6.1. (1) $\alpha_p=0.10dB \quad \alpha_s=43dB$

$\varepsilon=\sqrt{10^{\alpha_p/10}-1}=\sqrt{10^{0.01}-1}=0.1875 \quad A=10^{\alpha_s/20}=10^{\frac{43}{20}}=141.2538$

(2) $\alpha_p=0.04dB \quad \alpha_s=57dB$

$\varepsilon=\sqrt{10^{\frac{\alpha_p}{10}}-1}=\sqrt{10^{\frac{0.04}{10}}-1}=0.0962$，$A=10^{\frac{\alpha_s}{20}}=10^{\frac{57}{20}}=707.9458$

(3) $\alpha_p=0.23dB \quad \alpha_s=60dB$

$\varepsilon=\sqrt{10^{\frac{\alpha_p}{10}}-1}=\sqrt{10^{\frac{0.23}{10}}-1}=0.2332$，$A=10^{\frac{\alpha_s}{20}}=10^{\frac{60}{20}}=1000$

6.2. 解：因为 $H_1(s)=\frac{a}{s+a} \quad a>0$

则 $H_1(j\Omega)=\frac{a}{j\Omega+a} \quad a>0$

$|H_1(j\Omega)|=\frac{a}{\sqrt{\Omega^2+a^2}} \quad a>0$

由 $|H_1(j0)|=1$，$|H_1(j\infty)|=0$ 可得该滤波器具有单调下降的低通幅度响应特性

$\alpha_1(\Omega)=-20\lg|H_1(j\Omega)|=-10\lg|H_1(j\Omega)|^2=-10\lg\frac{a^2}{\Omega^2+a^2}$

则 $\Omega_c=0.9976a$，$a>0$

6.3 解：$H_h(s)=\frac{s}{s+a}$，$a>0$

则 $H_h(j\Omega)=\frac{j\Omega}{j\Omega+a} \quad a>0 \quad |H_h(j\Omega)|=\frac{\sqrt{\Omega^2}}{\sqrt{\Omega^2+a^2}}=\frac{|\Omega|}{\sqrt{\Omega^2+a^2}} \quad a>0$

由 $|H_h(j0)|=0$，$|H_h(j\infty)|=1$，得该滤波器具有单调上升的高通幅度响应特性。

$\alpha(\Omega)=-20\lg|H_h(j\Omega)|=-10\lg|H_h(j\Omega)|^2=-10\lg\frac{\Omega^2}{\Omega^2+a^2}=3$

$\Omega_c=1.0024a$，$a>0$

6.11 由 $\Omega_p=0.242$，$\alpha_p=0.5dB$，$N=2$，

得 $\varepsilon=\sqrt{10^{\frac{\alpha_p}{10}}-1}=\sqrt{10^{\frac{0.5}{10}}-1}=0.349$

$\Omega_c=\frac{\Omega_p}{\varepsilon^{\frac{1}{N}}}=\frac{2\pi\Omega_p}{\varepsilon^{\frac{1}{2}}}=2.126\ rad/s$

$\because H_p(s)=G(p)|_{p=\frac{s}{\Omega_c}}$

$\therefore G(p)=H_p(s)|_{s=p\Omega_c}=\frac{4.52}{p^2\Omega_c^2+3p\Omega_c+4.52}$

$=\frac{4.52}{4.52p^2+6.378p+4.52}$

$H_{hp}(s)=G(p)|_{p=\frac{\Omega_p\Omega_{ph}}{s}}=G(p)|_{p=\frac{2.52}{s}}=\frac{4.52s^2}{4.52s^2+16.025+28.52}$

6 1(1) I1. 解：冲激响应不变法设计的 H(z) 的系数为

(1) $s_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}j \quad s_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}j$

$$H_a(s) = \frac{-\frac{\sqrt{3}}{3}j}{s - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)} + \frac{\frac{\sqrt{3}}{3}j}{s - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)}$$

$$H(z) = \frac{-\frac{\sqrt{3}}{3}j}{1 - z^{-1}e^{-1+\sqrt{3}j}} + \frac{\frac{\sqrt{3}}{3}j}{1 - z^{-1}e^{-1-\sqrt{3}j}}$$

当 $T = 2s$，
$$H(z) = \frac{\frac{\sqrt{3}}{-3}j}{\cancel{1 - z^{-1}e^{-1+\sqrt{3}j}}\ 1 - z^{-1}e^{-1+\sqrt{3}j}} + \frac{\frac{\sqrt{3}}{3}j}{1 - z^{-1}e^{-1-\sqrt{3}j}}$$

$$= \frac{2\sqrt{3}}{3} \cdot \frac{z^{-1}e^{-1}\sin\sqrt{3}}{1 - 2z^{-1}e^{-1}\cos\sqrt{3} + e^{-2}z^{-2}}$$

(2) $$H_a(s) = \frac{1}{2s^2+3s+1} = \frac{1}{s+\frac{1}{2}} - \frac{1}{s+1}$$

$$H(z) = \frac{1}{1 - e^{-\frac{T}{2}}z^{-1}} - \frac{1}{1 - e^{-T}z^{-1}}\Bigg|_{T=2}$$

$$= \frac{1}{1 - e^{-1}z^{-1}} - \frac{1}{1 - e^{-2}z^{-1}}$$

$$= \frac{(e^{-1} - e^{-2})z^{-1}}{1 - (e^{-1}+e^{-2})z^{-1} + e^{-3}z^{-2}}$$

I2. 双线性变换法

(1) $$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}},\,T=2} = \frac{(1+z^{-1})^2}{(1-z^{-1})^2 + (1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}$$

$$= \frac{1+2z^{-1}+z^{-2}}{3+z^{-2}}$$

(2). $$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}},\,T=2} = \frac{(1+z^{-1})^2}{2(1-z^{-1})^2 + 3(1-z^{-2}) + (1+z^{-1})^2}$$

$$= \frac{1+2z^{-1}+z^{-2}}{6-2z^{-1}}$$

6.20 I. 用脉冲响应不变法，

$f_p=0.25kHz$ $T=0.5ms$

$\omega_p=\Omega T=2\pi f_p T=2\pi\times0.25\times10^{3}\times0.5\times10^{-3}$

$=0.5\pi$

$=1.57$

II. 用双线性变换法，

$\Omega_p=\frac{2}{T}\tan\frac{\omega_p}{2}$ $2\pi f_p=\frac{2}{T}\tan\frac{\omega_p}{2}$

$\omega_p=2\arctan\frac{\pi}{4}$ $=2\arctan\pi$.

6.22 I. 用脉冲响应不变法，

(1) 设采样周期为T.

$\Omega_p=\frac{\omega_p}{T}=\frac{0.2\pi}{T}rad/s$ $\Omega_s=\frac{\omega_s}{T}=\frac{0.3\pi}{T}rad/s$

$\alpha_p=1dB$ $\alpha_s=10dB$

(2) $\varepsilon=\sqrt{10^{\frac{\alpha_p}{10}}-1}=\sqrt{10^{\frac{1}{10}}-1}=0.508847$

$A=10^{\frac{\alpha_s}{20}}=10^{\frac{10}{20}}=3.1623$

$k=\frac{\Omega_p}{\Omega_s}=\frac{0.2\pi}{T}\div\frac{0.3\pi}{T}=\frac{2}{3}$.

$k_1=\frac{\varepsilon}{\sqrt{A^2-1}}=\frac{0.508847}{\sqrt{3.1623^2-1}}=0.193349$.

$N=\frac{\lg k_1}{\lg k}=4.0328$ $N=5$

$T=1s$, $\Omega_c=\frac{\Omega_s}{(A^2-1)^{\frac{1}{2N}}}=\frac{0.3\pi}{(3.1623^2-1)^{\frac{1}{10}}}=0.2408$.

查表归一化5阶巴特沃斯分母为 $D_5(p)=(p-p_1)(p-p_2)(p-p_3)(p-p_4)(p-p_5)$

得归一化系统函数 $G(p)=\frac{1}{(p-p_1)(p-p_2)(p-p_3)(p-p_4)(p-p_5)}$

$=\sum_{k=1}^{5}\frac{A_k}{p-p_k}$.

将Ω_c代入去归一化，得

$H_a(s)=G(p)|_{p=\frac{s}{\Omega_c}}=\sum_{k=1}^{5}\frac{\Omega_c A_k}{s-\Omega_c p_k}=\sum_{k=1}^{5}\frac{B_k}{s-s_k}$ $s_k=\Omega_c p_k$, $B_k=\Omega_c A_k$.

(3) $T=1s$, $H_a(s)\Rightarrow H(z)$

$$H(z)=\sum_{k=1}^{5}\frac{B_k}{1-e^{s_kT}z^{-1}}=\frac{2.2284\times10^{-6}+7.882\times10^{-5}z^{-1}+0.000227736z^{-2}+0.000023137z^{-3}+1.1855\times10^{-5}z^{-4}}{1-4.2232z^{-1}+7.1871z^{-2}-6.1541z^{-3}+2.6442z^{-4}-0.45872z^{-5}}$$

I7.22 双线性变换法

(1) 确定数字滤波器指标 $\omega_p=0.2\,rad$，$\omega_s=0.3\,rad$，$\alpha_p=1dB$，$\alpha_s=10dB$

(2) 非线性预畸变校正，将数字滤波器指标转换为相应模拟滤波器指标 $T=2s$

$\Omega_p=\frac{2}{T}\tan\frac{\omega_p}{2}=\tan\frac{0.2}{2}=0.01745$ $\alpha_p=1dB$

$\Omega_s=\frac{2}{T}\tan\frac{\omega_s}{2}=\tan\frac{0.3}{2}=0.02268$ $\alpha_s=10dB$

(3) $\varepsilon=\sqrt{10^{\frac{\alpha_p}{10}}-1}=\sqrt{10^{\frac{1}{10}}-1}=0.508847$

$A=10^{\frac{\alpha_s}{20}}=10^{\frac{10}{20}}=3.1623$

$k=\frac{\Omega_p}{\Omega_s}=\frac{0.01745}{0.02618}=0.6665$；$k_1=\frac{\varepsilon}{\sqrt{A^2-1}}=\frac{0.508847}{\sqrt{3.1623^2-1}}=0.193349$

$N=\frac{\lg k_1}{\lg k}=4.0503$，$N=5$

取 $T=2s$，$\Omega_c=\frac{\Omega_s}{(A^2-1)^{\frac{1}{2N}}}=\frac{0.02618}{(3.1623^2-1)^{\frac{1}{10}}}=1.2457$

查表得归一化5阶巴特沃斯模拟器的系统函数

为 $G(p)=\frac{1}{(p^2+0.618p+1)(p^2+1.618p+1)(p+1)}$

去归一化，得 $H_a(s)=G(p)|_{p=\frac{s}{\Omega_c}}$

(4) $H(z)=H_a(s)|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$

$$=\frac{1.7763\times10^{-5}+8.8815\times10^{-5}z^{-1}+0.0001776\,3z^{-2}+0.0001776z^{-3}+8.8815\times10^{-5}z^{-4}+1.7763\times10^{-5}z^{-5}}{1-4.2491z^{-1}+7.1728z^{-2}-6.13z^{-3}+2.64z^{-4}-0.4546z^{-5}}$$

7.23. $\omega_p=0.8rad$ $\omega_s=0.5rad$ $\alpha_p=3dB$ $\alpha_s=18dB$

1) $\Omega_{ph}=\tan\frac{\omega_p}{2}=\tan\frac{0.8}{2}=0.006981$ $\Omega_{sh}=\tan\frac{\omega_s}{2}=\tan\frac{0.5}{2}=0.004363$

2) 将 $H_a(s)\Rightarrow G(p)$

$\lambda_p=1$ $\alpha_p=3dB$ $\lambda_s=\frac{\Omega_{ph}}{\Omega_{sh}}=1.6$ $\alpha_s=18dB$

~~$\omega_p=$~~