 3.29\}$$

SO : $\bar{X} = 105.2$ AND $Z_{OBS} = -2.89$ WE DON'T REJECT H_0 WITH $\alpha = 0.001$,

FOR $\alpha = 0.005 \Rightarrow Z_{0.9975} = 2.81 \Rightarrow \{x : Z < -2.81 \cup Z > 2.81\}$

WE REJECT THE NULL HYPOTHESIS FOR $\alpha = 0.005$

THE CONFIDENCE INTERVAL : $1 - 0.005 \Rightarrow \left[105.2 \pm 2.81 \cdot \frac{2}{\sqrt{60}}\right]$

$= [104,92 ; 105,48]$