- **7.3** 线性支持向量机还可以定义为以下形式:

$\min_{w,b,\xi}\ \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{N}\xi_i^2$
$s.t. \quad y_i(w{\cdot}x_i + b) \geq 1 - \xi_i,\ i = 1, 2, \cdots, N$
$\xi_i \geq 0,\ i = 1, 2, \cdots, N$

试求其对偶形式。

解答：定义拉格朗日函数

$L(w, b, \xi, \alpha, \mu) = \frac{1}{2}\|\,w\|^2 + C\sum_{i=1}^{N}\xi_i^2 - \sum_{i=1}^{N}\alpha_i(y_i(w \cdot x_i + b) - 1 + \xi_i) - \sum_{i=1}^{N}\mu_i\xi_i$, 其中 $\alpha_i \geq 0, \mu_i \geq 0.$

求偏导:

$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{N}\alpha_i y_i x_i$

$\frac{\partial L}{\partial b} = -\sum_{i=1}^{N}\alpha_i y_i$

$\frac{\partial L}{\partial \xi_i} = 2C\xi_i - \alpha_i - \mu_i$

令偏导为0，可以得到： $w = \sum_{i=1}^{N}\alpha_i y_i x_i$

$\sum_{i=1}^{N}\alpha_i y_i = 0$

$\alpha_i + \mu_i = 2C\xi_i$

带回L中可得：

$L = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i\alpha_j y_i y_j x_i \cdot x_j)$
$+ C\sum_{i=1}^{N}\xi_i^2$
$- \sum_{i=1}^{N}(\alpha_i y_i(\sum_{j=1}^{N}\alpha_j y_j x_j \cdot x_i))$
$- b\sum_{i=1}^{N}\alpha_i y_i$
$- \sum_{i=1}^{N}\alpha_i(-1 + \xi_i)$
$- \sum_{i=1}^{N}\mu_i\xi_i$

$= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i\alpha_j y_i y_j x_i \cdot x_j)$
$+ \sum_{i=1}^{N}\alpha_i$
$+ C\sum_{i=1}^{N}\xi_i^2 - \sum_{i=1}^{N}\alpha_i\xi_i - \sum_{i=1}^{N}\mu_i\xi_i$

$= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i\alpha_j y_i y_j x_i \cdot x_j)$
$+ \sum_{i=1}^{N}\alpha_i$
$- C\sum_{i=1}^{N}\xi_i^2$