The Rectified-Leaking Competing Accumulator is More Plausible Than Multivariate Decision Field Theory for Explaining Decision-Making

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Word Count: 2195

**Abstract**

Real-life decision making has been found to violate the rationality axiom positing that the preferences of two options will not be affected by an independent third option. Therefore, recent cognitive models such as Multivariate Decision Field Theory (MDFT) and rectified and unrectified Leaking Competing Accumulator models (LCA) have been proposed to account for the three choice preference effects that violate the axiom: the similarity, attraction, and compromise effects. This study simulated similarity, five-choice compromise and dominance reversal effects to test the plausibility of these models. Simulations in unrectified-LCA fail on the similarity effect, while rectified-LCA is more plausible than MDFT in explaining the five-choice compromise effect and successfully avoiding dominance reversal. Overall, the results indicate that rectified-LCA is more plausible, suggesting that loss aversion might be the mechanism underlying decision-making.

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How individuals make decisions in multi-alternative and multi-attribute situations has long concerned psychologists. Traditional rationality theories have suggested that decision making is based on rationality axioms, such that a subject’s preferences between two options will not be affected by adding an independent third option. However, studies show that real-life choices violate this rationality axiom (Huber et al., 1982; Sjoberg, 1977; Tversky, 1972), suggesting that more comprehensive models are needed to account for irrational decision-making behaviours.

Multivariate Decision Field Theory (MDFT) and rectified and unrectified Leaking Competing Accumulator (LCA) have been proposed to explain three basic choice preference effects violating rationality axioms: the attraction effect, compromise effect and similarity effects (Huber et al., 1982; Sjoberg, 1977; Tversky, 1972). These effects are shown in Figure 1, which refers to a selection between two cars A and B that vary in quality and economy and are originally equally preferred. An attraction effect occurs when a decoy C – an option objectively worse than both A and B but is more similar to B - is introduced, making B preferred over A. In the compromise effect, compromise option D, the immediate of A and B, is introduced, D emerges as more preferable than both A and B. Finally, a similarity effect occurs when option E, equally valuable to A and B but is more similar to A, is added, making B more preferable than A.

**Figure 1**

*Options Represented in a Two-dimensional Space to Demonstrate Choice Preference Effects*

Chart, scatter chart

Description automatically generated

*Note*. The values of two attributes for each option on the dotted diagonal line sum up to 4, indicating that these options are objectively equally valuable. Option C is below the diagonal line with a summed value lower than 4, suggesting it is objectively worse than other options.

Although both MDFT and rectified-LCA were shown to successfully address these three effects, they also have limitations in explaining choice preferences in certain circumstances (Hotaling et al., 2010; Tsetsos et al., 2010). It is therefore important to further examine the strengths and limitations of these models to determine how prospective models can be constructed to account for and reveal the mechanisms underlying decision-making in multi-alternative situations.

**Cognitive Models of Decision-Making**

**MDFT**

The MDFT can be viewed as a four-layer neural network model (Figure 2; Usher & McClelland, 2004). The subject stochastically shifts attention between the two attributes (attention weight matrix; *W*(*t*)), therefore, only the values for the attended attribute are registered at time t (*ME* for economy and *MQ*for quality). To determine the relative advantage of each option, the average value of all other alternatives is subtracted from an option’s own value (*M* is multiplied the contrast matrix C). As other external factors (e.g., weather, mood) may also affect decision making, a noise term is further added. Thus, the valence (i.e., the relative advantage or disadvantage based on the momentary comparison) for each option at time t is calculated as:

|  |  |  |
| --- | --- | --- |
|  | *V*(*t*) = *CMW*(t) + *e*(*t*) | (1) |

**Figure 2**

*Connectionist Network Diagram for MDFT*

Diagram

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*Note*. The subject is choosing between three cars (i.e., A, B, C) that vary in their quality and economy. The value matrix M represents the subjective value of each option on each attribute. The attention weight matrix *W*(*t*) = [*W*E(*t*) *W*Q(*t*)]’ indicates which attribute that attention is allocated to. VA, VB and VC represent the valence matrices for each option. The *e*(*t*) represents the normally distributed random error. S is the feedback matrix. PA, PB and PC are the preference matrices for each option.

Following this, a 3-by-3 feedback matrix S is introduced, with the diagonal values indicating the memory strength for each item’s preference (e.g., a diagonal coefficient of 1 indicates that the subject at t+1 has perfect memory of the option’s state at time t). In contrast, the off-diagonal values in S correspond to the strength of inhibition between different alternatives. The off-diagonal values (i.e., the strength of inhibitory connections) are determined based on distance-dependent functions in which the further apart the two options, the stronger their inhibition to each other. Overall, a preference vector *P*(*t*) which indicates the summed advantage or disadvantage of each option is computed by:

|  |  |  |
| --- | --- | --- |
|  | *P*(*t*+1) = *S\*P*(t) + *V*(*t*+1) | (2) |

**LCA**

LCA (Figure 3) has the same assumption of stochastic attentional shift as MDFT and, similarly, only the value for the attended attribute of each option is registered at time t. However, in LCA the relative advantage of each option is computed based on the pairwise differences of all options. LCA is distinguished by its intrinsic mechanism of loss aversion, in which losses are weighted more heavily than gains. This is achieved by transforming *d*ij through a non-symmetrical loss-aversion function which has a lower slope for gains (equation 3) and a higher slope for losses (equation 4). A positive constant I0will also be added to the transformed input value, as the input will otherwise always be negative (equation 5). The input will then be sent into the leaky, competing accumulators to compute the activation values for each alternative (equation 6, with a neural decay constant λ representing the decay in memory throughout time, a normally distributed noise term ξ, a global inhibition parameter β). Only activations greater than or equal to 0 will be propagated to interact with other options in rectified-LCA (as shown by the + sign).

**Figure 3**

*Connectionist Network Diagram for LCA*

Diagram

Description automatically generated

*Note*. *d*ij represents the pairwise difference in the value of option i and j (e.g., d*12* is the difference between option 1 and 2). *A*1, *A*2 and *A*3 are the activations for each alternative at time t+1.

|  |  |  |
| --- | --- | --- |
|  | *V*(*x*) = *z*(*x*), *d*ij > 0 | (3) |
|  | *V*(*x*) = - {*z*(|*x*|) + [*z*(|*x*|)]2}, *d*ij < 0 | (4) |
|  | *I*1 = *V*(*d*12) + *V*(*d*13) + *I*0 | (5) |
|  | *A*i(*t* + 1) = [λ*A*i(t) + (1 - λ)[*I*i(*t*) – βΣ*A*j(t) + ξ ⋅ (*t*)]]+. | (6) |

The unrectified-LCA is identical to rectified-LCA except for the absence of half-wave rectification, meaning that it allows for activations with all values, including negative activations, to be propagated, (equation 7).

|  |  |  |
| --- | --- | --- |
|  | *A*i(*t* + 1) = λ*A*i(t) + (1 - λ)[*I*i(*t*) – βΣ*A*j(*t*) + ξ ⋅ (*t*)]. | (7) |

**Simulations**

To examine their plausibility, models were simulated for similarity, five-choice compromise, and dominance reversal effects. Greater focus was placed on the latter two effects, which have been controversial in previous literature. Default parameter values were used for the MDFT (Roe et al., 2001) and LCA (Usher & McClelland, 2004) simulations, unless otherwise specified.

***Similarity Effect***

Both MDFT and rectified-LCA could explain the similarity effect through vote splitting (Roe et al., 2001; Usher & McClelland, 2004), as more similar options A and C share votes when attention is shifted to quality, making B preferable to A or C. However, unrectified-LCA fails to account for the similarity effect (Figure 4) due to the loss-aversion mechanism penalising B, which was more distant than the other alternatives. Lack of rectification allows activities to be lower than zero, therefore, option B is strongly suppressed and has negative activation when attention is shifted to quality, making its preference overly low even when attention is shifted back to economy. As unrectified-LCA cannot explain similarity effect - an important effect that cognitive models were originally proposed to account for - it is considered not plausible. Therefore, the following discussions focus on rectified-LCA and MDFT.

**Figure 4**

*Simulation of Unrectified-LCA for the Similarity Effect*

Graphical user interface

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*Note*. A = [0.25, 0.75], B = [0.75, 0.25], C = [0.2, 0.8].

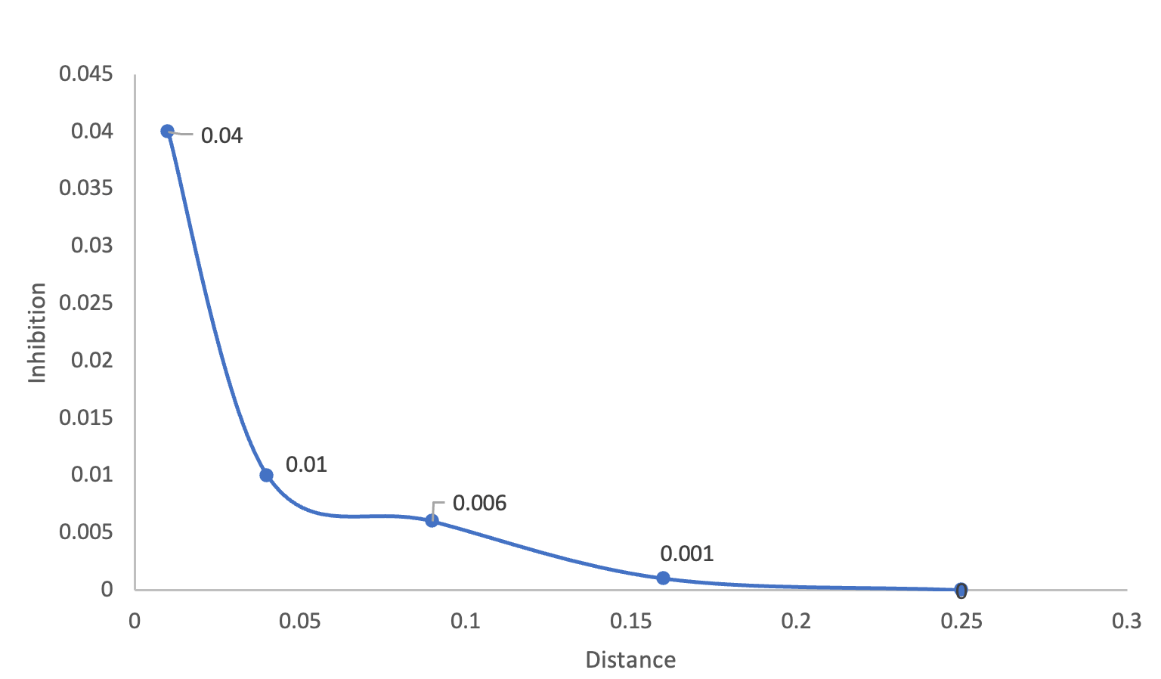
***Five-choice Compromise Effect***

A decision-making effect that remains controversial when comparing rectified-LCA and MDFT is the five-choice compromise effect, in which two extreme alternatives equally valuable to all other options are introduced to an original three-choice compromise effect (Hotaling et al., 2010; Tsetsos et al., 2010). Recent research consistently finds robust compromise effects in five-choice, nine-choice and fifteen-choice conditions (Fischer, 2016; Park et al., 2021; Yoo et al., 2018). Moreover, increasing the extremeness of the alternatives has been found to strengthen the compromise effect (Yoo et al., 2018), consistent with Simon and Tversky’s (1992) loss aversion account that consumers weigh losses more than gains, putting more extreme options at a disadvantage when compared to compromise options. Thus, a model that successfully simulates the five-choice compromise effect needs to satisfy two requirements: 1) the immediate option is preferred over more extreme alternatives; and 2) preference for the immediate option increases as alternatives become more extreme.

**MDFT*.*** MDFT explains the compromise effect through a decorrelation mechanism and noises. In a three-choice situation, the compromise option is nearer to and thus more strongly inhibits the two extreme alternatives, making these extreme alternatives correlate with each other but decorrelate with the compromise. Thus, the extreme alternatives share votes, becoming less preferred than the compromise. Although Tsetsos et al. (2010) suggest that this decorrelation mechanism fails in the five-choice condition, the current simulation could obtain a five-choice compromise effect using MDFT (Figure 6) by deriving and using a new distance-dependent function (Figure 5).

**Figure 5**

*An exponential-distribution-like distance-dependent function*



*Note*. This distance function was developed based on Tsetsos et al.’s (2010) finding that the inhibition needs to be high when two options are extremely close but reduces abruptly when options become further apart. For X = [a, b] and Y = [c, d], the distance between X and Y is calculated as [(a-c)2 + (b-d)2]1/2.

**Figure 6**

*Simulation of MDFT for the Five-choice Compromise Effect*

Chart

Description automatically generated

*Note*. A = [1.8, 2.2], B = [1.9, 2.1], C = [2, 2], D = [2.1, 1.9], E = [2.2, 1.8], S = [0.94 -0.04 -0.01 -0.006 -0.001; -0.04 0.94 -0.04 -0.01 -0.006; -0.01 -0.04 0.94 -0.04 -0.01; -0.001 -0.01 -0.04 0.94 -0.04; -0.001 -0.006 -0.01 -0.04 0.94]. The distance function in Figure 5 was used.

However, when increasing the distances among options, MDFT can no longer produce the five-choice compromise effect and compromise C becomes the least preferred option (Figure 7). Because MDFT assumes that the inhibitions decrease as options recede, the decorrelation mechanism no longer works when options are far apart and only weakly or minimally inhibit each other. Therefore, the magnitude of the compromise effect decreases as the extremeness of alternative options increases in MDFT. This violates the second requirement of a successful five-choice compromise effect, meaning that MDFT cannot adequately account for the five-choice compromise effect.

**Figure 7**

*Simulation of MDFT for the Five-choice Compromise Effect with Increased Extremeness*

Chart

Description automatically generated

*Note*. A = [1.6, 2.4], B = [1.8, 2.2], C = [2, 2], D = [2.2, 1.8], E = [2.4, 1.6], S = [0.94 -0.01 -0.001 -0.001 -0.001; -0.01 0.94 -0.01 -0.001 -0.001; -0.001 -0.01 0.94 -0.01 -0.001; -0.001 -0.001 -0.01 0.94 -0.01; -0.001 -0.001 -0.001 -0.01 0.94]. The distance function in Figure 5 was used.

**Rectified-LCA*.*** The rectified-LCA model with default parameters (Usher & McClelland, 2004) cannot explain the five-choice compromise effect (Figure 8). Due to the loss aversion assumption of LCA that inhibitions or penalisations increase as distances between options increase, extreme alternatives are more strongly inhibited than the compromise option. The rectification therefore provides extra boosts for extreme options, which have more negative activations by specifying the lower bound of the activation to be 0, making extreme alternatives more preferred.

**Figure 8**

*Simulation of Rectified-LCA for the Five-choice Compromise Effect*

Chart

Description automatically generated

*Note*. A = [0.25, 0.75], B = [0.375, 0.625], C = [0.5, 0.5], D = [0.625, 0.375], E = [0.75, 0.25].

The failure of rectified-LCA in explaining the five-choice compromise effect due to strong distance-dependent penalisation can be compensated by increasing the positive constant I0, making the inputs of all options higher and slightly weakening the effects of initial distance-based loss aversion. After increasing I0, rectified-LCA can successfully produce the five-choice compromise effect, with compromise C clearly the most preferred (Figure 9) and best fitting the empirical data.

**Figure 9**

*Simulation of Rectified-LCA for the Five-choice Compromise Effect with Increased I0*

Chart, scatter chart

Description automatically generated

*Note*. I0 = 2, A = [0.25, 0.75], B = [0.375, 0.625], C = [0.5, 0.5], D = [0.625, 0.375], E = [0.75, 0.25].

Furthermore, when increasing the extremeness of the alternatives, the magnitude of the compromise effect increases in the rectified-LCA (Figure 10). These results are highly consistent with the theoretical account and evidence regarding the compromise effect in real-life decision making (Park et al., 2021; Yoo et al., 2018), supporting the robustness of rectified-LCA in explaining compromise effect with different sizes of choice sets and levels of alternative extremeness. That said, it is notable that increasing I0 in rectified-LCA may weaken other effects that rely on loss aversion, such as the similarity effect. Thus, whether it is justifiable to increase I0 only in the five-choice condition may be a focus for future studies. Despite this limitation of rectified-LCA, this essay argues that the rectified-LCA is more plausible than MDFT in explaining the five-choice compromise effect, as the intrinsic mechanism of MDFT violates the property of compromise effect as discussed beforehand.

**Figure 10**

*Simulation of Rectified-LCA for the Five-choice Compromise Effect with Increased I0 and Distances Among Options*

Chart

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*Note*. I0 = 2, A = [0.15, 0.85], B = [0.325, 0.675], C = [0.5, 0.5], D = [0.675, 0.325], E = [0.85, 0.15].

***Dominance Reversal***

When one of the options has higher values on all attributes as compared to other alternatives (i.e., C in Figure 11), it is expected that every decision maker will choose dominant option C.

**Figure 11**

*Example of a Dominant Option in a Two-dimensional Space*

Chart, scatter chart

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**MDFT*.*** Tsetsos et al. (2010) suggest that MDFT’s negated inhibition mechanism (i.e., the negative activation of an option that can boost neighbouring options) can cause dominance reversal. For example, B may be preferred over C in some simulations due to the strong boosts B receives from close, inferior option A, which has a negative preference. However, Tsetsos et al. (2010) use unproportionally strong inhibition values between A and B in simulations, and it is unclear whether dominance reversal will still exist when more reasonable, smaller inhibitions values are used. Therefore, the current simulation uses the newly developed distance-dependent function (Figure 7) that specifies a maximum inhibition value of 0.04 to further explore whether dominance reversal will occur. Simulation results confirm the occurrence of dominance reversal (Figure 12), even when smaller inhibition values between A and B are used.

**Figure 12**

*Simulation of MDFT for Dominance Reversal*

Graphical user interface

Description automatically generated with medium confidence

*Note*. A = [0, 0], B = [0.1, 0.1], C = [0.4, 0.4], S = [0.94 -0.04 -0.001; -0.04 0.94 -0.001; -0.001 -0.001 0.94].

Hotaling et al. (2010) propose a distance function to address dominance reversal, specifying that inhibitions are more sensitive to changes in distances that vary on an indifference direction (i.e., the negative slope in the economy– quality space), as compared to dominance direction (i.e., the positive slope in the economy-quality space), since people may not pay much attention to clearly dominated options. Therefore, this study further implements Hotaling et al.’s (2010) distance function (two functions, due to inconsistencies in their paper) using the parameters provided in example 1 (p.1295) to examine whether dominance reversal still occurs. The results displayed in Figures 13 and 14 show that there is still dominance reversal, meaning that the simulations suggest that MDFT is implausible, as it cannot avoid the dominance reversal effect.

**Figure 13**

*Simulation of MDFT for Dominance Reversal Using Hotaling et al.’s (2010) function with* **√**2

Chart

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*Note*. A = [0, 0], B = [0.1, 0.1], C = [0.4, 0.4], S = [0.94 -0.1 -0.0903; -0.1 0.94 -0.0968; -0.0903 -0.0968 0.94]. As there was an inconsistency in Hotaling et al.’s (2010) paper regarding whether distances along each direction should be divided by **√**2, this study examined two distance functions. The distance function used in this simulation divided the distance by **√**2.

**Figure 14**

*Simulation of MDFT for Dominance Reversal Using Hotaling et al.’s (2010) function without* **√**2

A picture containing chart

Description automatically generated

*Note*. A = [0, 0], B = [0.1, 0.1], C = [0.4, 0.4], S = [0.94 -0.0998 -0.0664; -0.0998 0.94 -0.0878; -0.0664 -0.0878 0.94]. This simulation used Hotaling et al.’s (2010) distance function simulation without dividing the distance by **√**2.

Notably, current simulations were only based on one set of parameters, therefore, using other parameters for Hotaling et al.’s (2010) function may possibly avoid dominance reversal. However, to avoid dominance reversal, the inhibition between close alternatives A and B must be relatively small, which may cause the failures of other effects relying on strong inhibitions between close alternatives (e.g., the attraction effect). This leads to difficulties in proposing a distance function for MDFT that avoids dominance reversal while retaining other effects. While future studies may seek to find distance functions satisfying all effects, the aforementioned finding makes this outcome unlikely.

**Rectified-LCA*.*** In contrast, as rectified-LCA does not allow negated inhibition, it does not produce dominance reversal in current simulations (Figure 15). The rectification does not allow for an inferior option that has a negative activation to boost other options. Thus, B will not be preferred due to its close, inferior alternative A. The rectification is also more theoretically justified based on evidence from neural network models where negative activations can lead to undesirable consequences (McClelland & Rumelhart, 1981). Therefore, this study argues that LCA is more plausible than MDFT according to both theoretical and empirical evidence.

**Figure 15**

*Simulation of LCA for Dominance Reversal*

Chart

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*Note*. A = [0, 0], B = [0.025, 0.025], C = [0.1, 0.1].

**Conclusion of the Choice**

To conclude, this study’s simulations found that rectified-LCA is the most plausible decision-making model for three main reasons: 1) unrectified-LCA cannot simulate the similarity effect; 2) rectified-LCA shows that compromise effect increases with the extremeness of alternatives while MDFT shows the opposite pattern; and 3) rectified-LCA does not lead to dominance reversal while MDFT produces dominance reversal with all three distance functions used in the simulations. This essay discusses the difficulties to find distance functions for MDFT that can avoid dominance reversal while retaining other effects. As the rectified-LCA is more plausible, it may indicate that more distant options have stronger influences on each other than closer options, suggesting that loss aversion may be the mechanism underlying multivariate decision-making. Still, the theoretical validity of increasing I0 in LCA is debatable, which may be an important focus for future research.

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