• 7.3 线性支持向量机还可以定义为以下形式:

$$egin{aligned} \min_{w,b,\xi} rac{1}{2} \|w\|^2 + C \sum_{i=1}^N \xi_i^2 \ s.\,t. \quad y_i(w\cdot x_i + b) \geq 1 - \xi_i, \ i = 1, 2, \cdots, N \ \xi_i \geq 0, \ i = 1, 2, \cdots, N \end{aligned}$$

试求其对偶形式。

解答: 定义拉格朗日函数

$$L(w,b,\xi,\alpha,\mu) = \frac{1}{2} \|w\|^2 + C \sum_{i=1}^N \xi_i^2 - \sum_{i=1}^N \alpha_i (y_i(w \cdot x_i + b) - 1 + \xi_i) - \sum_{i=1}^N \mu_i \xi_i, \, \sharp \, \forall \, \alpha_i \geq 0, \, \mu_i \geq 0.$$

求偏导:

$$egin{aligned} rac{\partial L}{\partial w} &= w - \sum_{i=1}^{N} lpha_i y_i x_i \ & rac{\partial L}{\partial b} &= - \sum_{i=1}^{N} lpha_i y_i \end{aligned}$$

$$\frac{\partial L}{\partial \xi_i} = 2C\xi_i - \alpha_i - \mu_i$$

令偏导为0,可以得到: $w = \sum_{i=1}^N \alpha_i y_i x_i$

$$\sum_{i=1}^N lpha_i y_i = 0$$

$$\alpha_i + \mu_i = 2C\xi_i$$

带回L中可得:

 $-C\sum_{i=1}^{N}\xi_i^2$

$$\begin{split} L &= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{i} \alpha_{j} y_{i} y_{j} x_{i} \cdot x_{j}) \\ &+ C \sum_{i=1}^{N} \xi_{i}^{2} \\ &- \sum_{i=1}^{N} (\alpha_{i} y_{i} (\sum_{j=1}^{N} \alpha_{j} y_{j} x_{j} \cdot x_{i})) \\ &- b \sum_{i=1}^{N} \alpha_{i} y_{i} \\ &- \sum_{i=1}^{N} \alpha_{i} (-1 + \xi_{i}) \\ &- \sum_{i=1}^{N} \mu_{i} \xi_{i} \\ &= -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{i} \alpha_{j} y_{i} y_{j} x_{i} \cdot x_{j}) \\ &+ \sum_{i=1}^{N} \alpha_{i} \\ &+ C \sum_{i=1}^{N} \xi_{i}^{2} - \sum_{i=1}^{N} \alpha_{i} \xi_{i} - \sum_{i=1}^{N} \mu_{i} \xi_{i} \\ &= -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{i} \alpha_{j} y_{i} y_{j} x_{i} \cdot x_{j}) \\ &+ \sum_{i=1}^{N} \alpha_{i} \end{split}$$