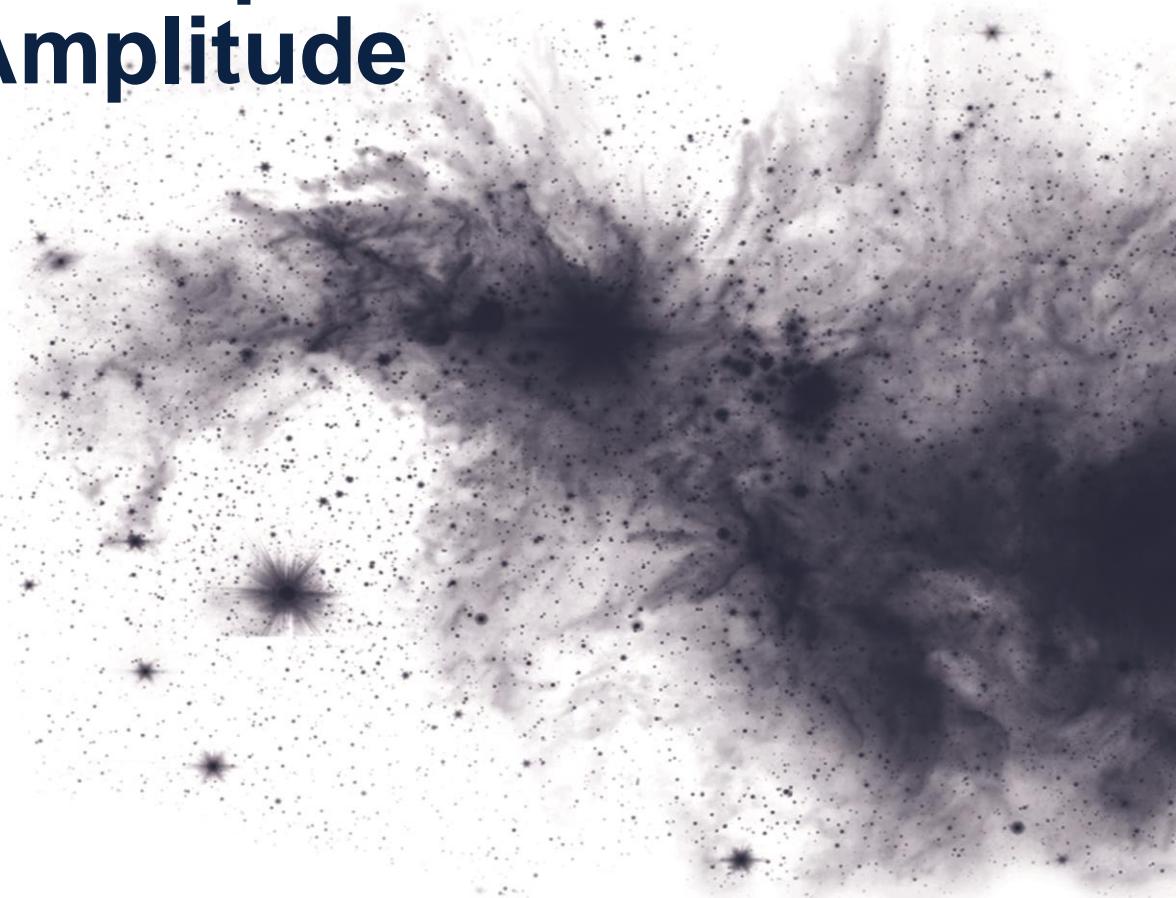
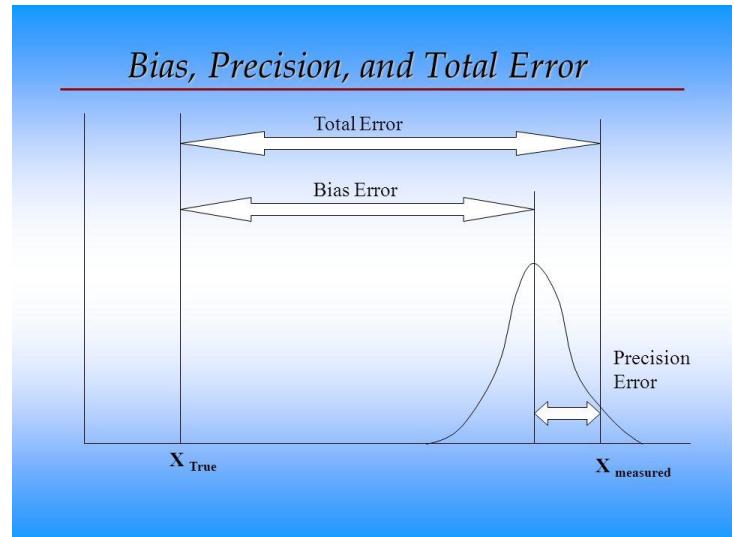
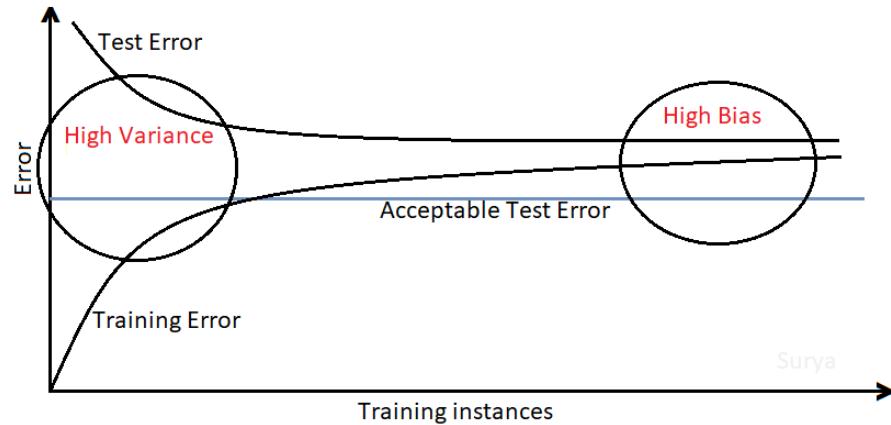
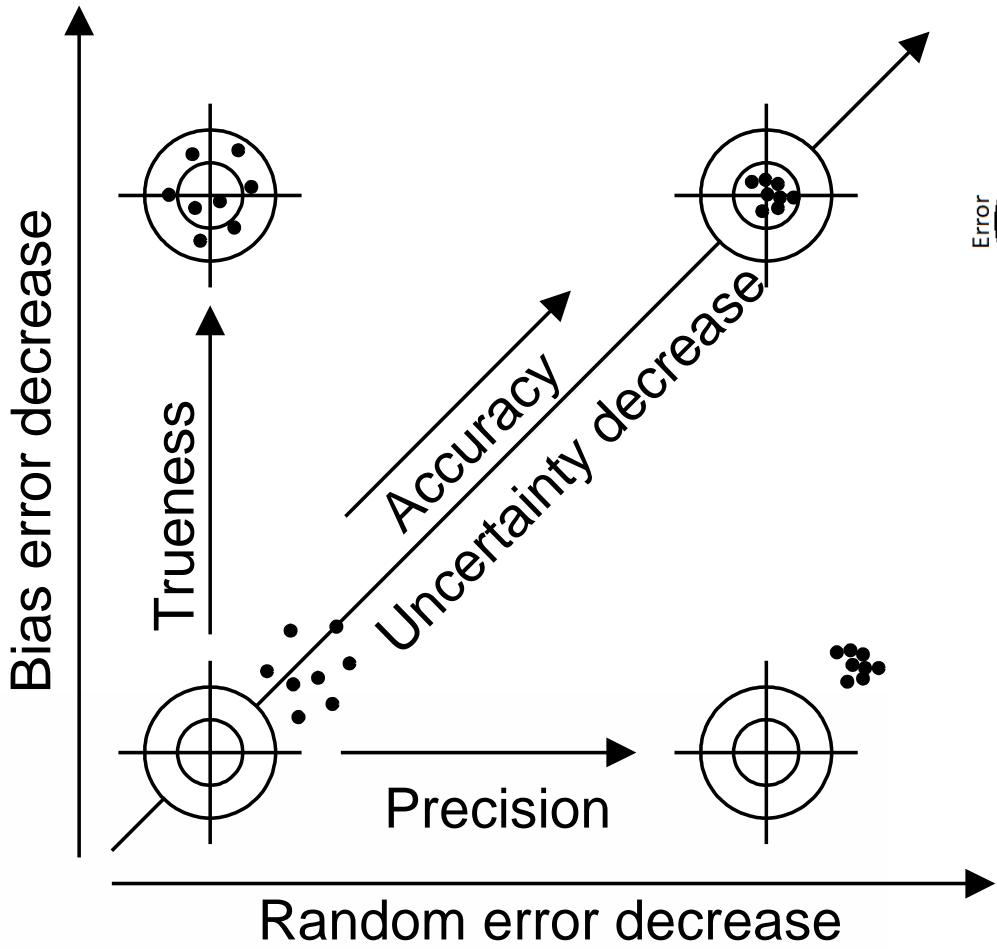


# Estimation problem: Amplitude

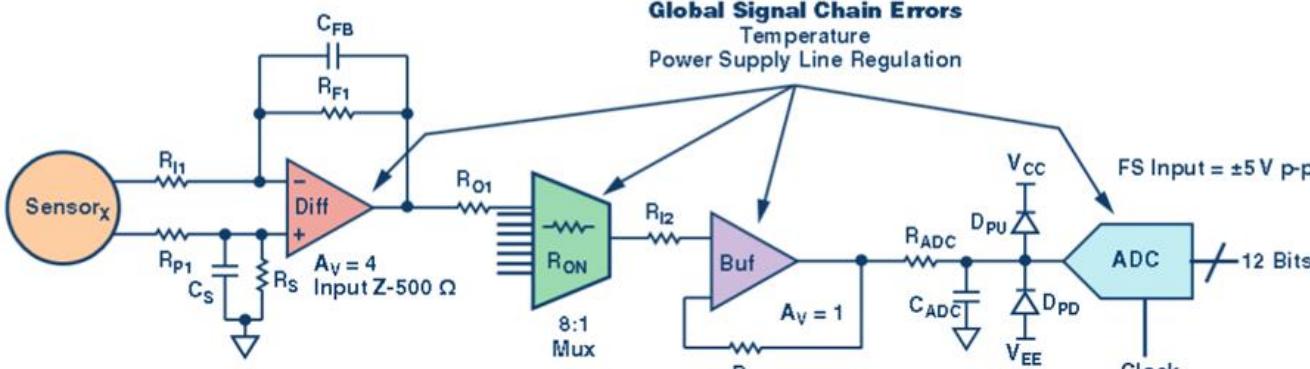
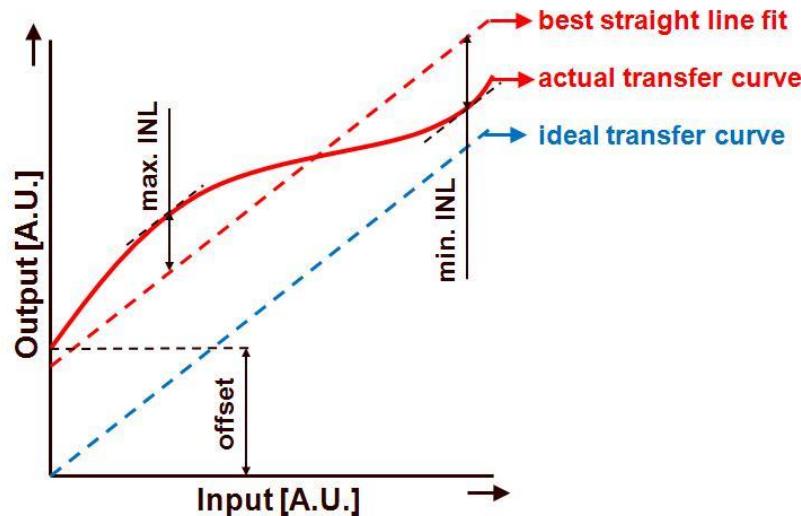
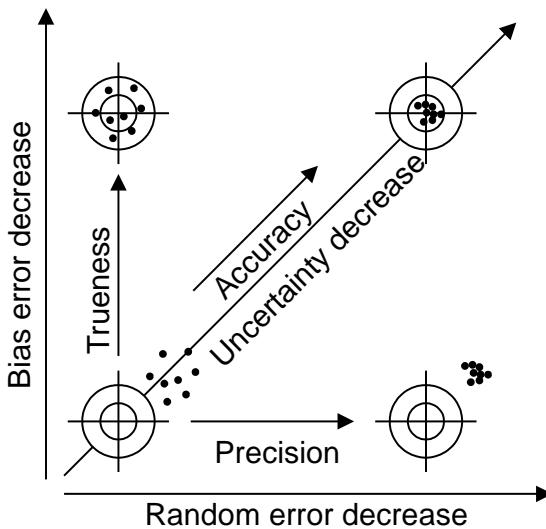


Prof. L. Svilainis

# Measurement errors



# Bias errors



**Amplifier Errors**

- Input Offset Voltage Drift
- Input Bias Current Drift
- Input Bias Current
- Long-Term Drift (1000 hrs)
- Input Offset Current
- PSRR
- Input Offset Voltage
- CMRR

**Multiplex Errors**

- On Resistance ( $R_{ON}$ )
- Resistor Coefficient
- Resistor Tolerance
- Channel Isolation

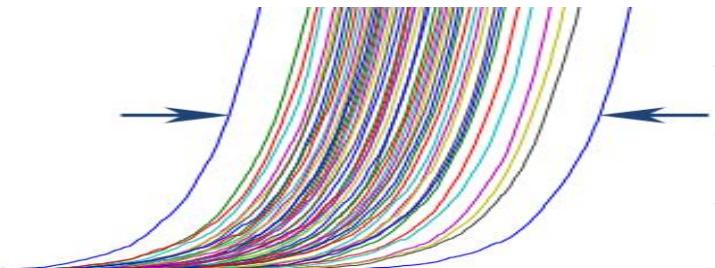
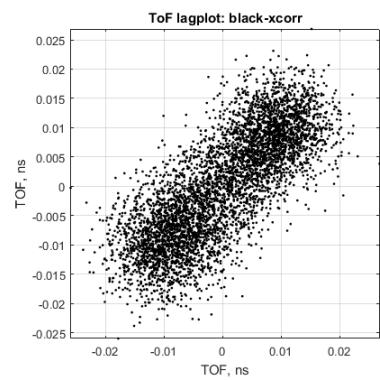
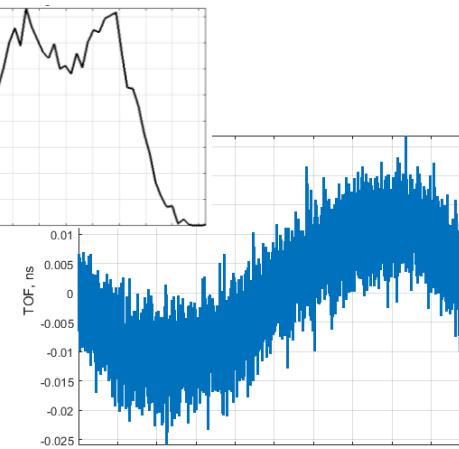
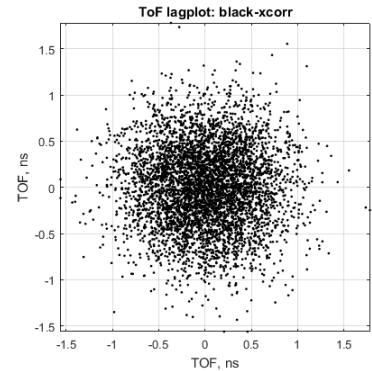
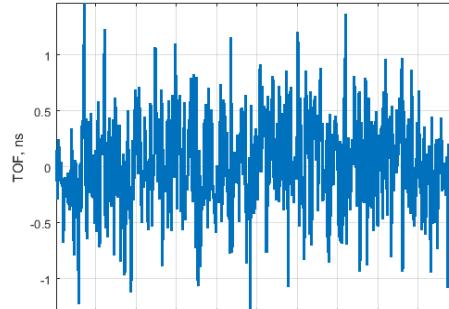
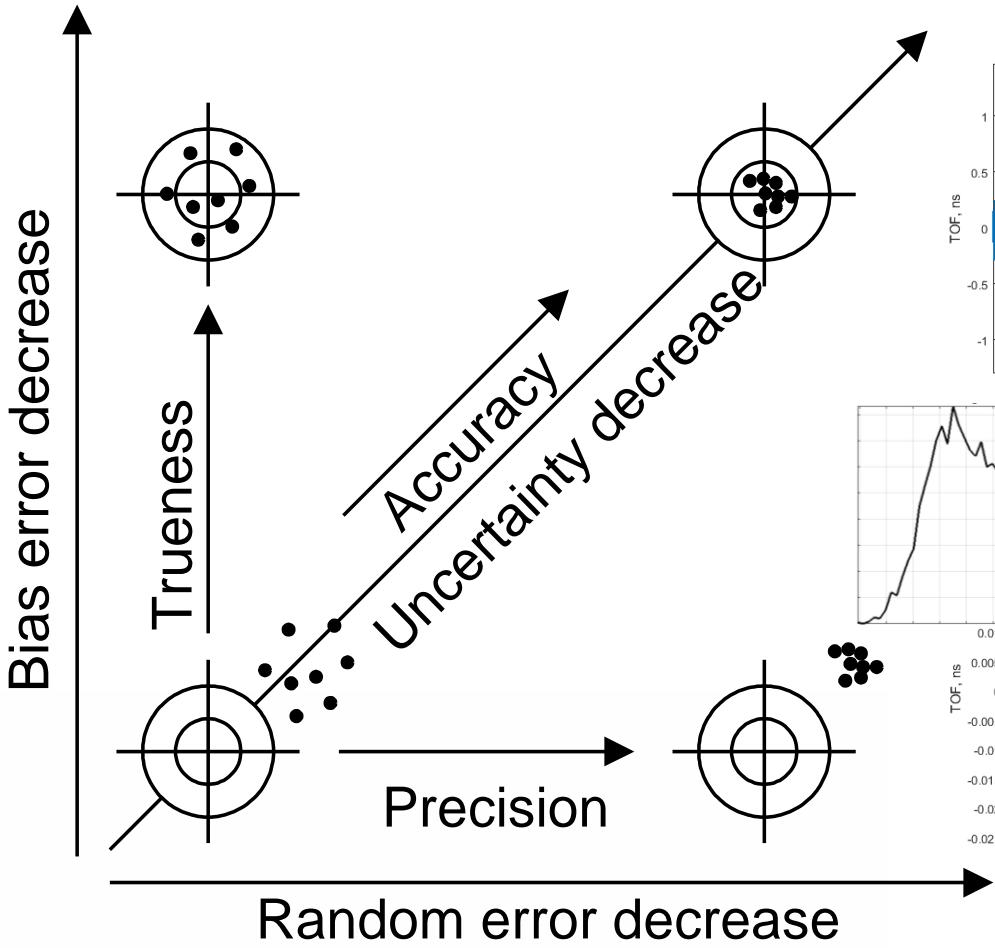
**Amplifier Errors**

- Input Offset Voltage Drift
- Input Bias Current Drift
- Input Bias Current
- Long-Term Drift (1000 hrs)
- Input Offset Current
- PSRR
- Input Offset Voltage
- CMRR

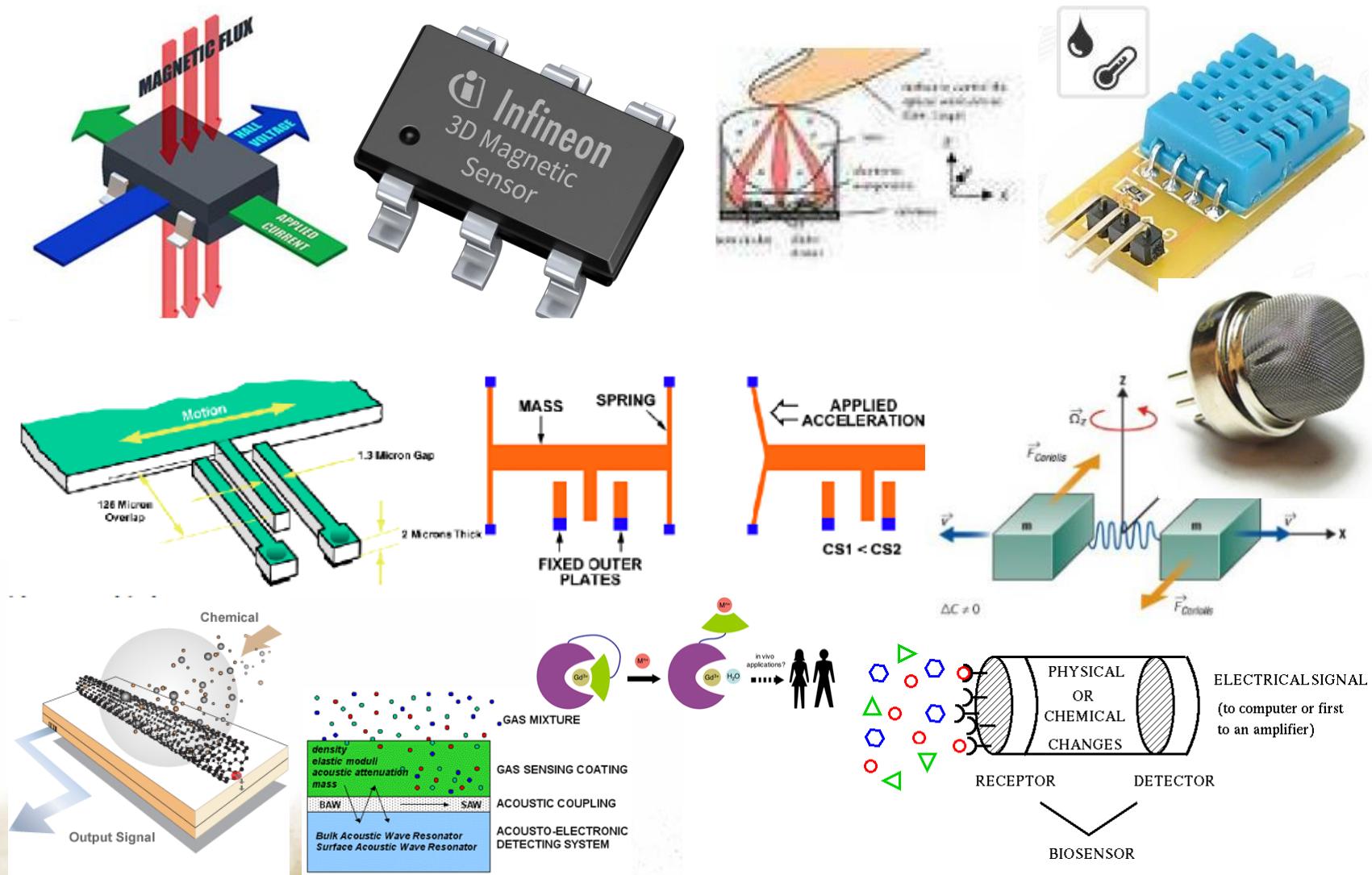
**Converter Errors**

- Differential Nonlinearity (DNL)
- Offset Error
- Gain Error
- Offset Drift
- Gain Drift
- PSRR

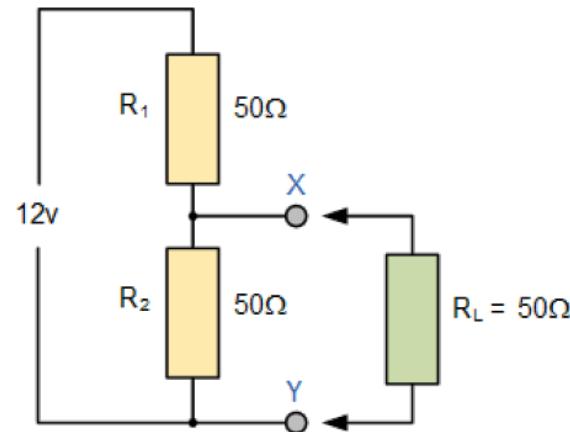
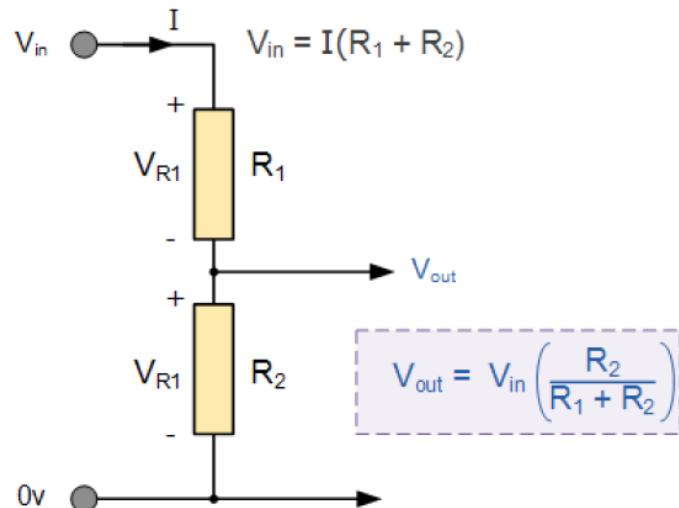
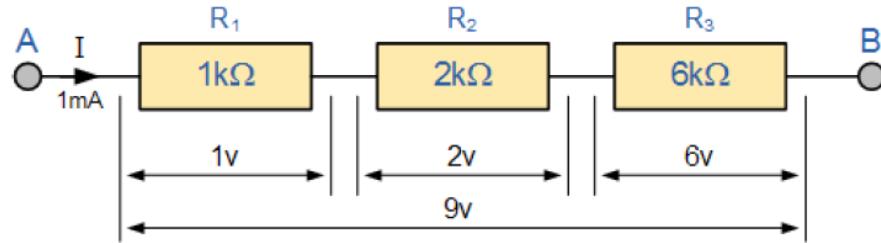
# Random errors



# Sensors



# Amplitude estimation: divider



a) Without  $R_L$  connected

$$R_{X-Y} = 50\Omega$$

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

$$V_{out} = 12v \times \frac{50}{50 + 50} = 6.0v$$

b) With  $R_L$  connected

$$R_{X-Y} = 25\Omega \text{ (Resistors in Parallel)}$$

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

$$V_{out} = 12v \times \frac{25}{50 + 25} = 4.0v$$

# Amplifier

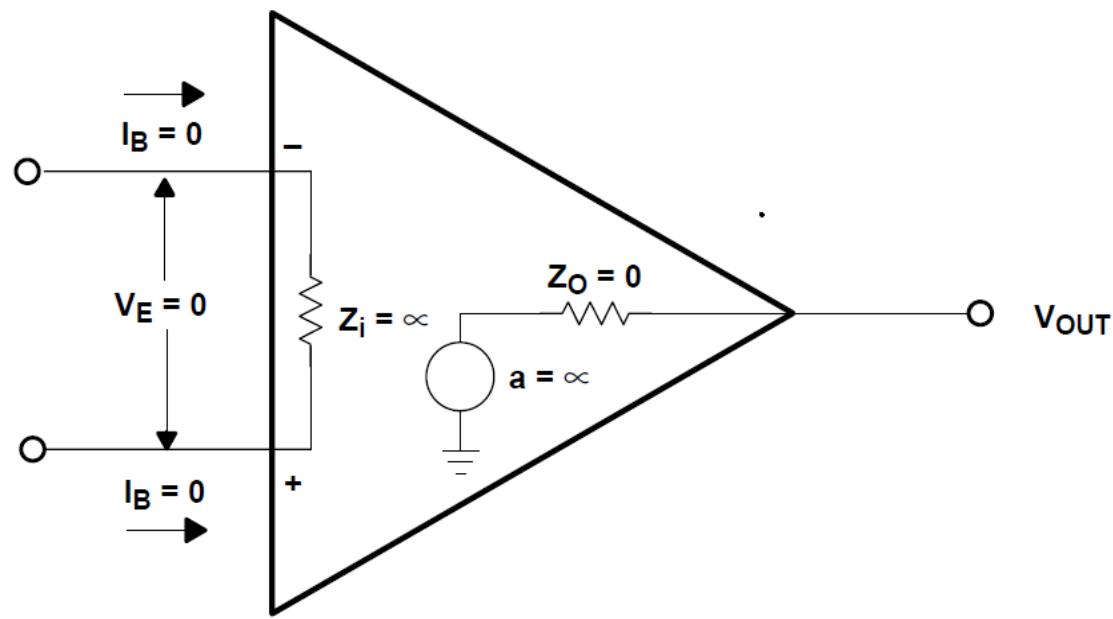
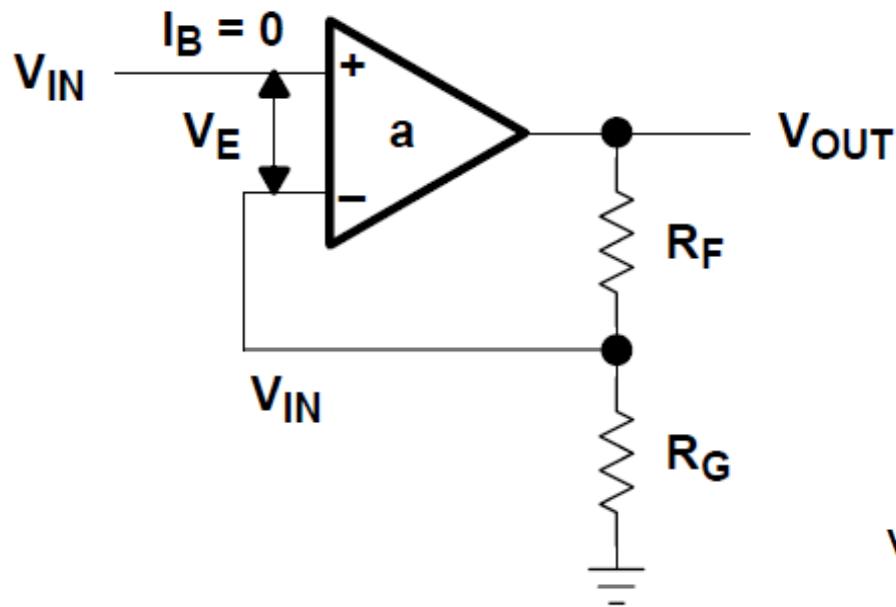


Figure 3–1. The Ideal Op Amp

# Amplifier

Noninverting amplifier

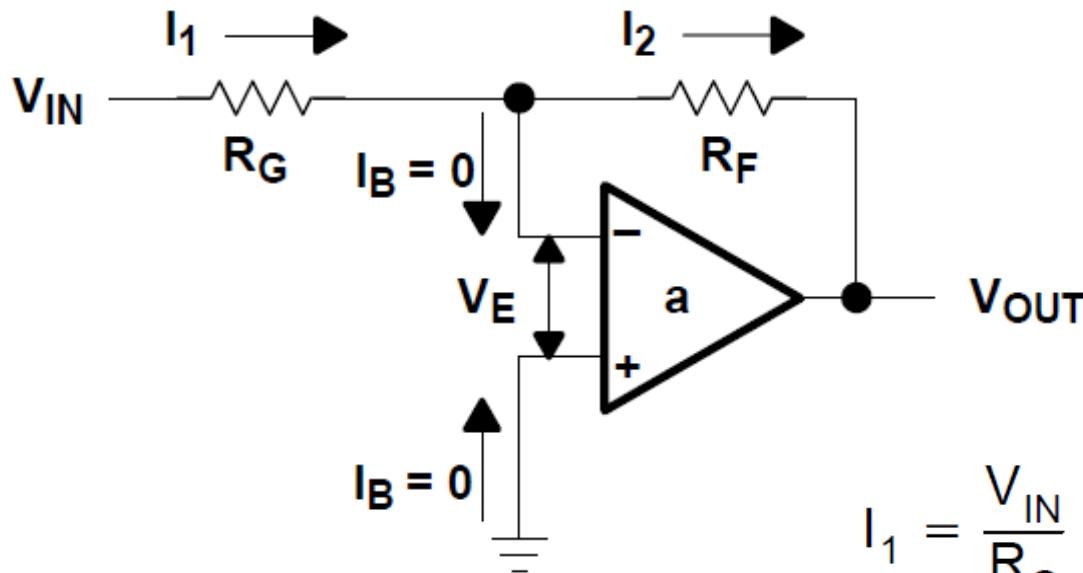


$$V_{IN} = V_{OUT} \frac{R_G}{R_G + R_F}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{R_G + R_F}{R_G} = 1 + \frac{R_F}{R_G}$$

# Amplifier

Inverting amplifier

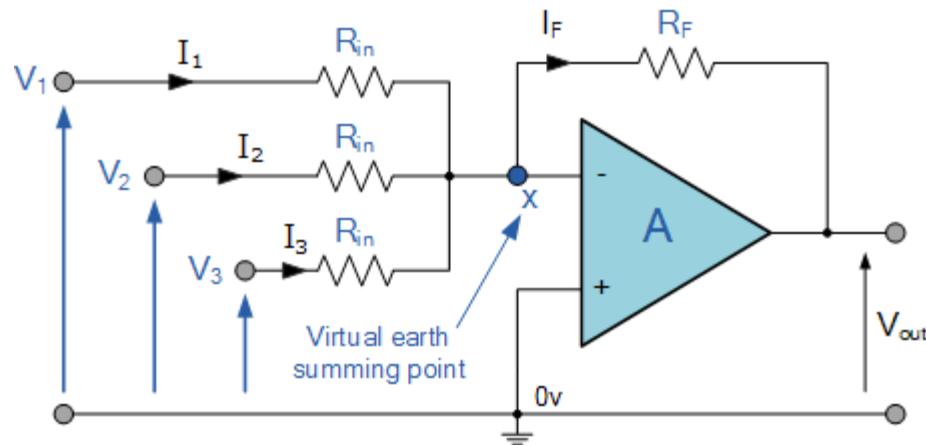


$$I_1 = \frac{V_{IN}}{R_G} = - I_2 = - \frac{V_{OUT}}{R_F}$$

$$\frac{V_{OUT}}{V_{IN}} = - \frac{R_F}{R_G}$$

# Amplifier

Current summation



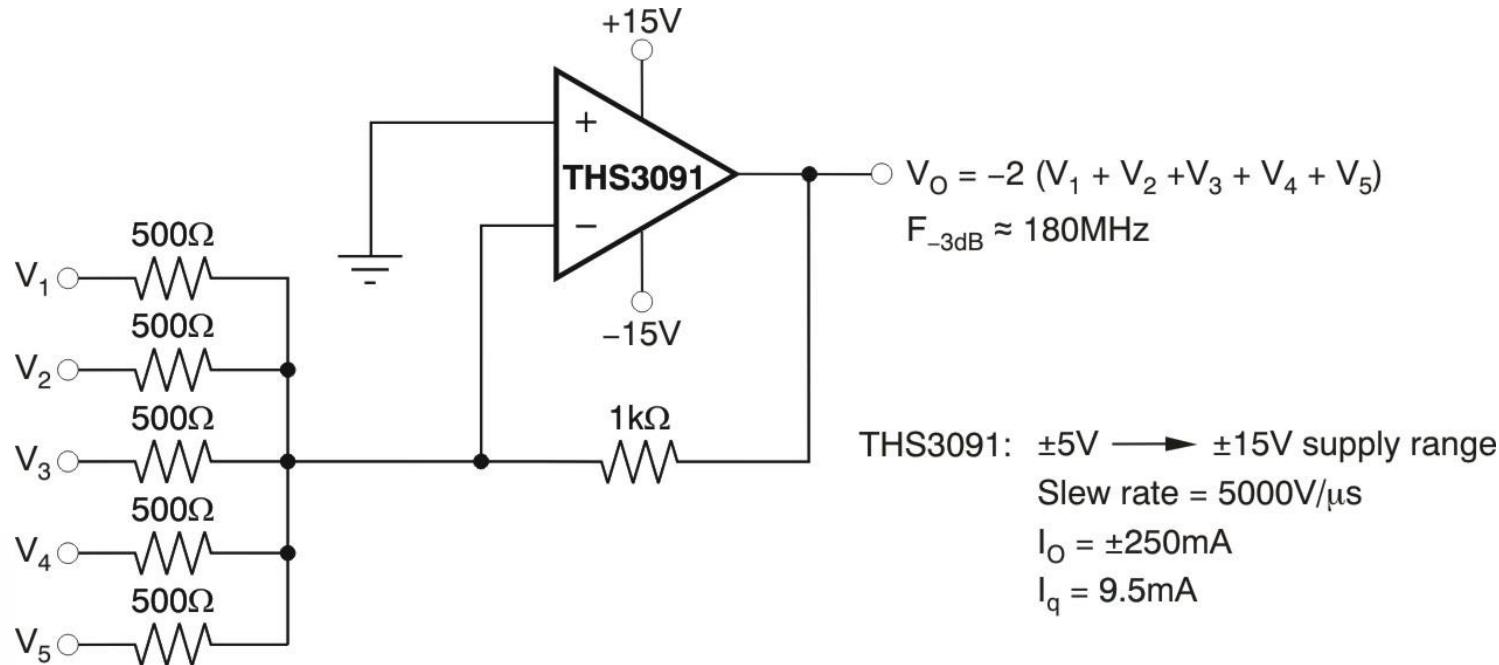
$$I_F = I_1 + I_2 + I_3 = - \left[ \frac{V_1}{R_{in}} + \frac{V_2}{R_{in}} + \frac{V_3}{R_{in}} \right]$$

$$\text{Inverting Equation: } V_{out} = - \frac{R_F}{R_{in}} \times V_{in}$$

$$\text{then, } -V_{out} = \left[ \frac{R_F}{R_{in}} V_1 + \frac{R_F}{R_{in}} V_2 + \frac{R_F}{R_{in}} V_3 \right]$$

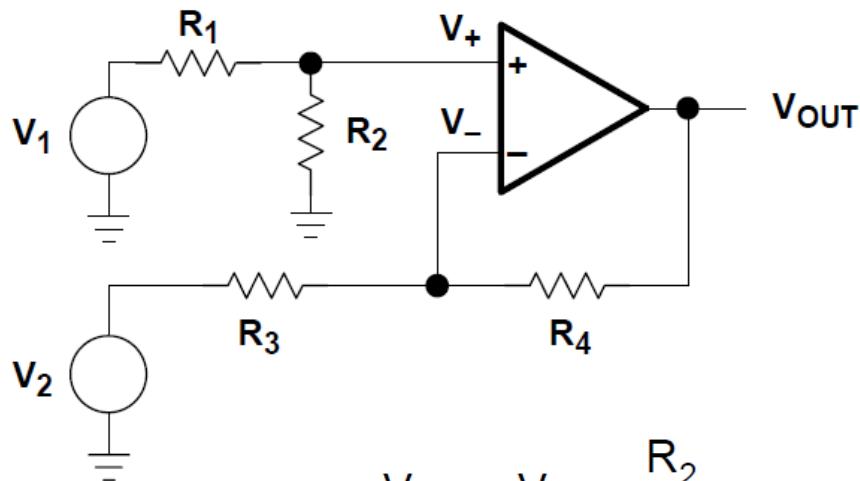
# Amplifier

Voltage summation



# Amplifier

Differential amplifier



$$V_+ = V_1 \frac{R_2}{R_1 + R_2}$$

$$V_{\text{OUT}1} = V_+ (G_+) = V_1 \frac{R_2}{R_1 + R_2} \left( \frac{R_3 + R_4}{R_3} \right)$$

$$R_2 = R_4 \text{ and } R_1 = R_3$$

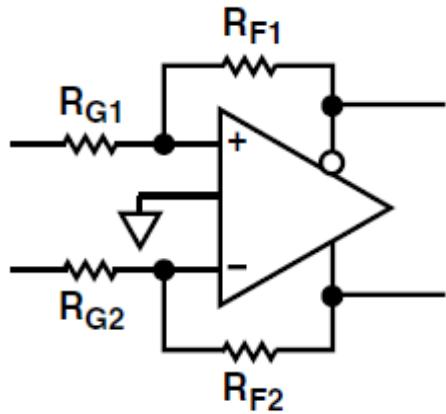
$$V_{\text{OUT}} = (V_1 - V_2) \frac{R_4}{R_3}$$

$$V_{\text{OUT}2} = V_2 \left( - \frac{R_4}{R_3} \right)$$

# Amplifier



## Fully differential amplifier



$$\text{Gain} = R_f / R_{in}$$

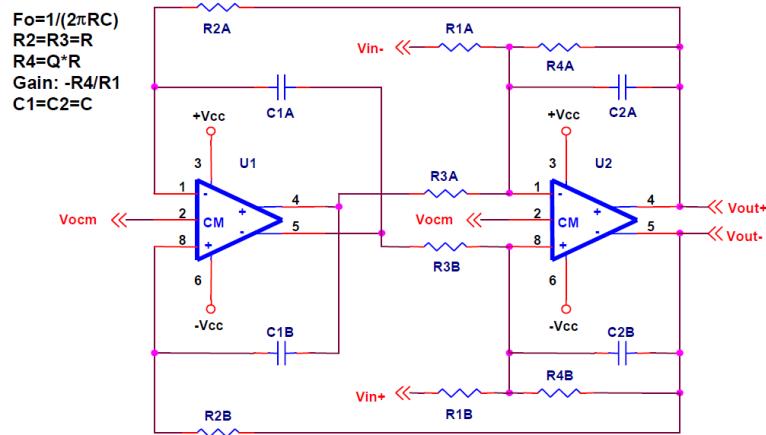
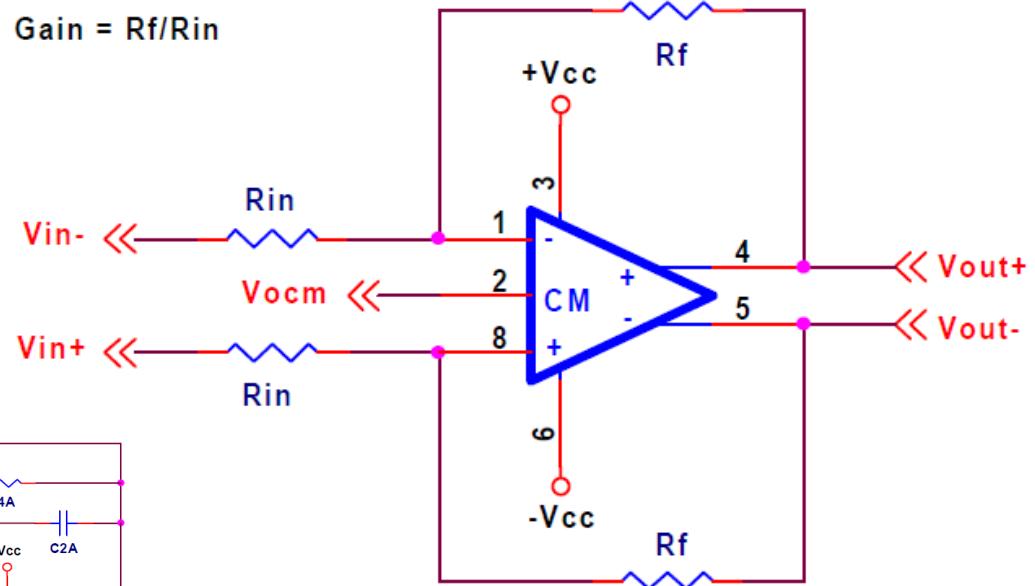
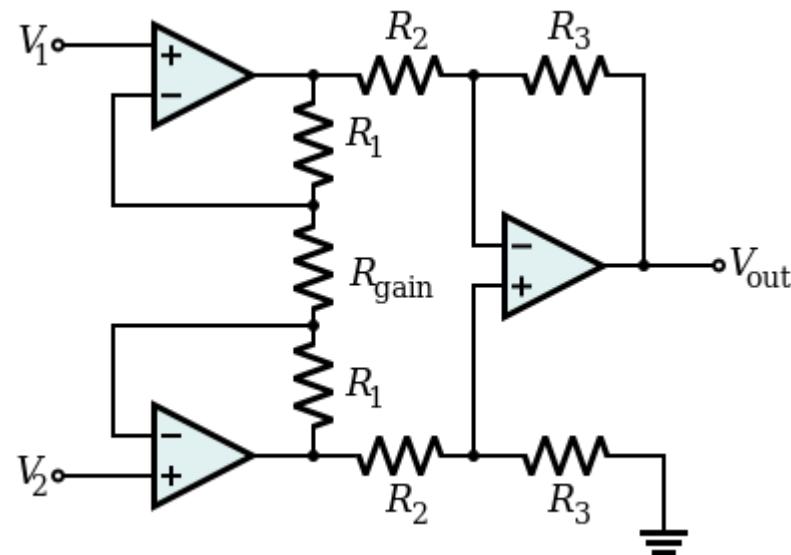


Figure 11: Akerberg Mossberg Band Pass Filter

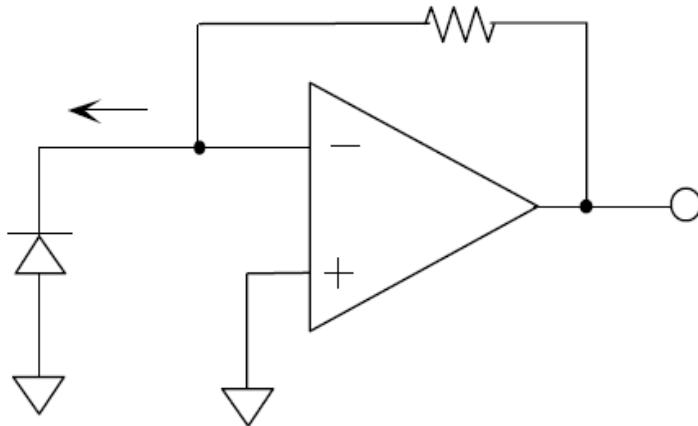
# Amplifier

Instrumental amplifier



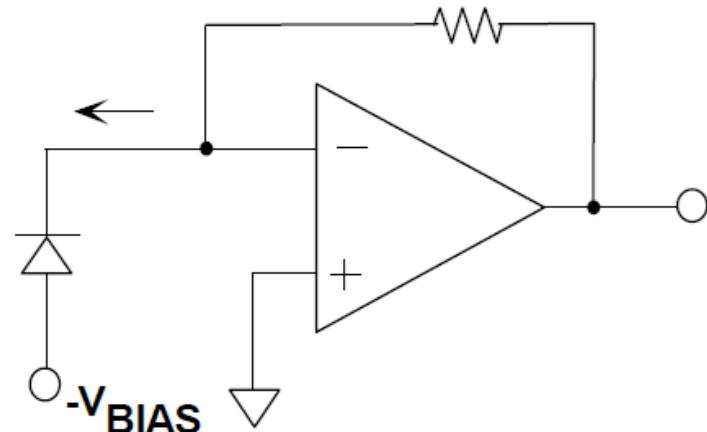
$$\frac{V_{\text{out}}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{\text{gain}}}\right) \frac{R_3}{R_2}$$

# Amplifier



## PHOTOVOLTAIC

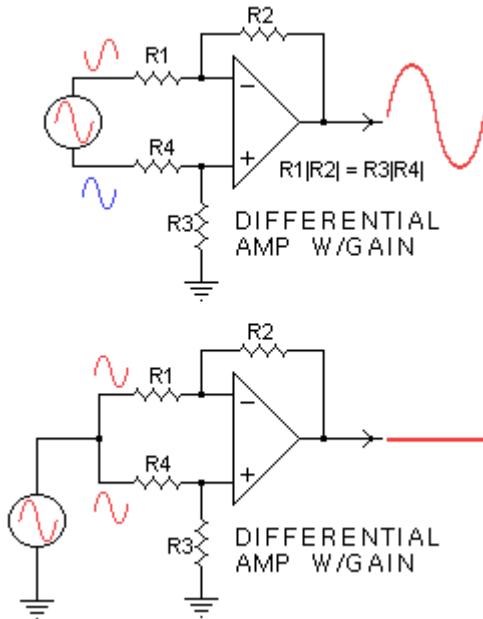
- Zero Bias
- No Dark Current
- Precision Applications
- Low Noise (Johnson)



## PHOTOCODUCTIVE

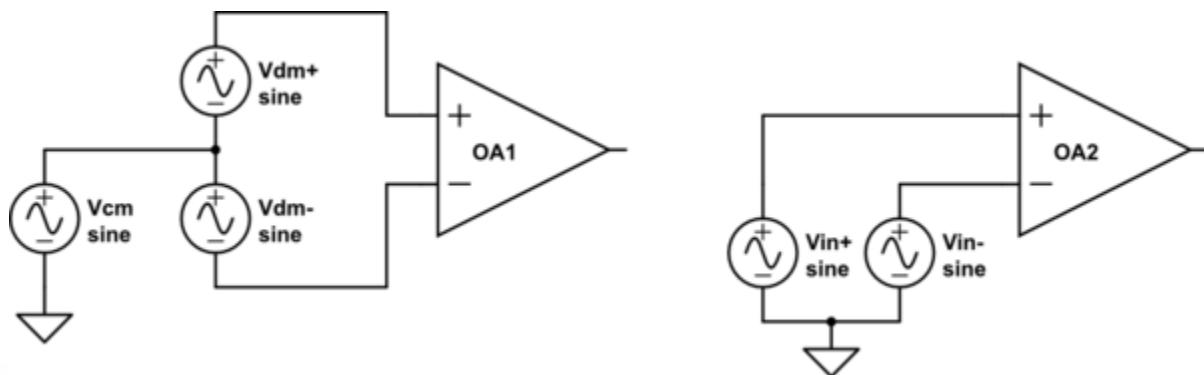
- Reverse Bias
- Dark Current Exists
- High Speed Applications
- Higher Noise (Johnson + Shot)

# Amplifier



## Common-Mode Rejection Ratio

$$\text{CMRR} = \Delta V_{\text{COM}} / \Delta V_{\text{os}}$$



CMRR, as published in the data sheet, is a dc parameter when graphed vs. frequency, falls off as the frequency increases.

# Amplifier

## Supply Voltage Rejection Ratio

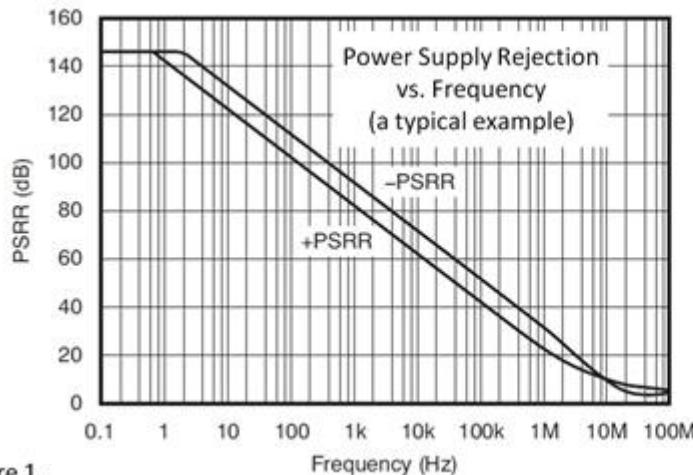


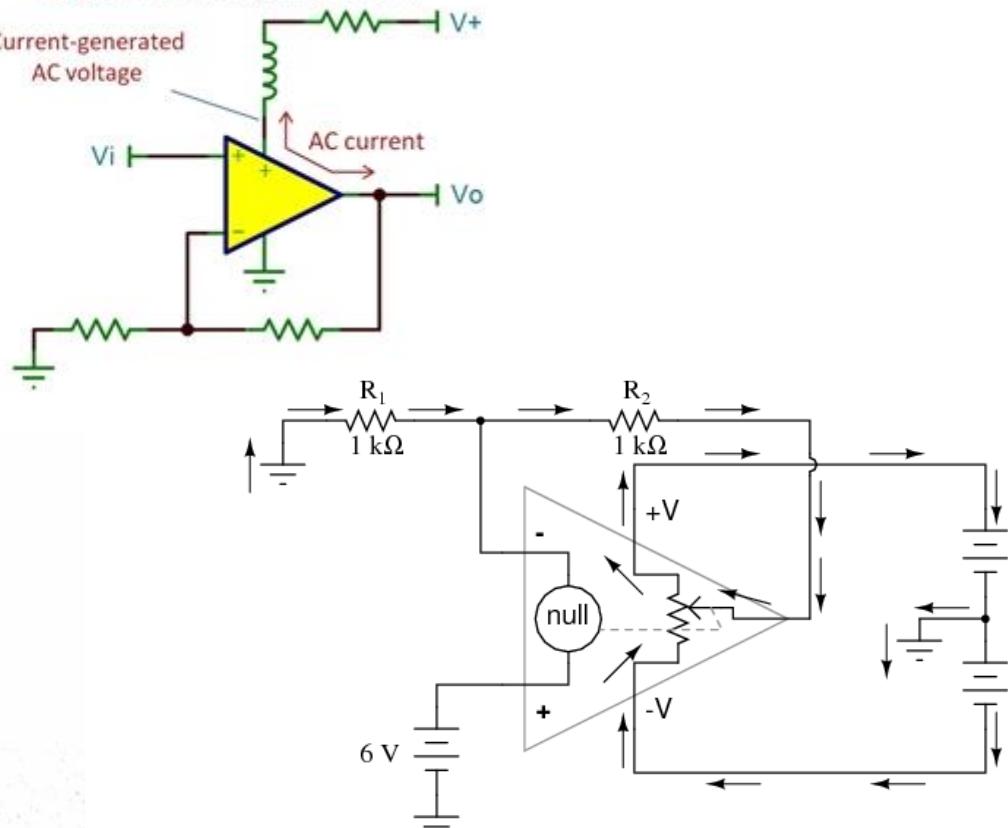
Figure 1.

$$K_{SVR} = \frac{\Delta V_{CC\pm}}{\Delta V_{OS}}$$

$$K_{SVR} = \frac{\Delta V_{CC}}{\Delta V_{OS}}$$

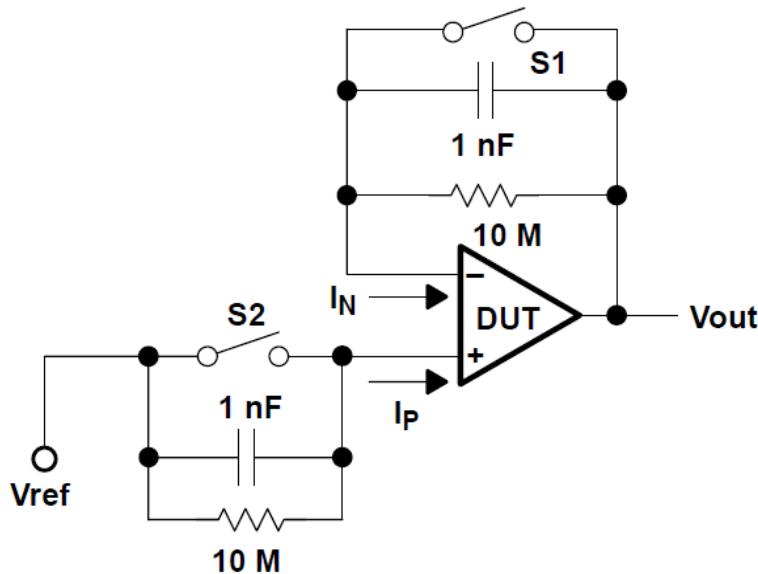
Power supply without proper bypass capacitor to ground creates high impedance.

Current-generated AC voltage



# Amplifier

## Bias



S1 Closed

$$I_P = \frac{V_{out} - V_{ref}}{10^7}$$

S2 Closed

$$I_N = \frac{V_{out} - V_{ref}}{10^7}$$

*Test Circuit –  $I_{IB}$*

input bias current,  $I_{IB}$ ,

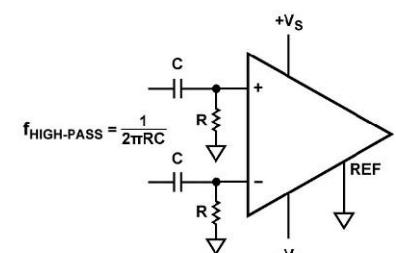
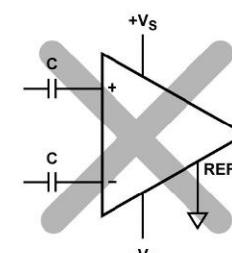
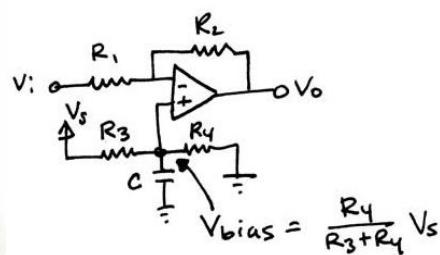
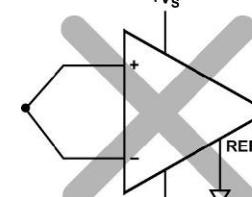
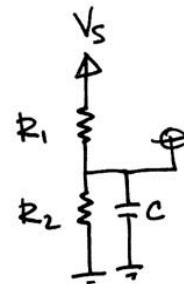
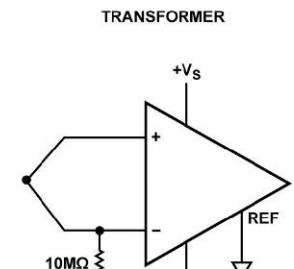
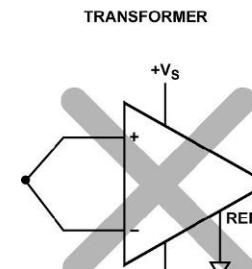
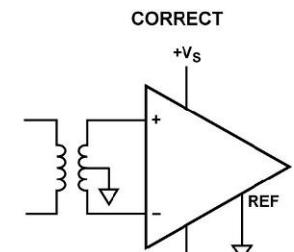
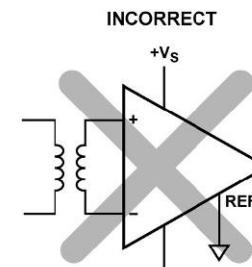
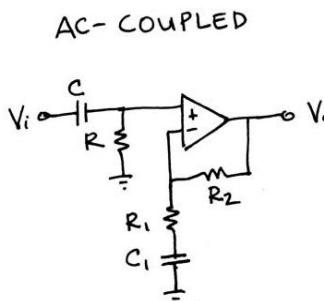
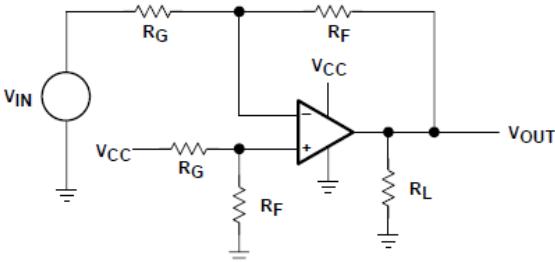
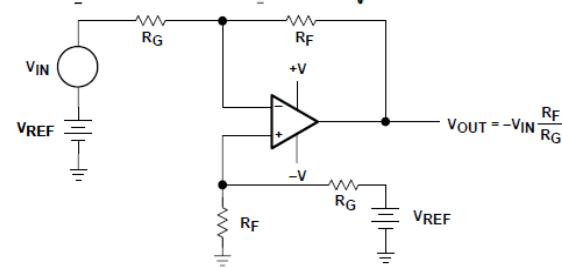
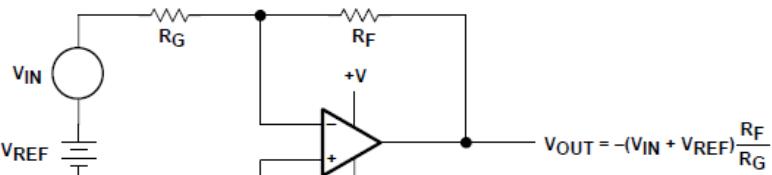
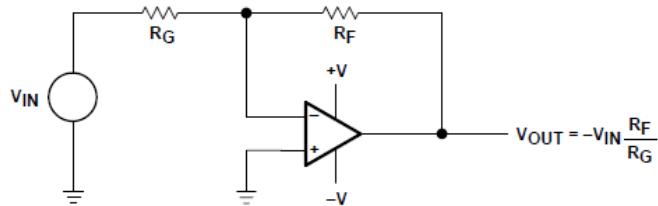
input offset current,  $I_{IO} = I_N - I_P$ .

$$I_{IB} = \frac{(I_N + I_P)}{2}$$

# Amplifier



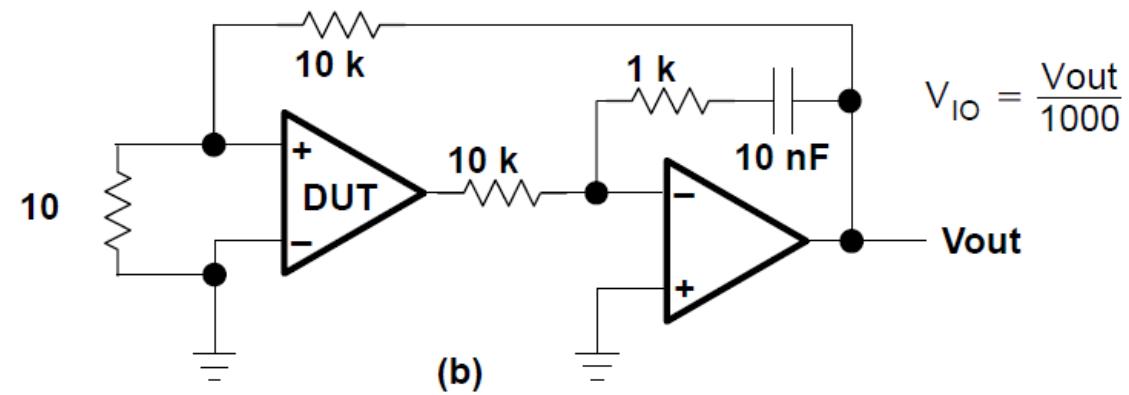
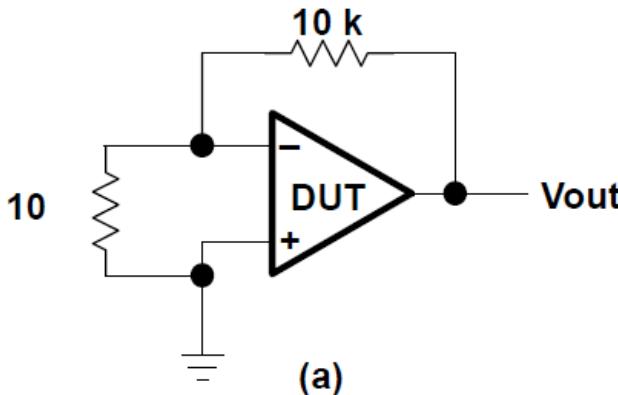
## Bias



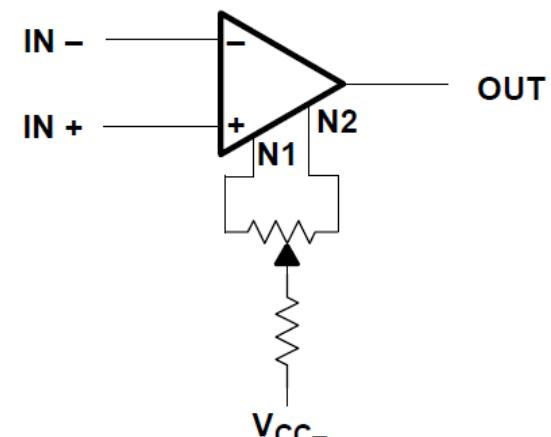
CAPACITIVELY COUPLED

# Amplifier

Zero offset



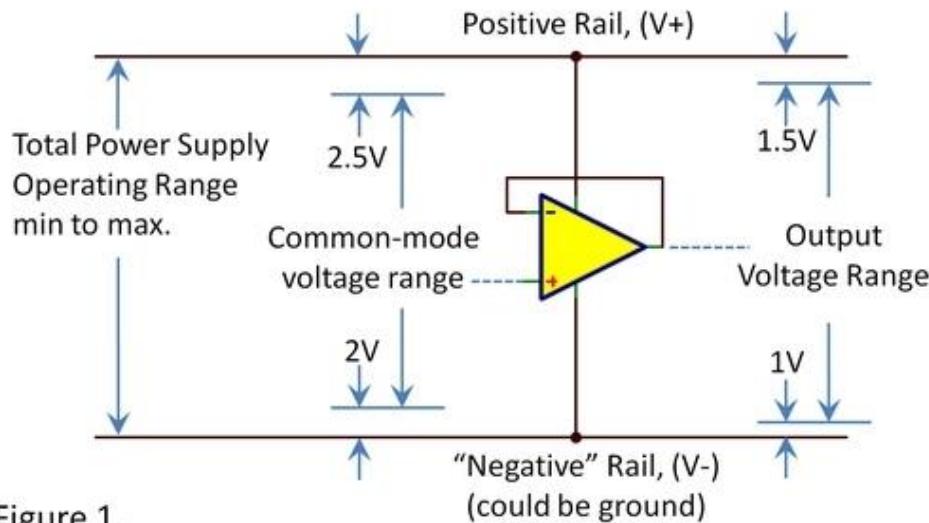
## -1. Test Circuits for Input Offset Voltage



# Amplifier

1922

## Input / output voltage range



Voltage range of this  $G=1$  circuit:  
 $(V_-)+2V$  to  
 $(V_+)-2.5V$

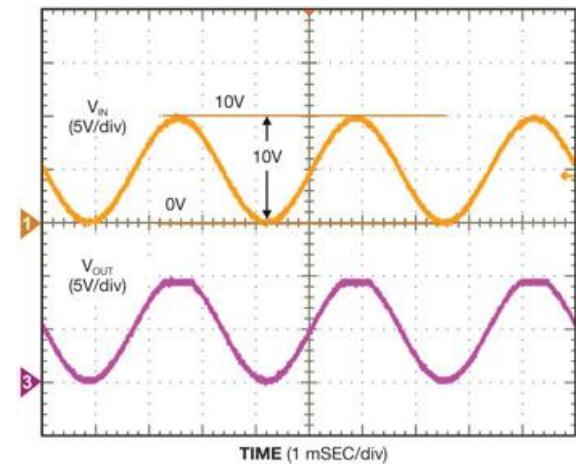


Figure 4 The TLC2272's  $V_{OUT}$  shows clipping when  $V_{IN}$  (Channel 1) exceeds 9.2V.

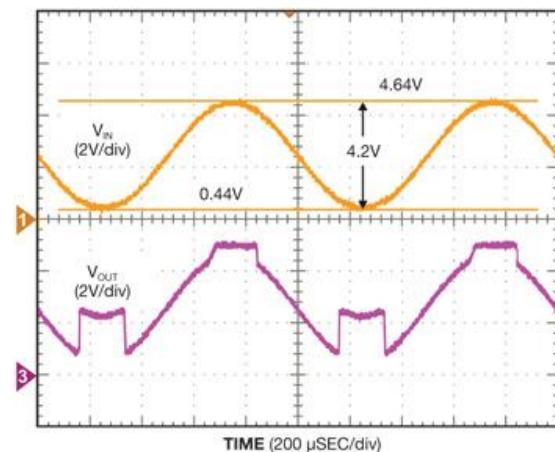
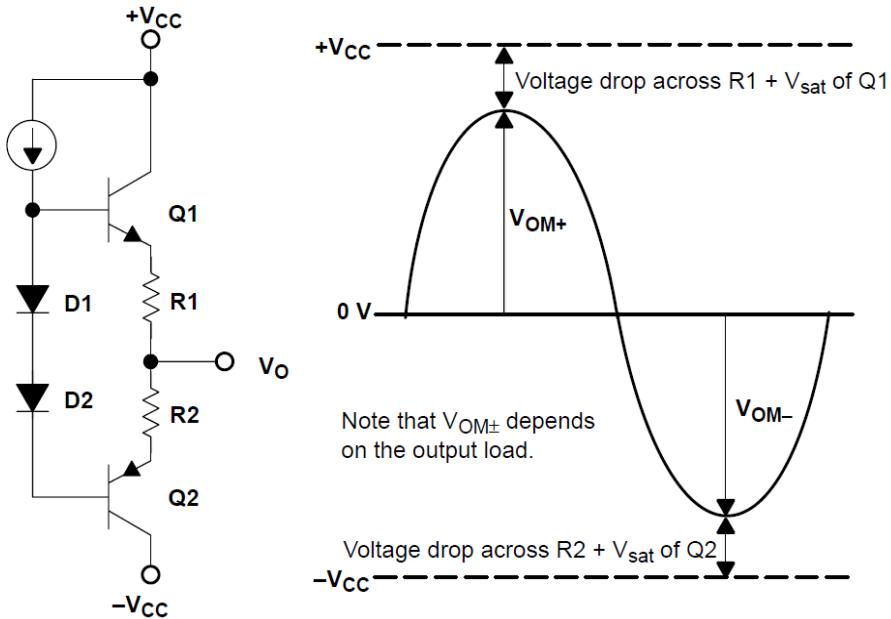


Figure 6 Nonlinear output behavior occurs when the TL971's  $V_{IN}$  is 4.2V p-p.

# Amplifier

## Output stage



## Supply current, $I_{DD}$

# Amplifier

1922

Input / output voltage range (SS op amp)

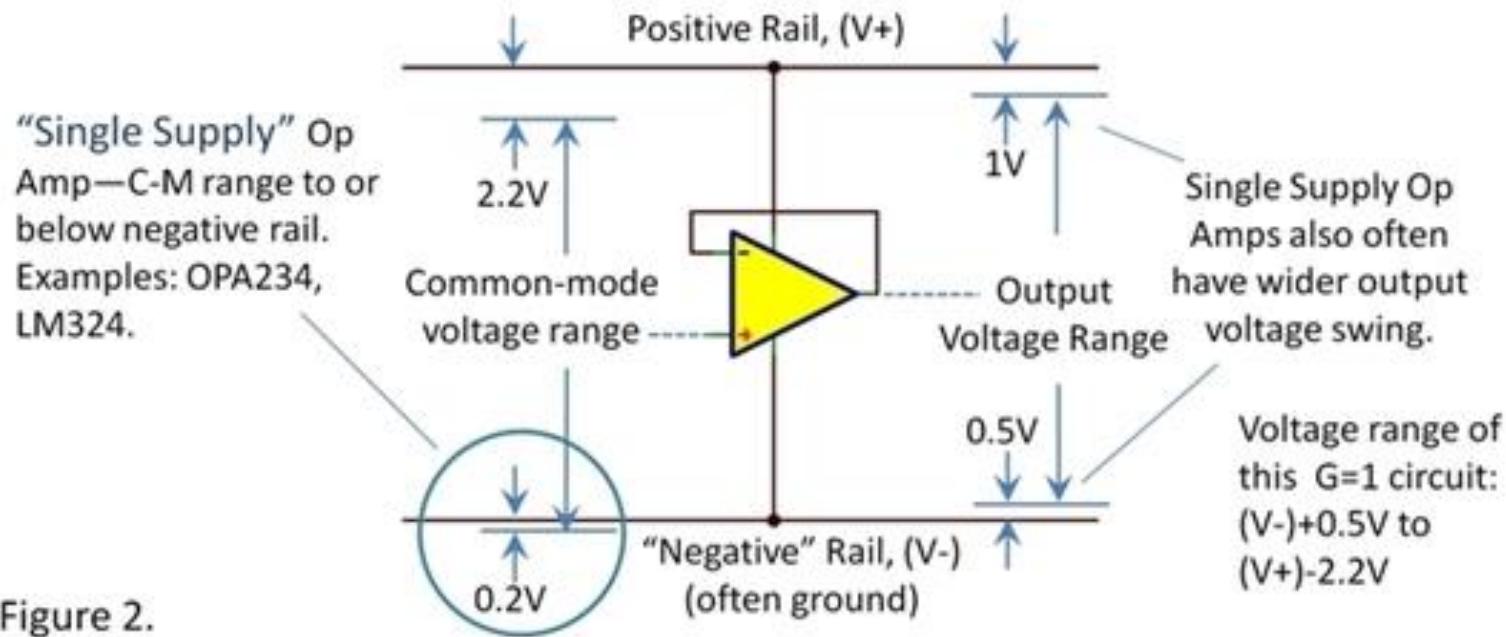


Figure 2.

# Amplifier

Input / output voltage range (RR op amp)

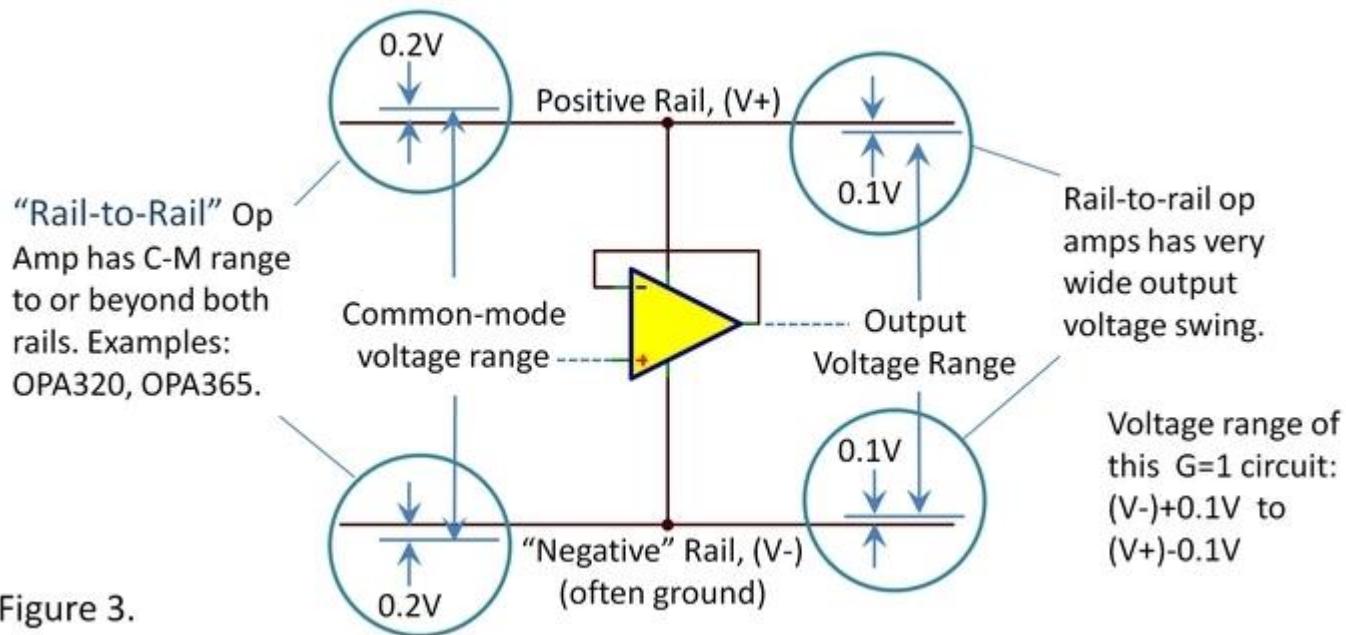
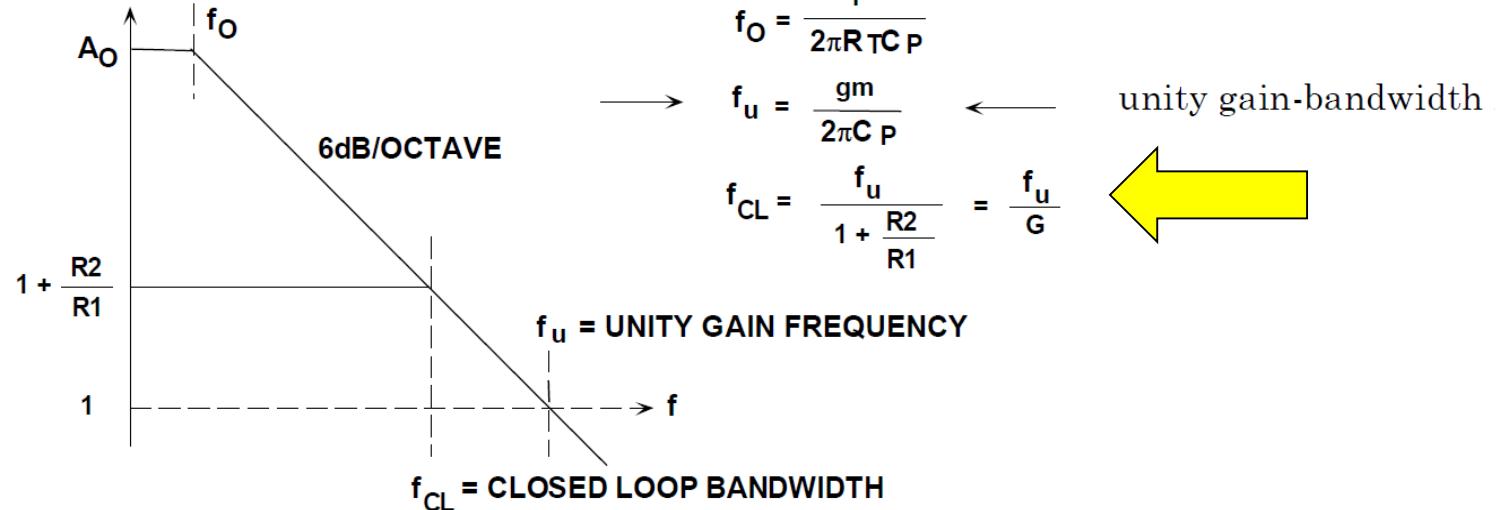
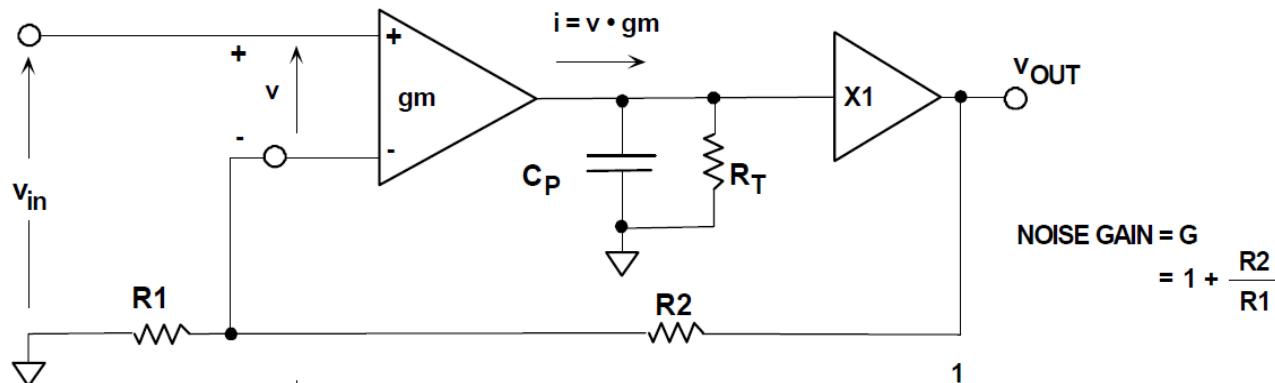


Figure 3.

# Amplifier

Small signal bandwidth and gain

## MODEL AND BODE PLOT FOR A VFB OP AMP

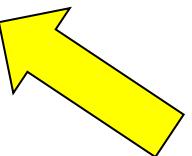


# Amplifier

Large signal bandwidth

full-power bandwidth

$$\text{FPBW} = \frac{\text{SR}}{2\pi A} = \frac{25\text{V} / \mu\text{s}}{2\pi \cdot 1\text{V}} = 4\text{MHz}$$

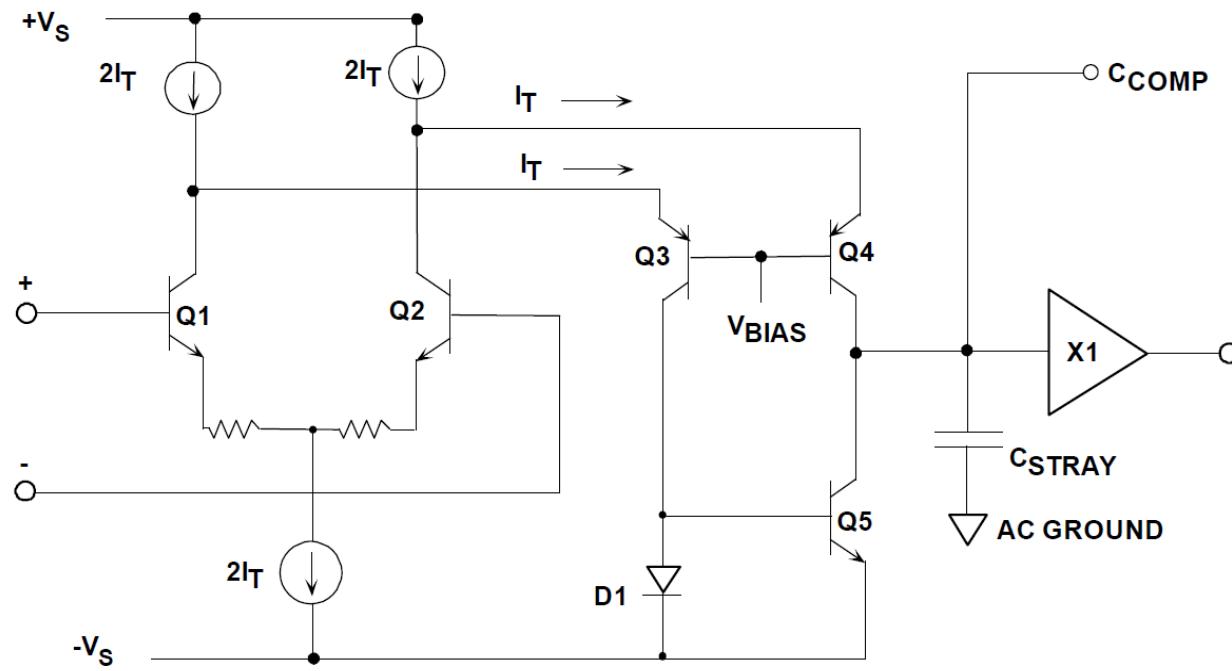
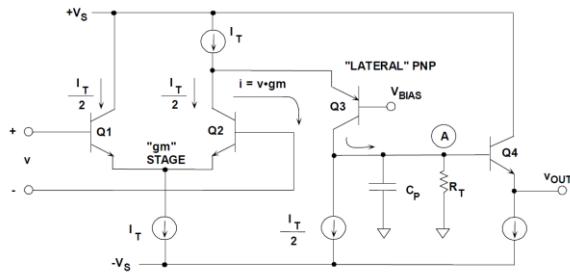


2V peak-to-peak

A is the peak amplitude of the output signal

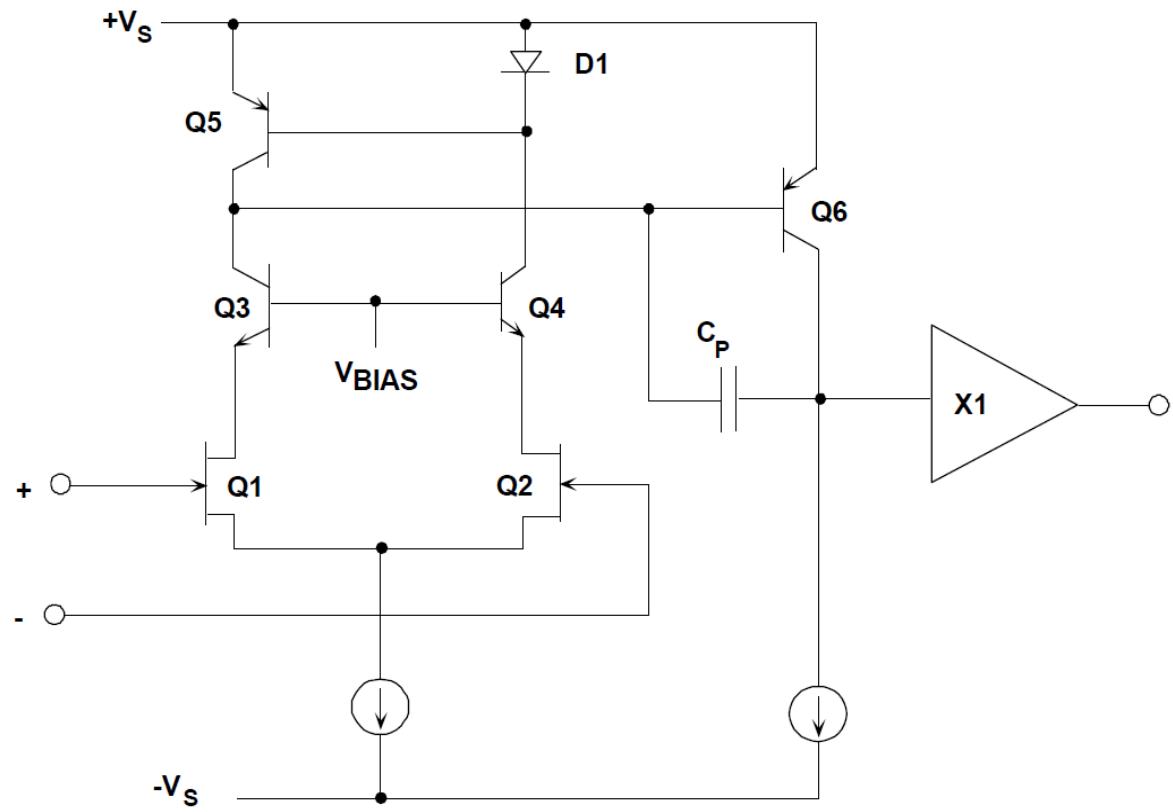
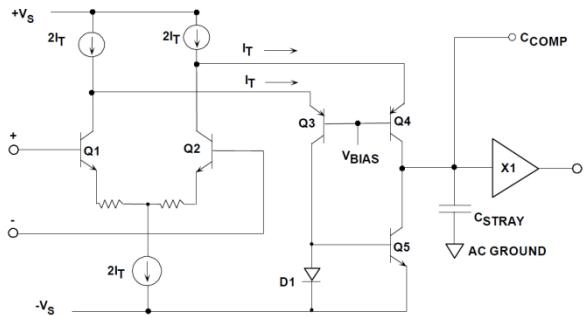
# Amplifier

## Voltage feedback amplifier



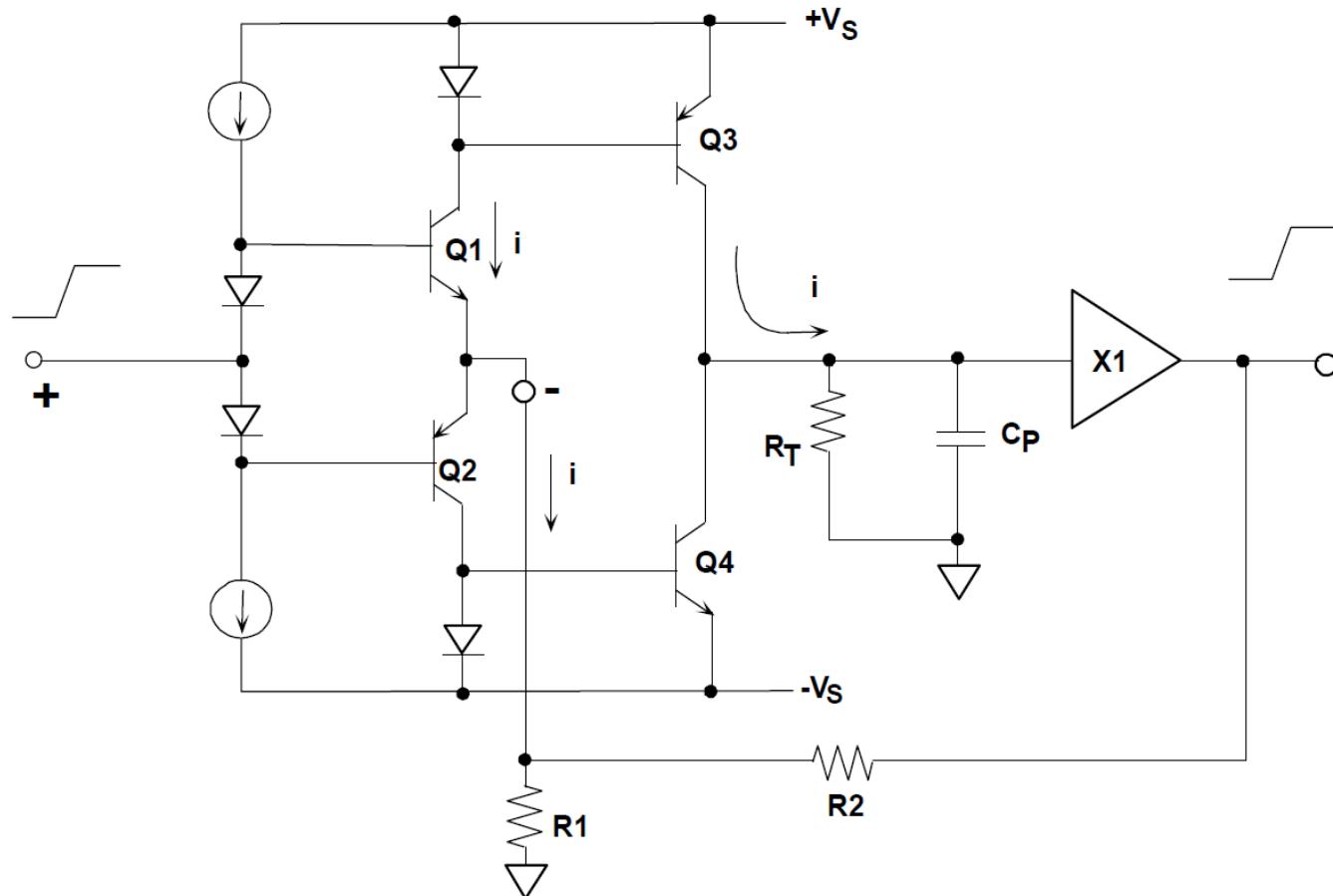
# Amplifier

Voltage feedback amplifier: BiFET



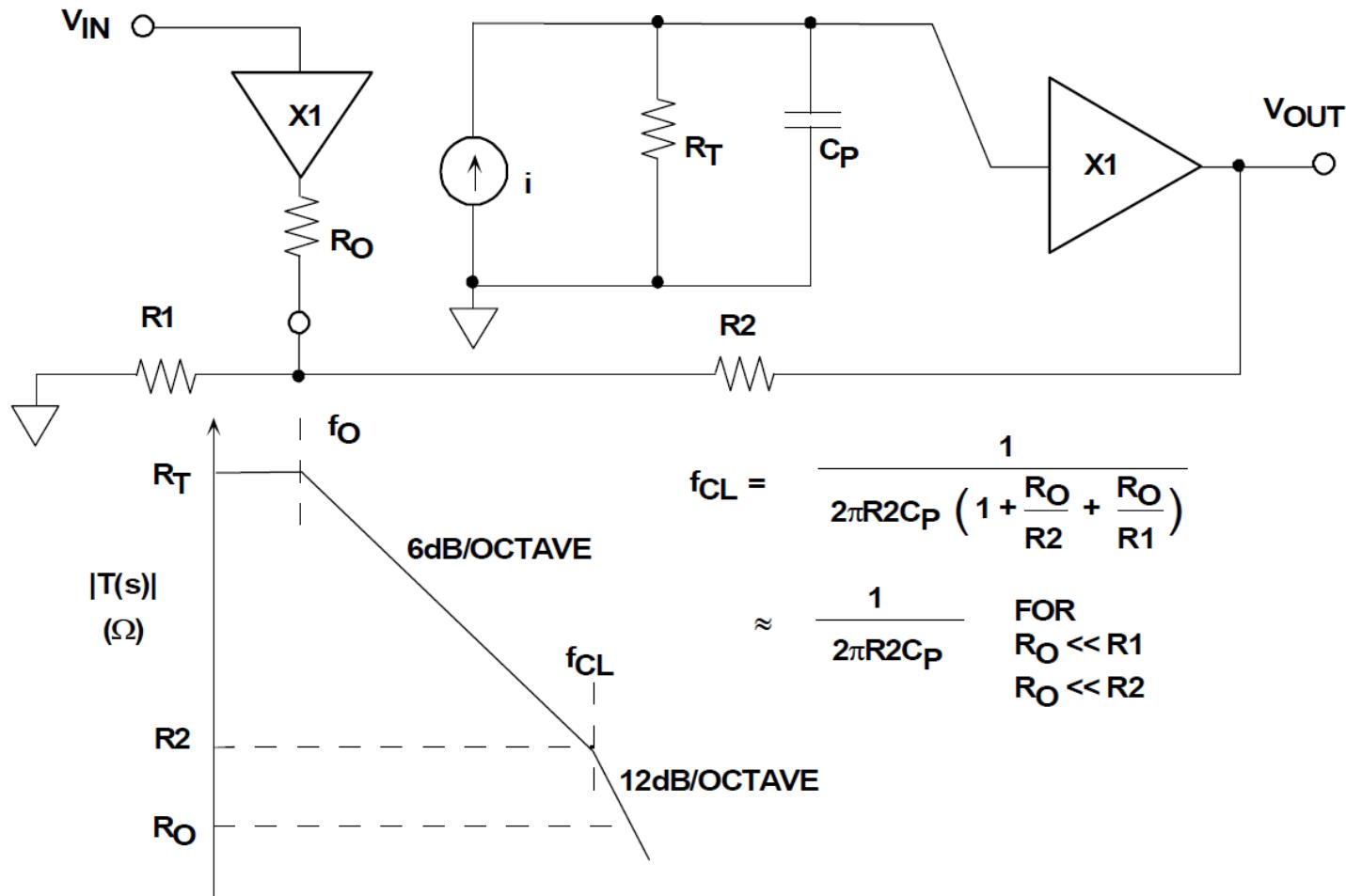
# Amplifier

Current feedback amplifier



# Amplifier

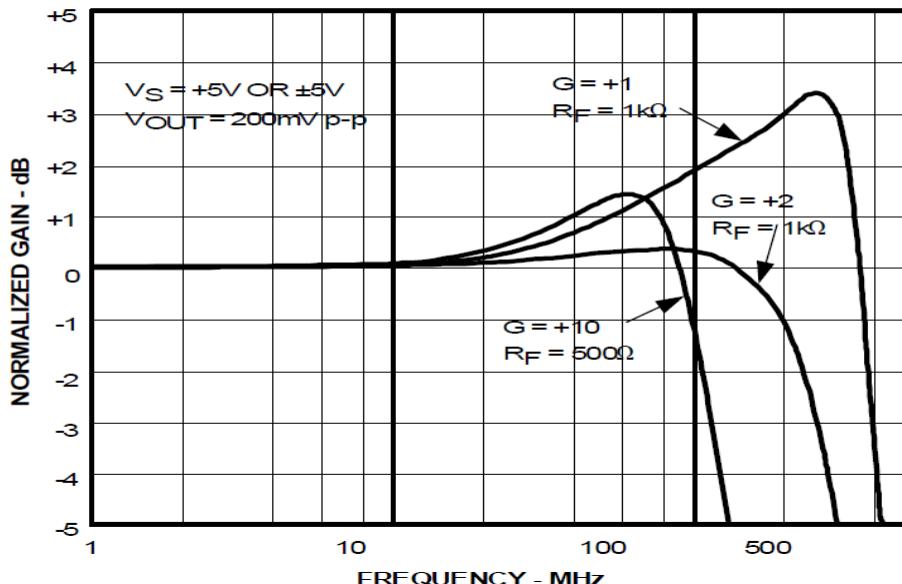
Current feedback amplifier - CFA



# Amplifier

## Current feedback amplifier - CFA

### AD8011 FREQUENCY RESPONSE $G = +1, +2, +10$



keep the amplifier's quiescent power low, yet are capable of supplying *current-on-demand* for wide current excursions required during fast slewing.

Closed-loop bandwidth of a CFB op amp is determined by the internal dominant-pole capacitance and the external feedback resistor, independent of the gain-setting resistor

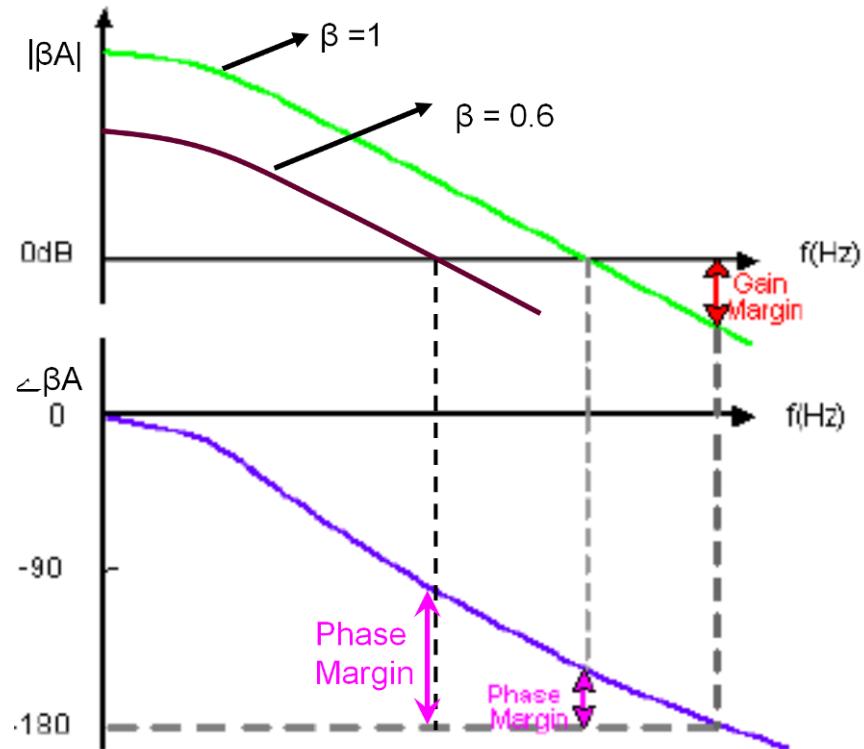
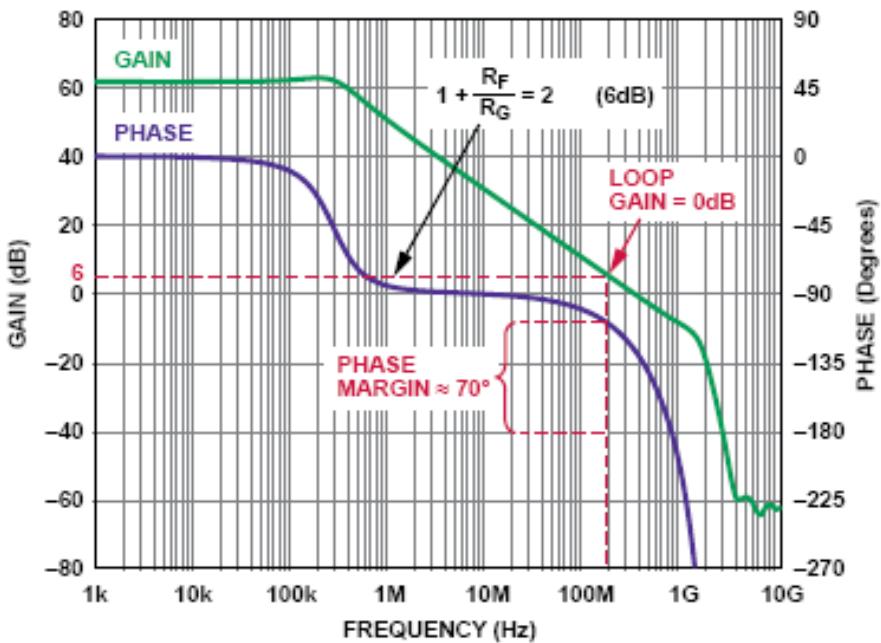
CFB yields higher FPBW and lower distortion than VFB for the same process and power dissipation

Inverting input impedance of a CFB op amp is low, non-inverting input impedance is high

Examination of this equation quickly reveals that *the closed-loop bandwidth of a CFB op amp is determined by the internal dominant pole capacitor,  $C_p$ , and the external feedback resistor  $R_2$ , and is independent of the gain-setting resistor,  $R_1$ .* This ability to maintain constant bandwidth independent of gain makes CFB op amps ideally suited for wideband programmable gain amplifiers.

# Amplifier

## Phase



Phase margin at unity gain ( $\phi_m$ ) is the difference between the amount of phase shift a signal experiences through the op amp at unity gain and  $180^\circ$ :

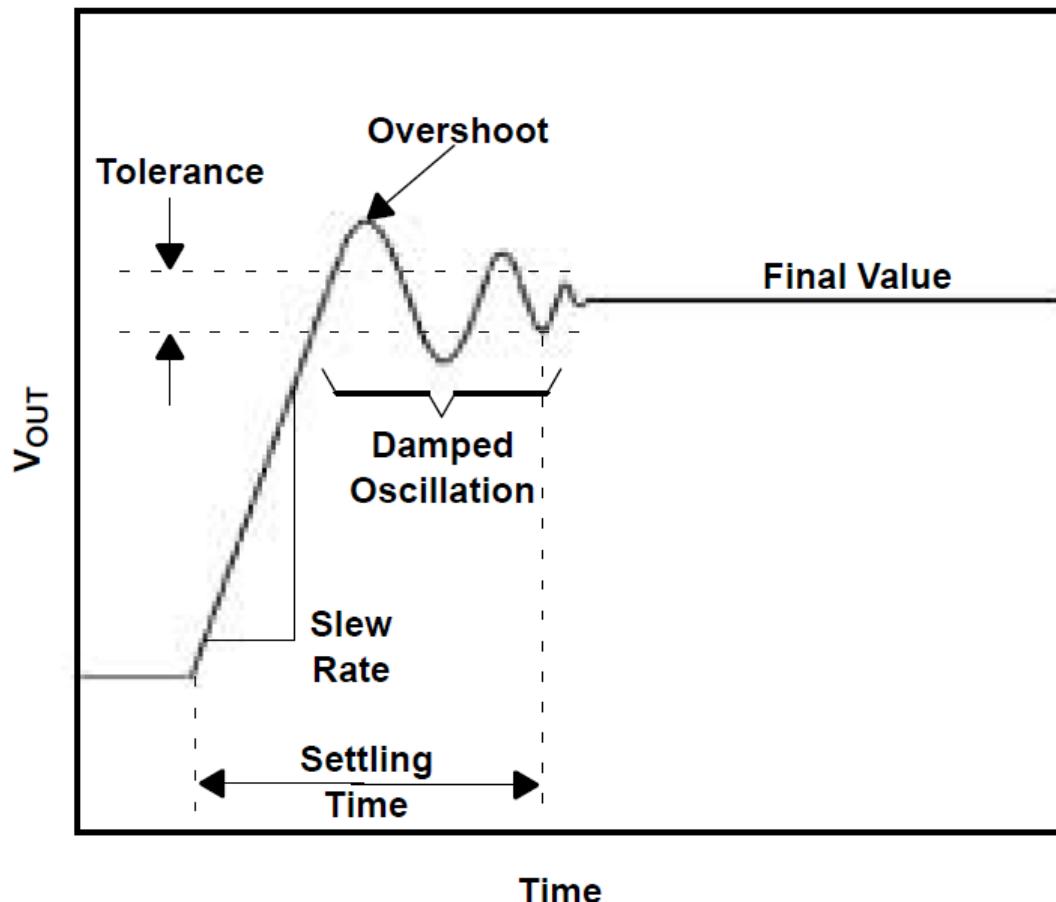
$$\phi_m = 180^\circ - \phi @ B1 \quad (11-6)$$

Gain margin is the difference between unity gain and the gain at  $180^\circ$  phase shift:

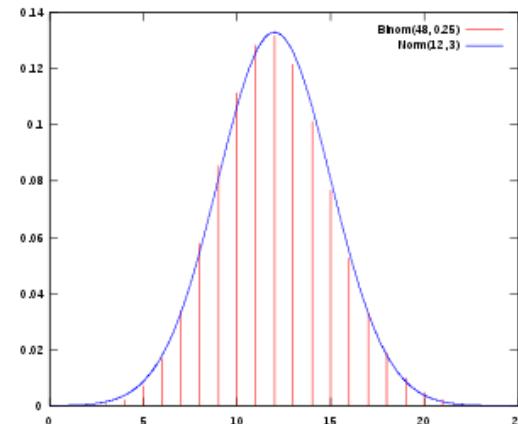
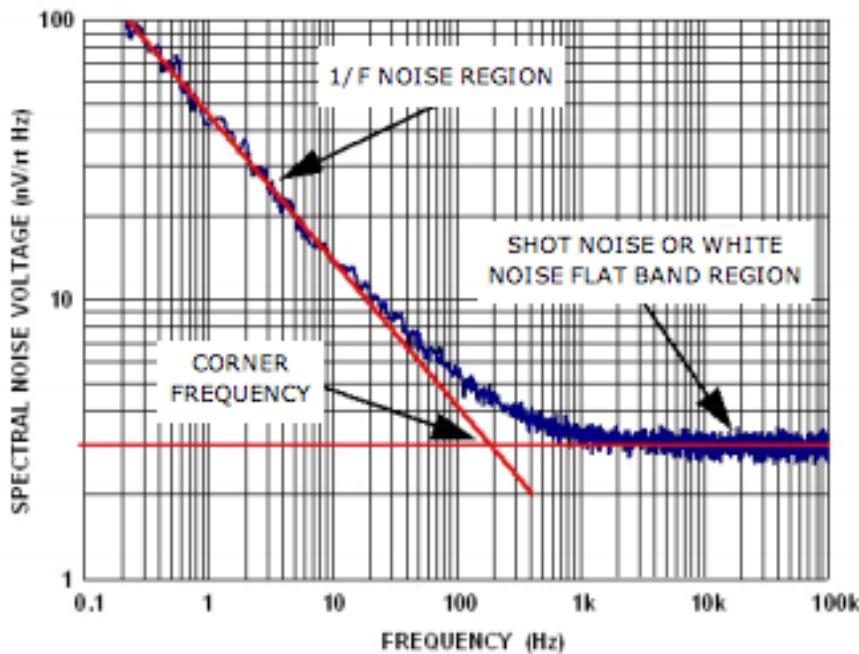
$$\text{Gain margin} = 1 - \text{Gain} @ 180^\circ \text{ phase shift} \quad (11-7)$$

# Amplifier

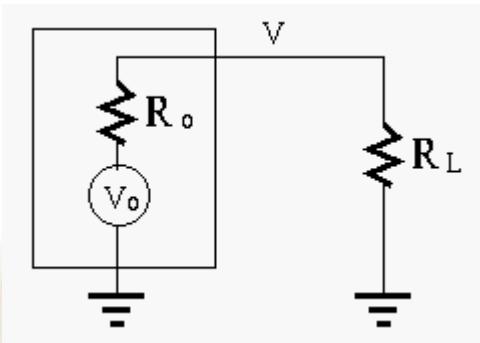
Settling time



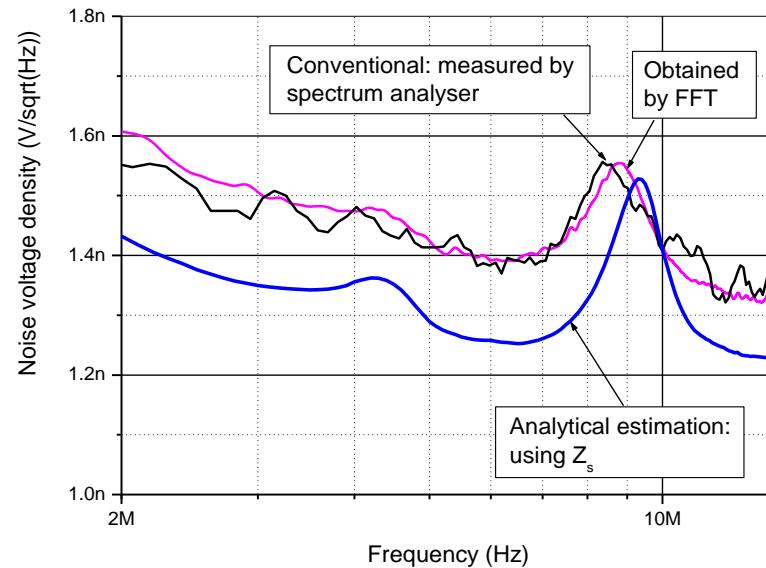
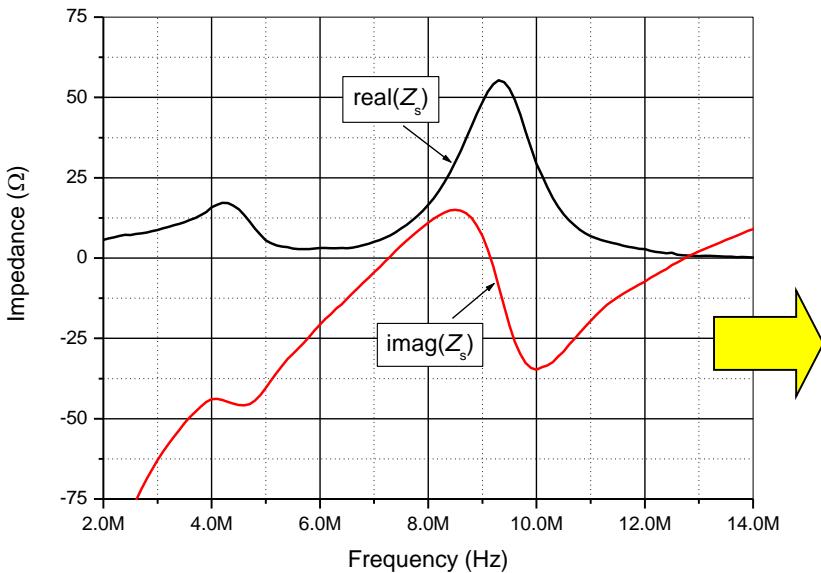
# Noise



$$v_n = \sqrt{\bar{v}_n^2} \sqrt{\Delta f} = \sqrt{4k_B T R \Delta f}$$

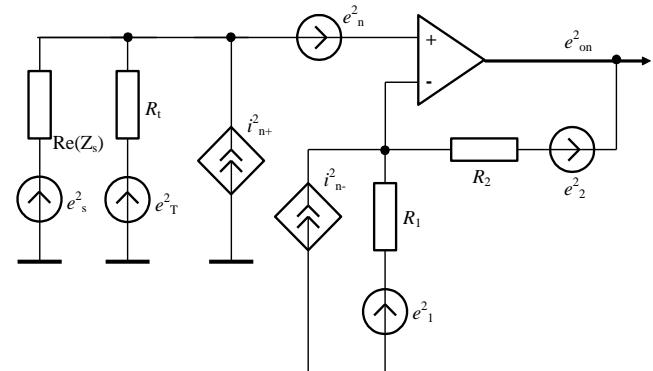


# Noise



$$e_{ntotA}^2 = \left| \frac{GR_t}{R_t + Z_s} \right|^2 e_s^2 + \left| \frac{GZ_s}{R_t + Z_s} \right|^2 e_T^2 + \left| \frac{GR_t Z_s}{R_t + Z_s} \right|^2 i_{n+}^2 + G^2 e_n^2 + (G-1)^2 e_1^2 + e_2^2 + R_2^2 i_{n-}^2$$

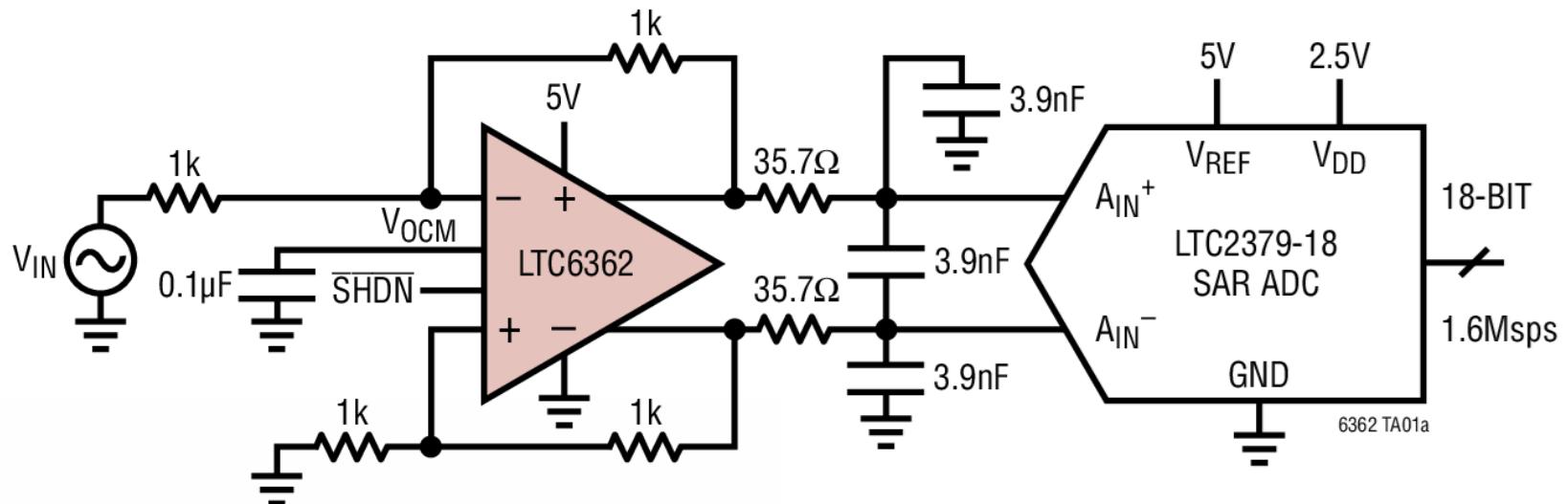
Amplifier adds voltage and current noise sources



# Amplitude estimation: ADC



**DC-Coupled Interface from a Ground-Referenced Single-Ended Input to an LTC2379-18 SAR ADC**



# ADC sampling

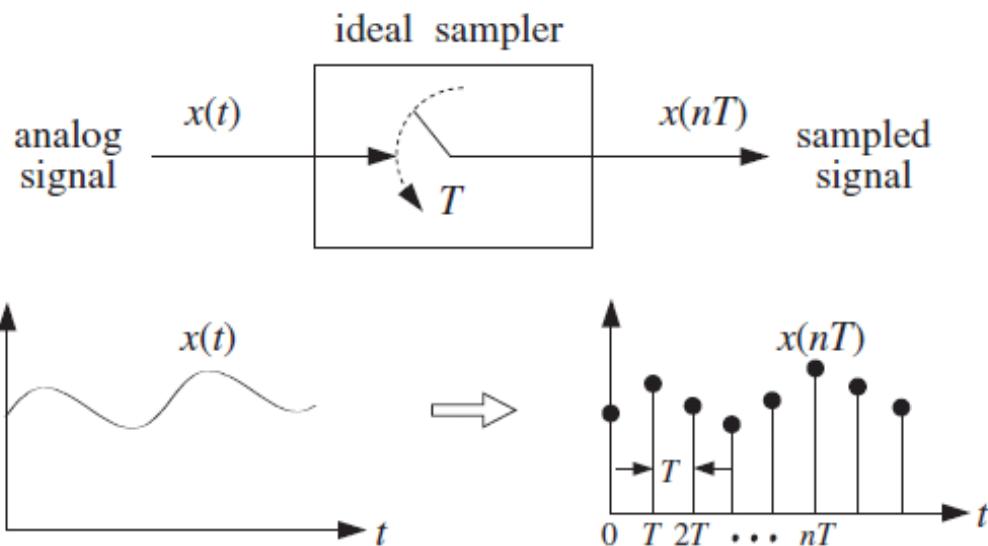
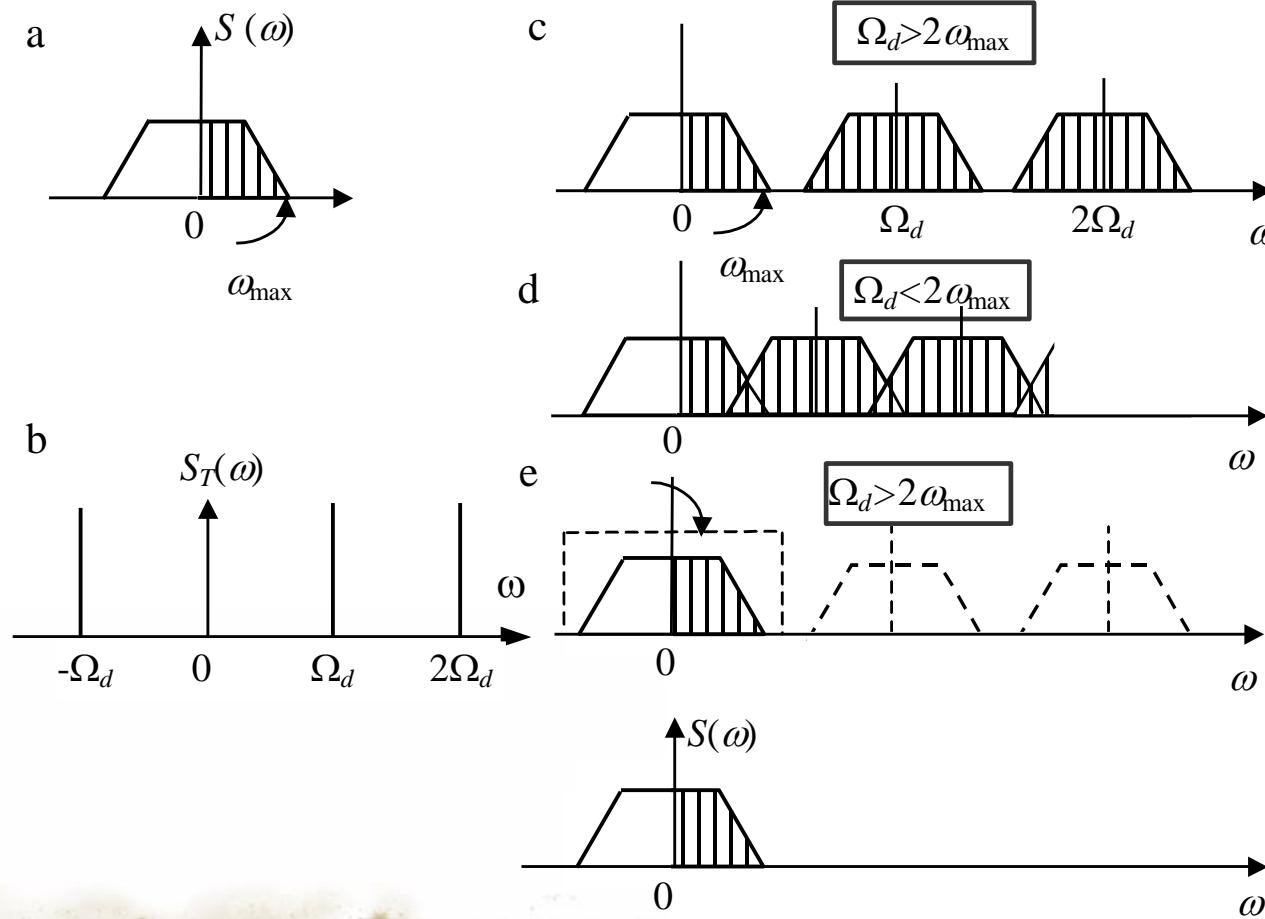
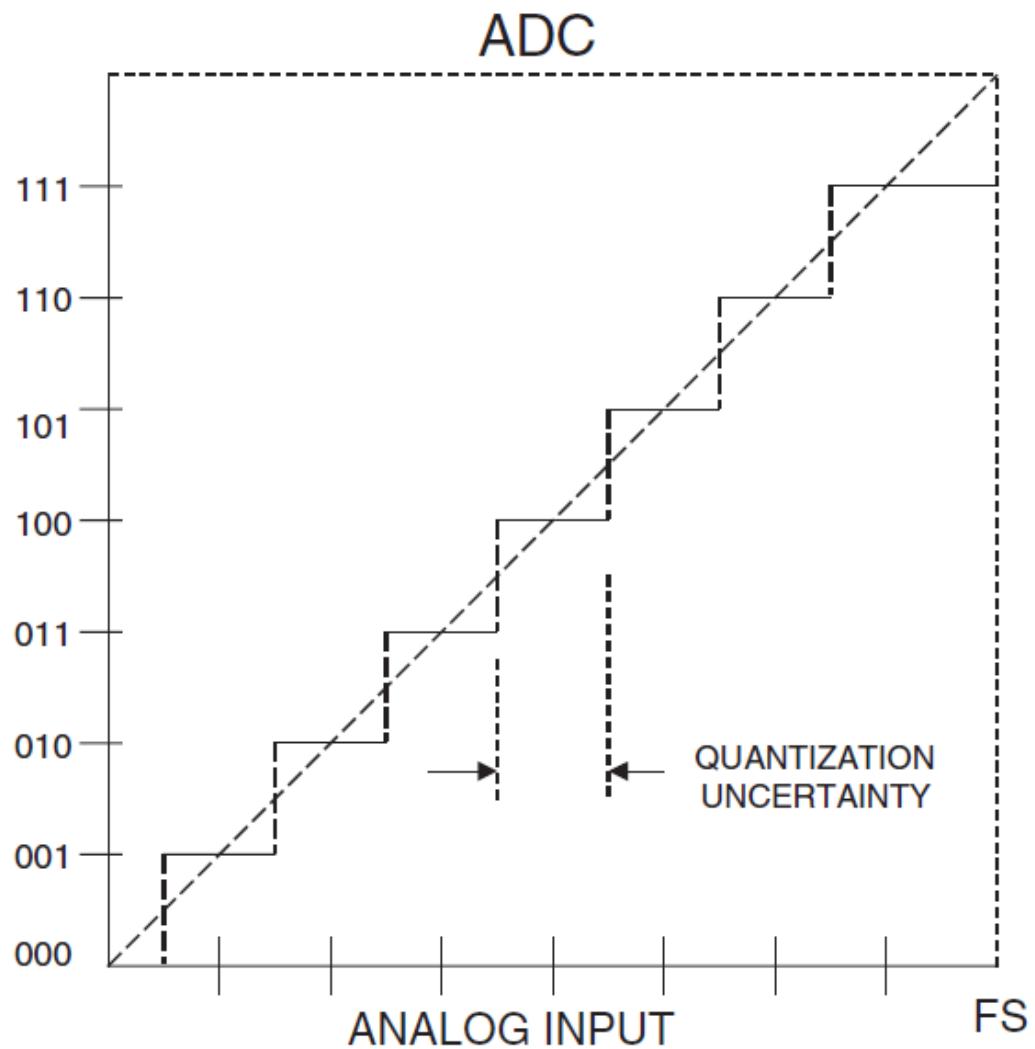


Fig. 1.3.1 Ideal sampler.

# ADC aliasing

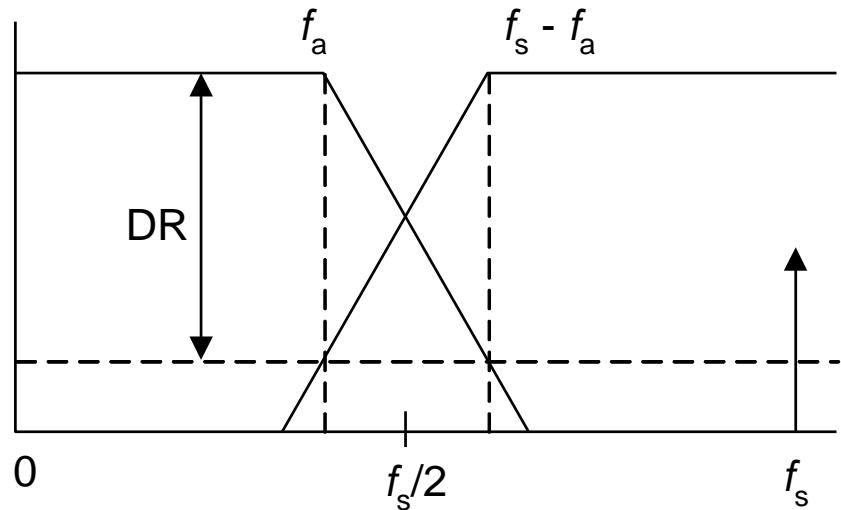


# ADC quantization noise



$$e_{nQ}^2 = \frac{V_{\text{ref}}^2}{0.5f_s \cdot 2^{2 \cdot \text{bits}} 12}$$

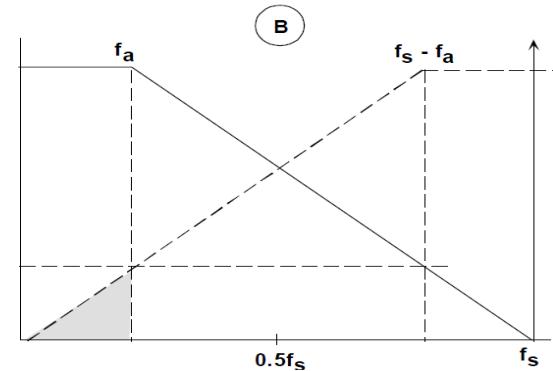
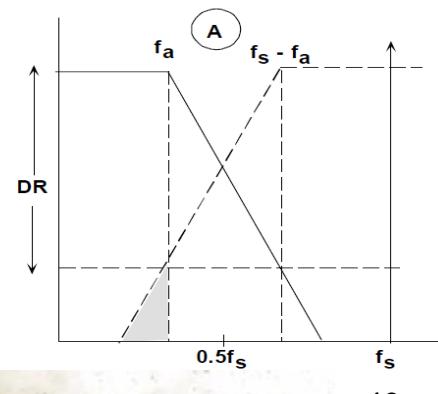
# ADC antialiasing filter selection



$$\text{DR} = 6.021 * N + 1.763 \text{ [dB]}$$

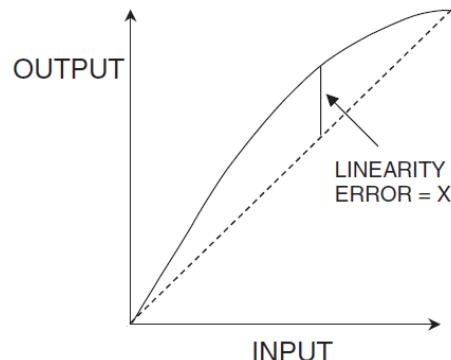
$$\text{DR} = 20 \lg(V_{\text{maxRMS}}/\text{std(noise)})$$

INCREASING SAMPLING FREQUENCY RELAXES REQUIREMENT ON ANTIaliasING FILTER

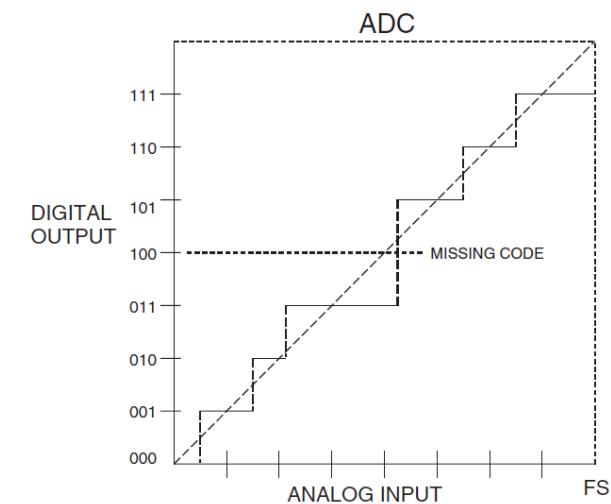
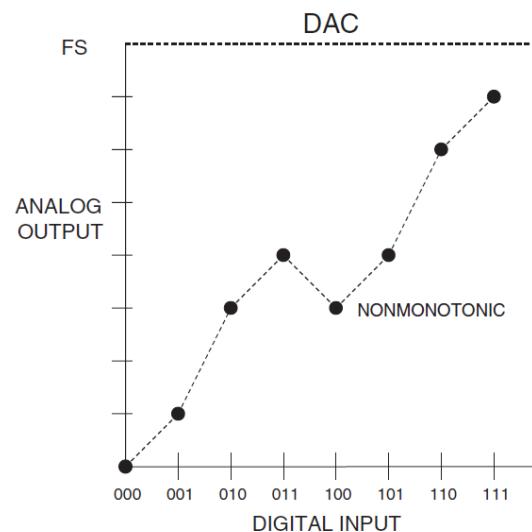
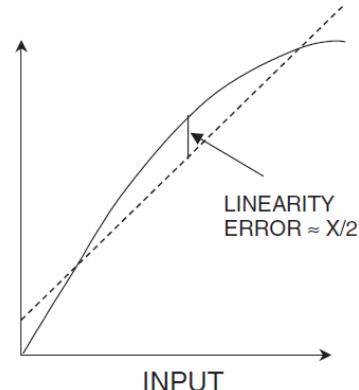


# ADC nonlinearity

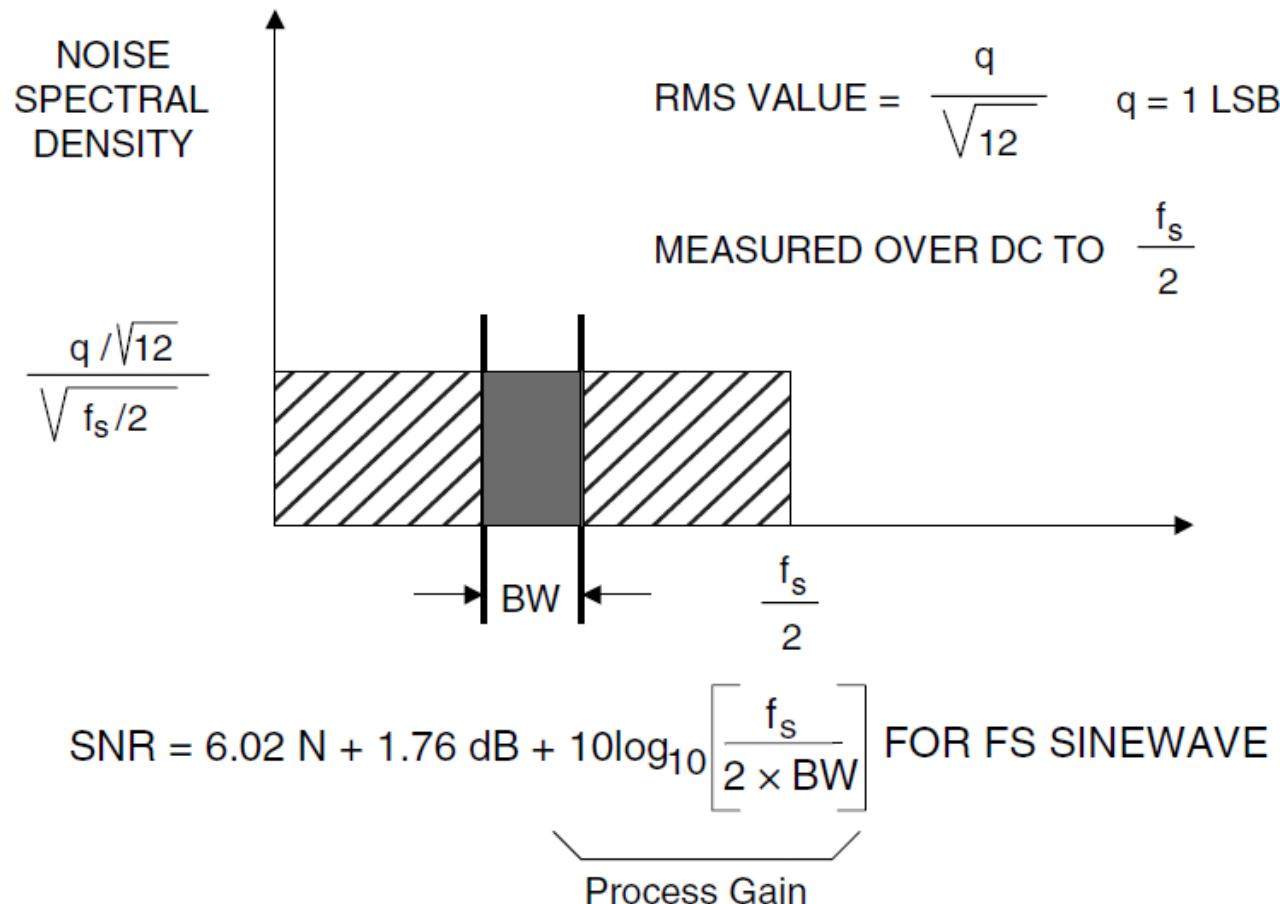
END POINT METHOD



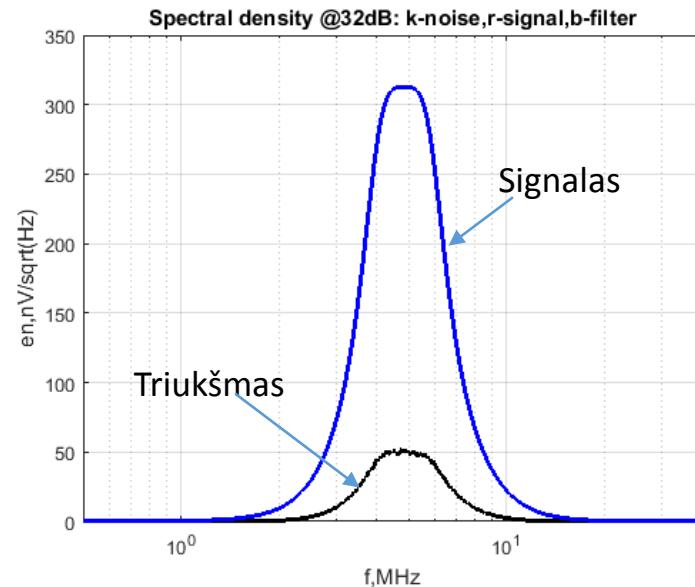
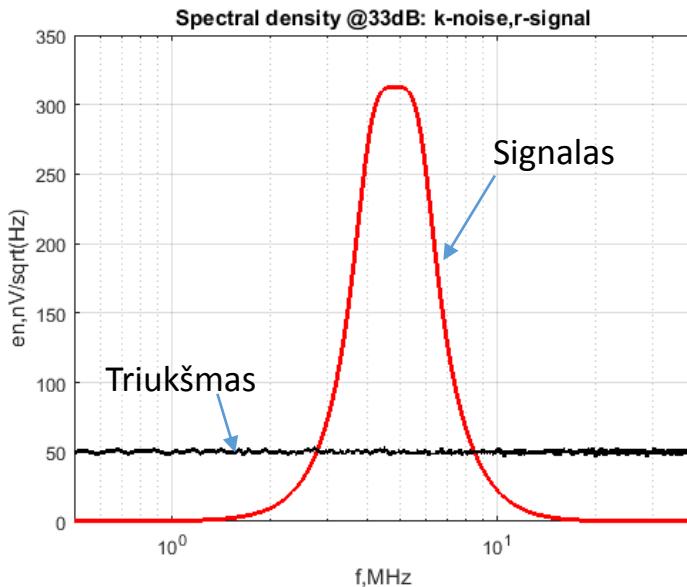
BEST STRAIGHT LINE METHOD



# Digital filtering gain / oversampling

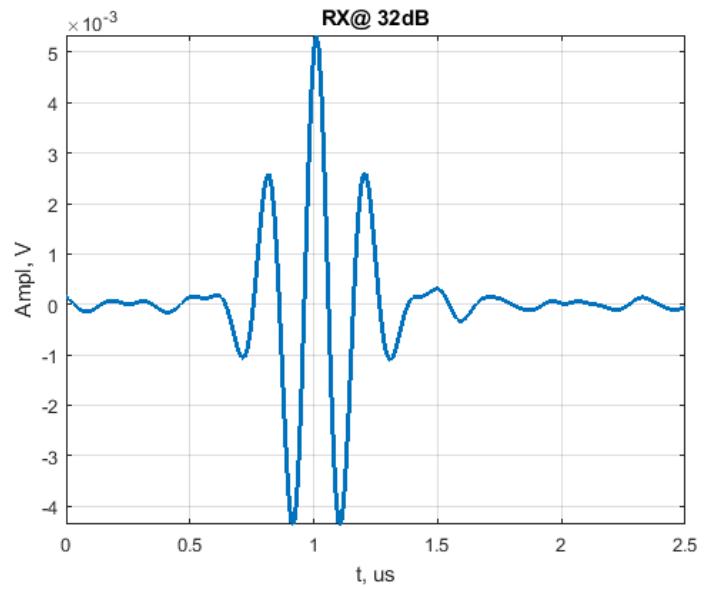
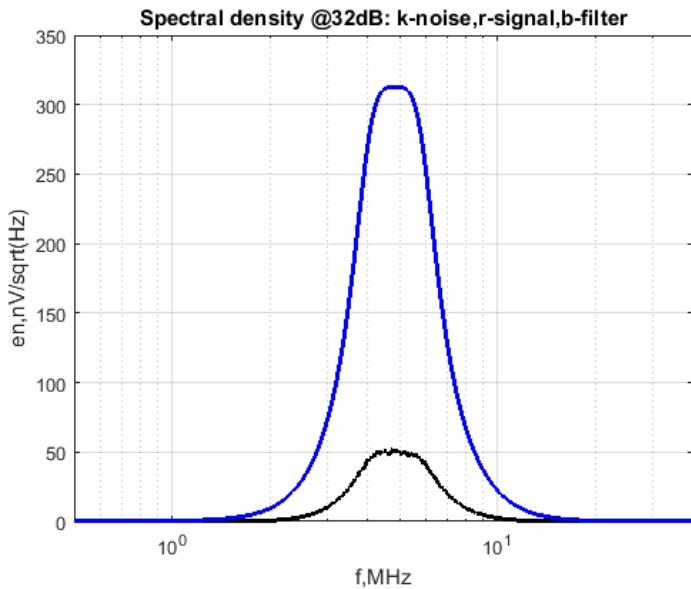


# Filtering



Kuriuo iš pavaizduotų atvejų galime taikyti filtra?

# Filtering types



Passfilter

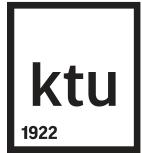
# End: DC measurements



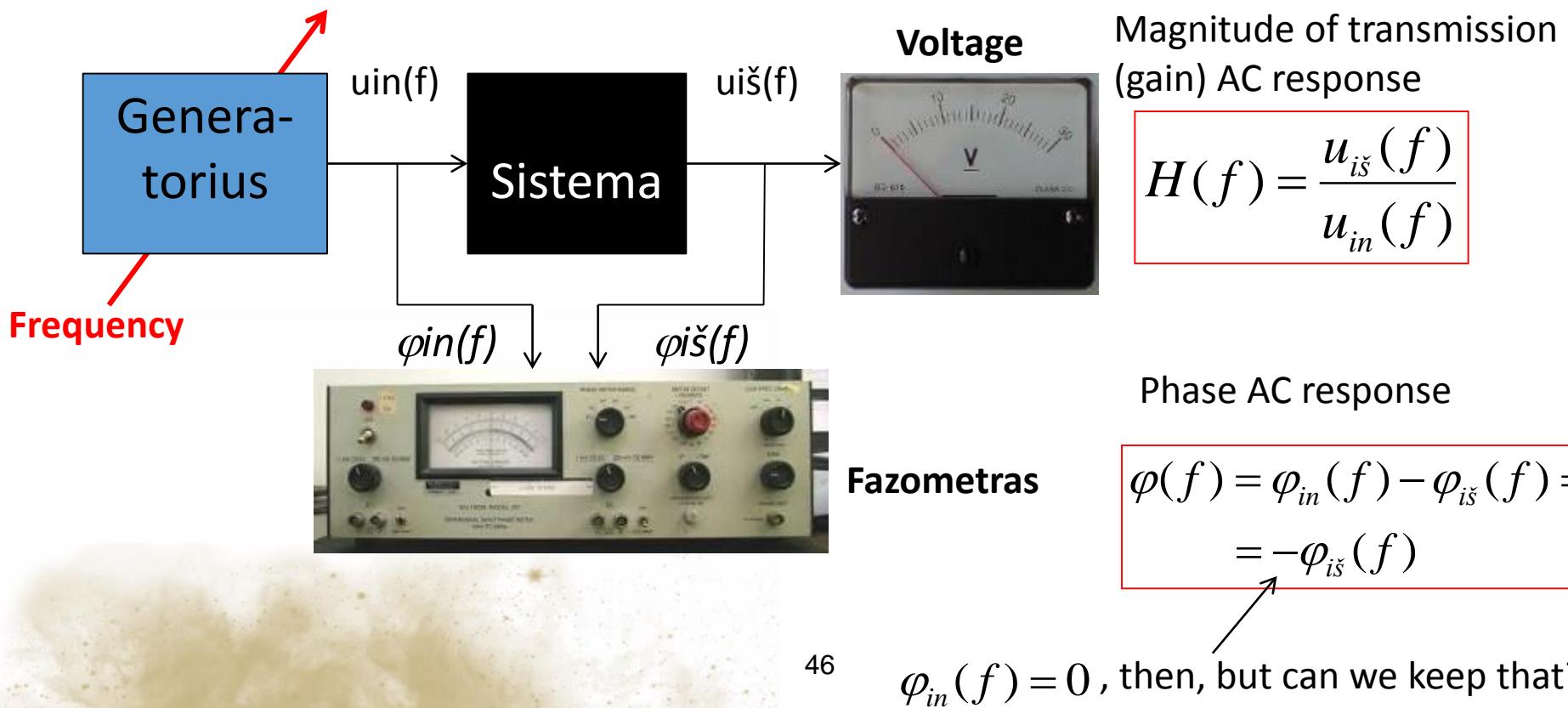
## Conclusions:

- Bias and random errors exist in measurements
- Source of Bias errors – deviation of the system model from real situation
- Source of random errors – noise in electronics
- Careful consideration of the dividers, attenuators, and amplifier modes is needed in order to avoid bias errors
- Even amplifier biasing requires special attention, as well the gain bandwidth evaluation and settling time
- Use of ADC in measurements could add additional sources of errors
- Required bandwidth and dynamic range define the ADC antialiasing filter requirements which can be relaxed by the increase of the sampling frequency
- Filtering can reduce random errors, one of the simplest examples - oversampling

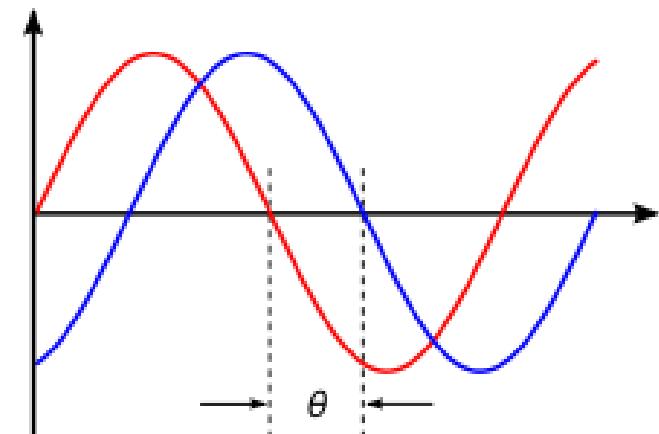
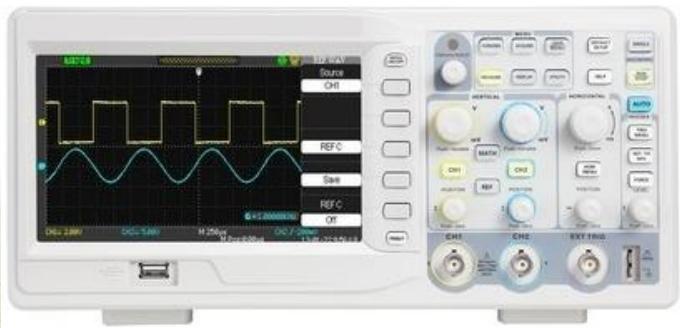
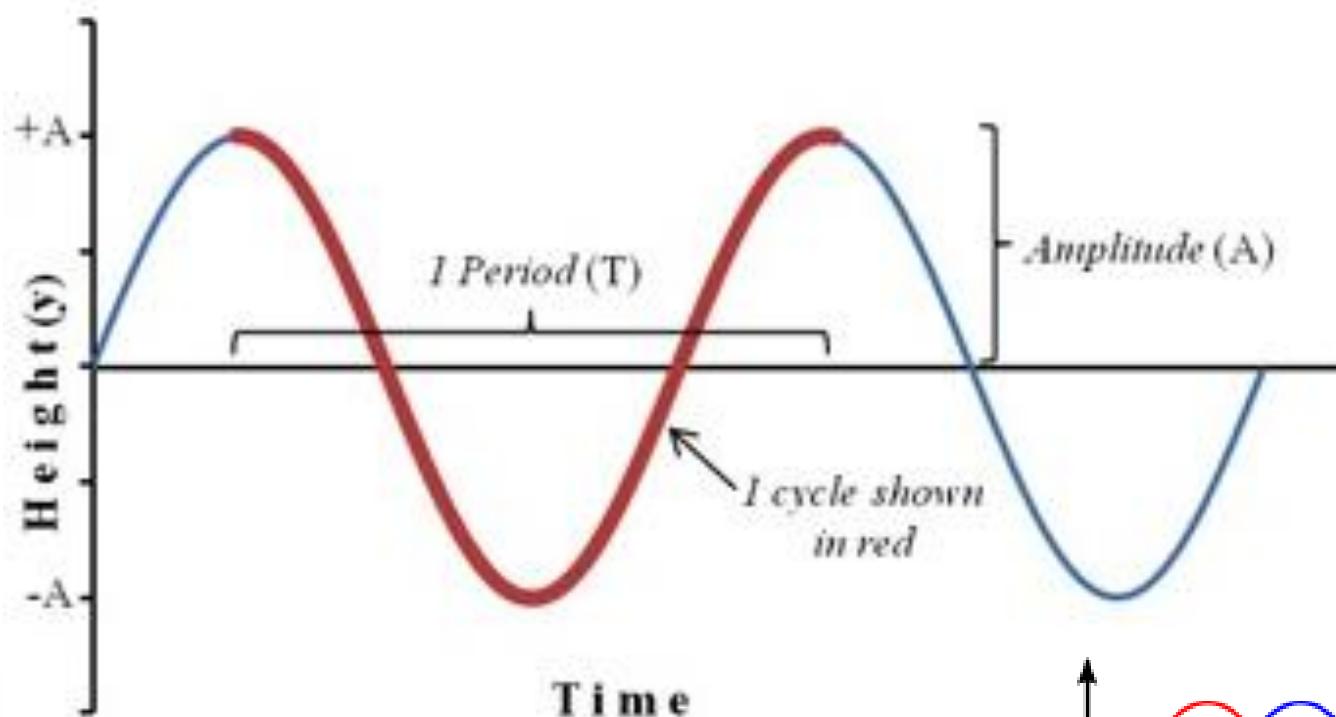
# AC response measurement



Probing signal – variable frequency sinusoid

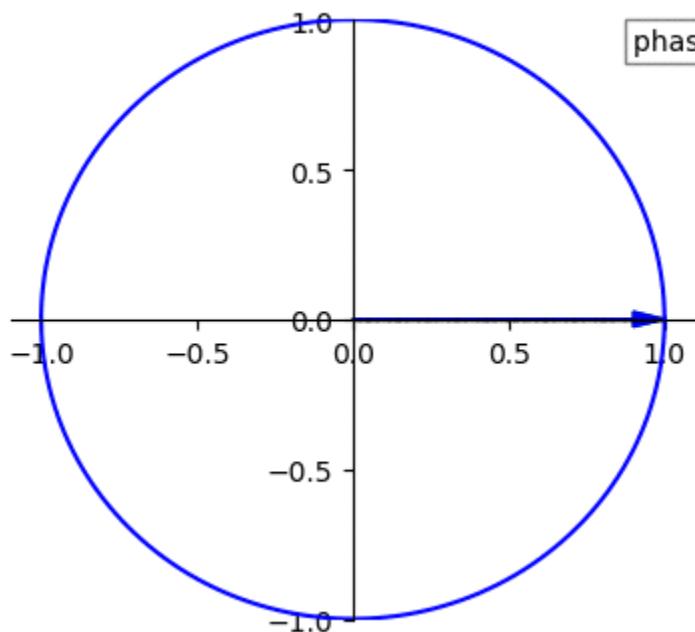


# Sinusoid

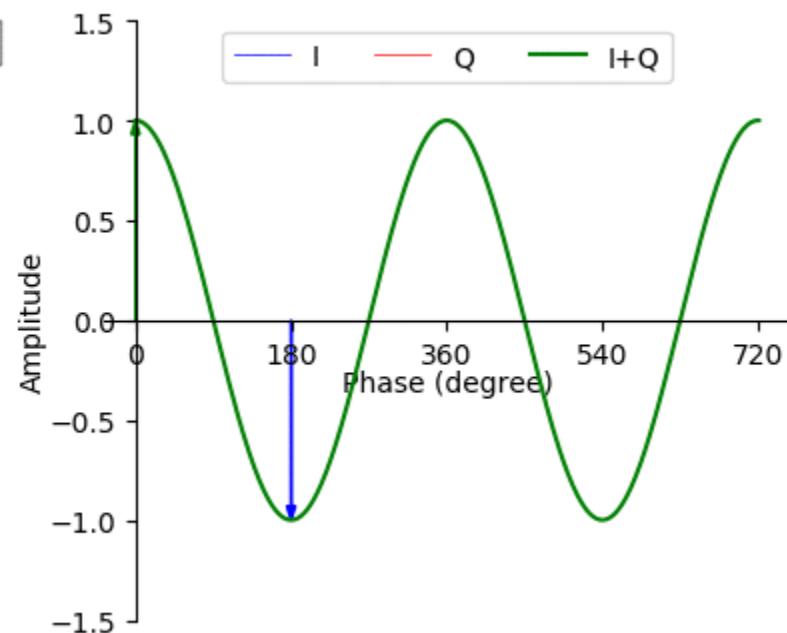


# Sinusoid

Phasor diagram



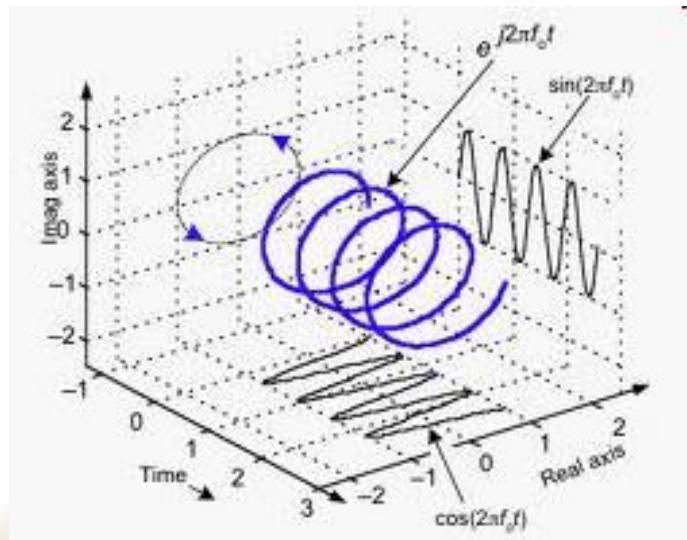
Wave diagram



# Sinusoid

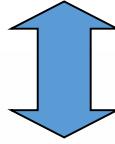
- Complex exponent is periodical and has real and imaginary parts:

$$x[n] = e^{j\omega n} = \cos(\omega n) + j \sin(\omega n)$$

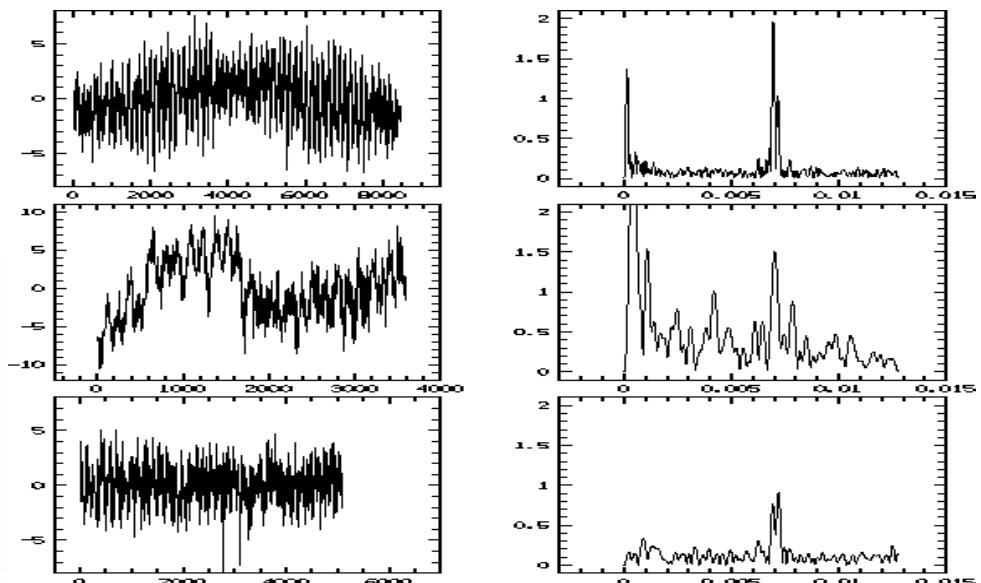


- Continuous time and frequency signal has ALL the values presented (sampled). Such signal is not periodic both in time and frequency domain
- Fourier transform (FT) is used to describe continuous, non-periodic (single) signals

$$S(\omega) = \int_{-\infty}^{+\infty} s(t) e^{-j\omega t} dt$$



$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) e^{j\omega t} d\omega$$

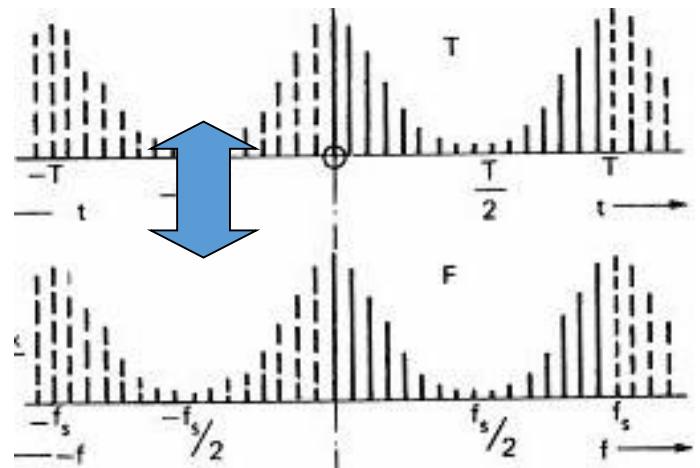
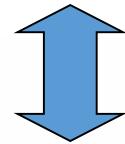


# DFT

Discrete time AND frequency signal is presented only at discrete time AND frequency samples. Such signal is periodic both in time AND frequency domain. Briefly: such signal is both discrete and periodic;

**DFT**-Discrete Fourier transform is used to transforms the siignal between time and frequency axes. **FFT** is the **separate DFT case** where signal length N is log2 (4,16,32,64,128,256,512,1024 etc.)

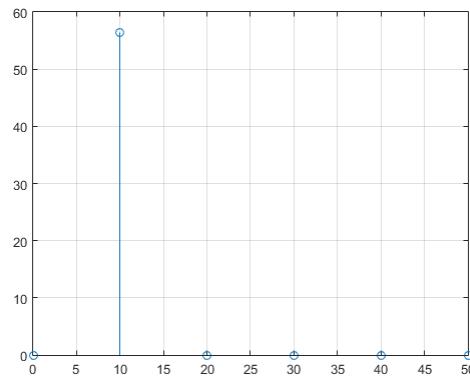
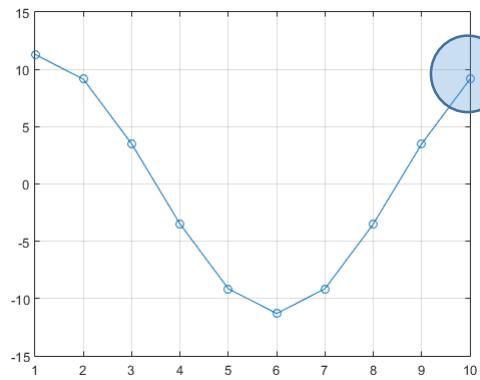
$$S_k = \sum_{n=0}^{N-1} s_n e^{-j2\pi nk/N} = S_{k+N}$$
$$s_n = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{j2\pi nk/N} = s_{n-N}$$



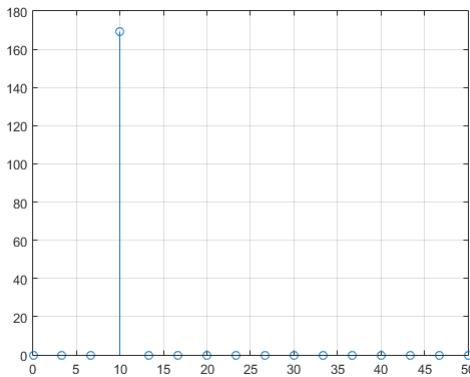
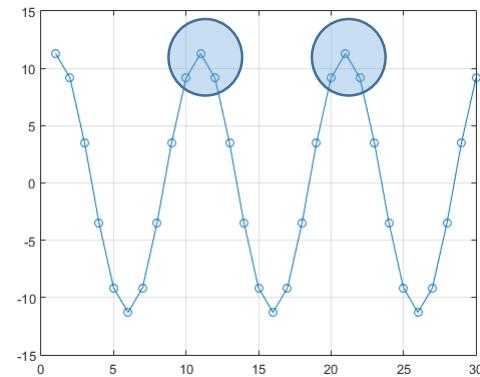
# Sinusoid parameters estimation DFT



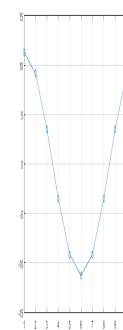
DFT is assuming signal as periodical in time domain  
Everything is OK while record length matches period:



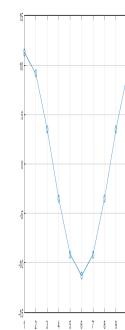
$$F_s = 100\text{MHz}, f_0 = 10\text{MHz}, N = 10$$



+



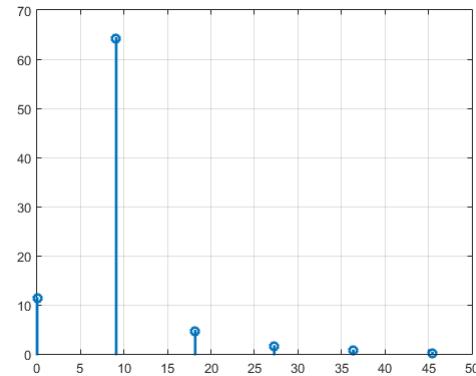
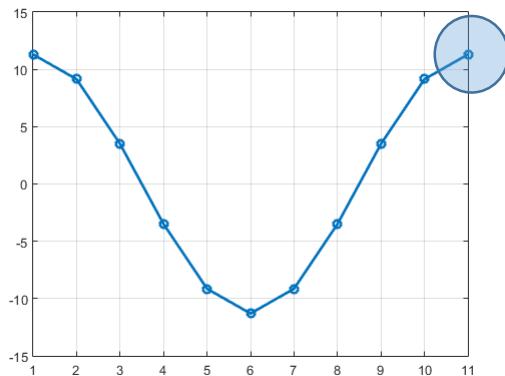
+



# Sinusoid parameters estimation DFT

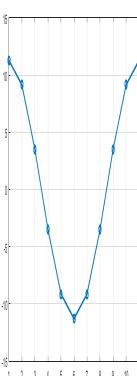
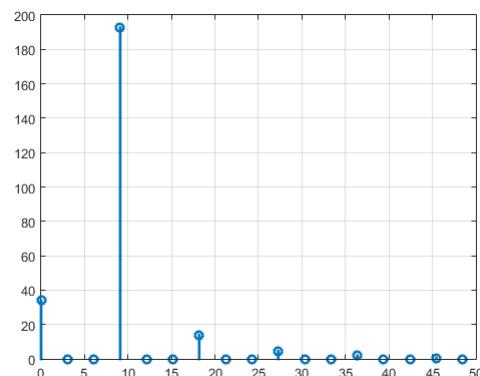
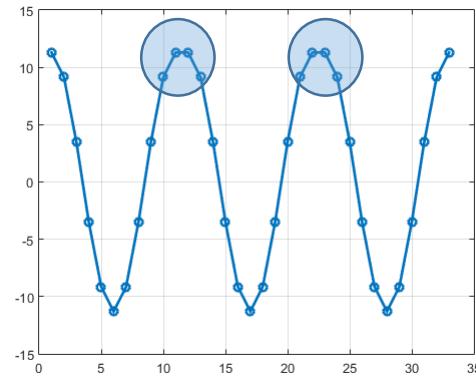


Tačiau vos tik įrašo ilgis nebeatitinka lyginio periodų skaičiaus,  
Spektras keičiasi

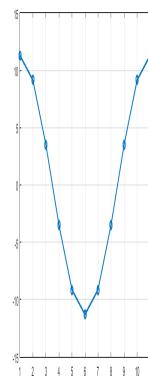


$$F_s = 100\text{MHz}, f_0 = 10\text{MHz}, N = 11$$

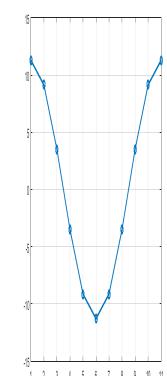
Kodėl?



+



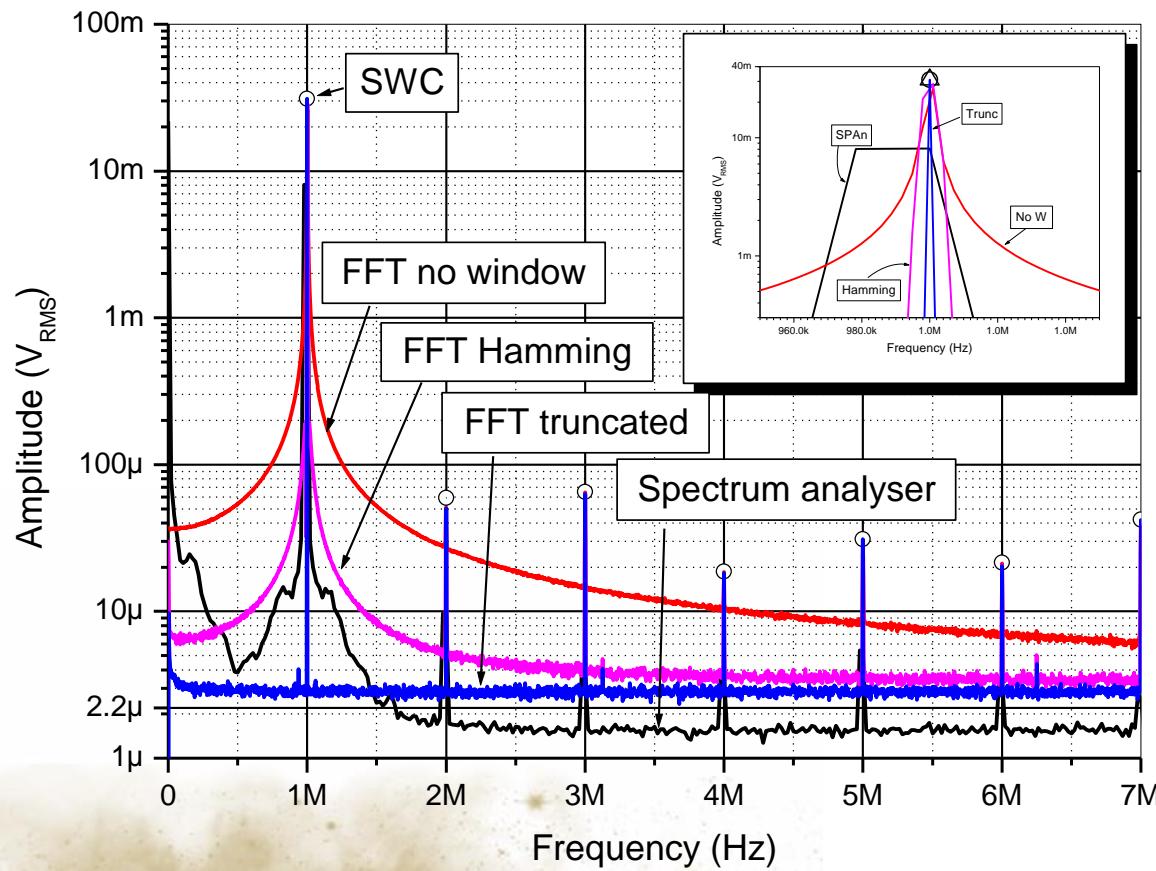
+



# Sinusoid parameters estimation DFT

1922

Problem: not all frequencies' magnitude can be accurately estimated using DFT/FFT



# Solution-DTFT

Discrete time signal is described (sampled) at discrete time instances. Spectrum of such signal is periodic

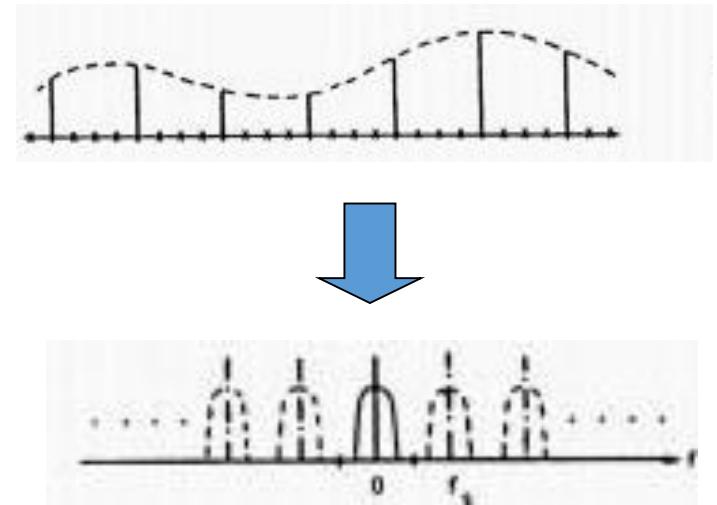
Discrete time Fourier transform **DTFT** is used to analyze sampled single/non-periodic signals

$$S(\Omega) = \sum_{n=-\infty}^{+\infty} s_n e^{-j\Omega n}$$

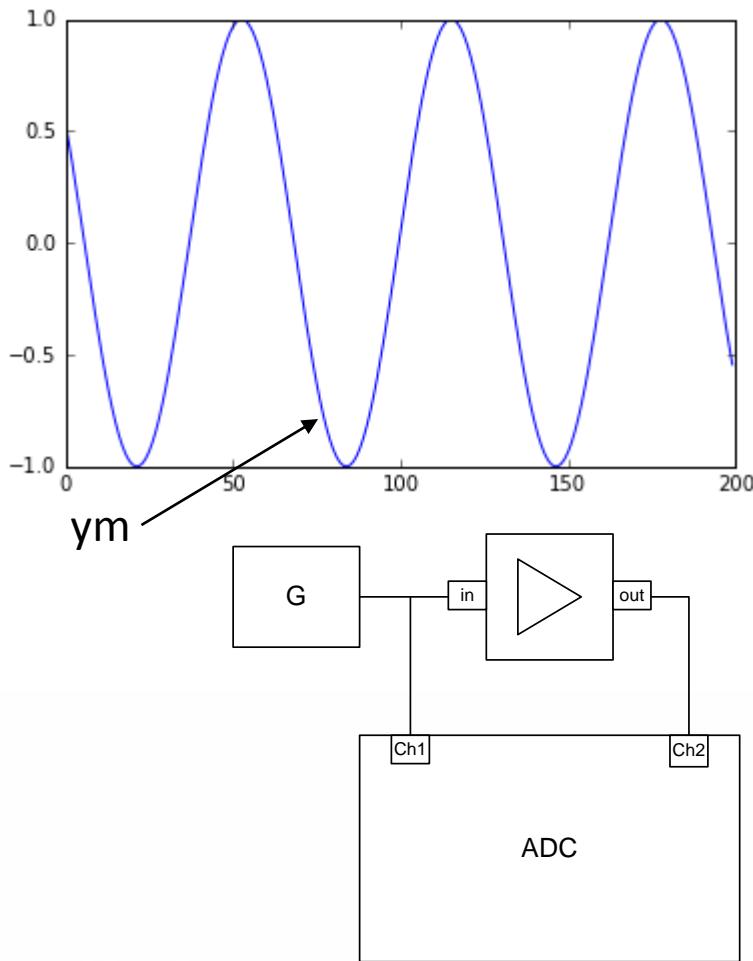
Frequency here is Continuous variable

Time is discrete

$$s_n = \frac{1}{2\pi} \int_0^{2\pi} S(\Omega) e^{j\Omega n} d\Omega$$



# Sinusoid parameters estimation SWC



**SWC=single frequency DTFT**

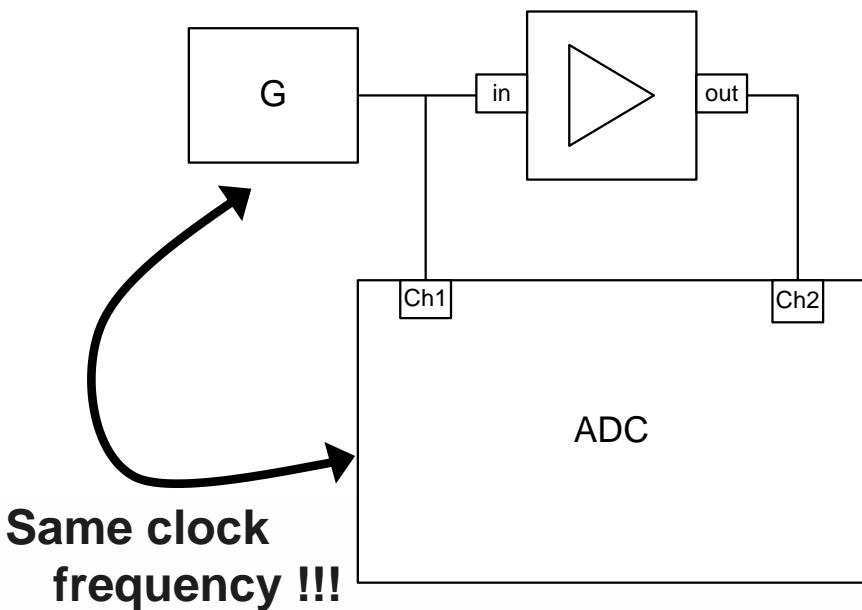
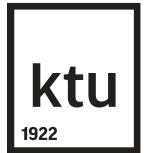
$$u(t) = U_c \cos(2\pi f t) + U_s \sin(2\pi f t) + U_{DC}$$

$$U_c = \frac{\sum_{m=1}^M [\cos(2\pi f t_m) \cdot y_m]}{\sum_{m=1}^M [\cos(2\pi f t_m)]^2}$$

$$U_s = \frac{\sum_{m=1}^M [\sin(2\pi f t_m) \cdot y_m]}{\sum_{m=1}^M [\sin(2\pi f t_m)]^2}$$

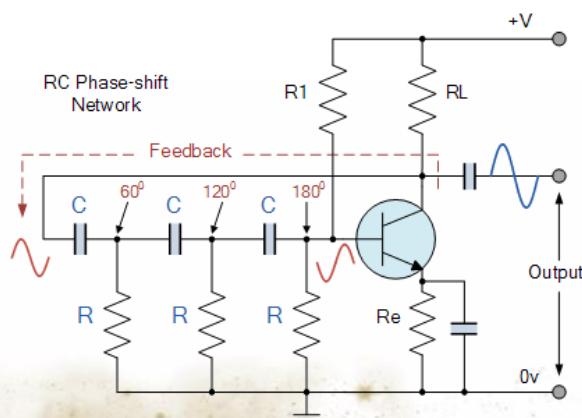
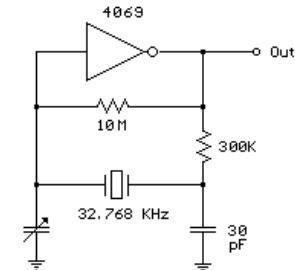
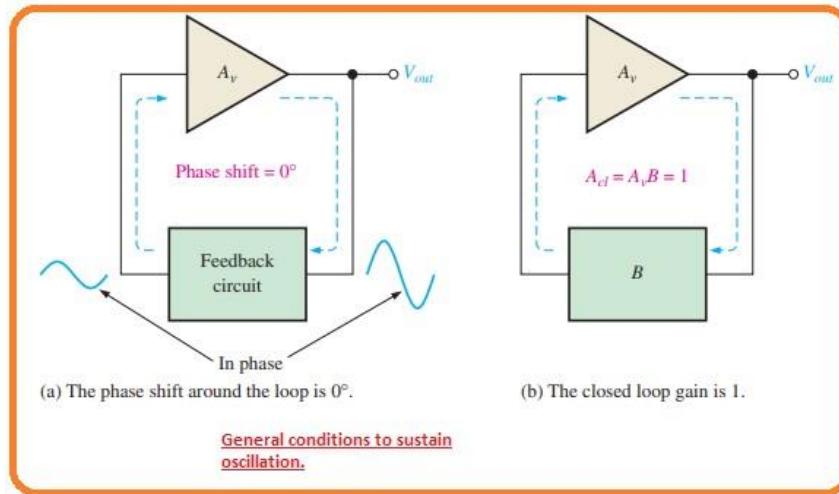
<https://se.mathworks.com/matlabcentral/fileexchange/68174-swctruncated>

# Sinusoid parameters estimation SWC

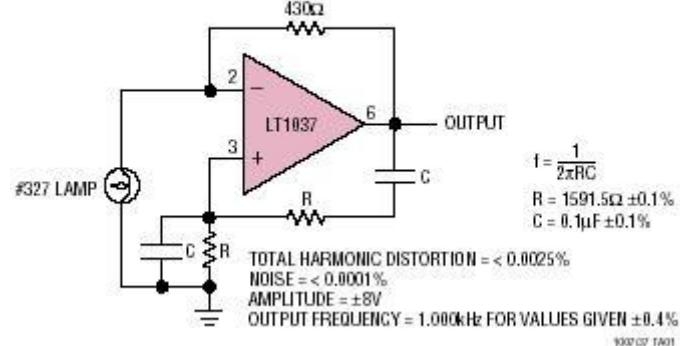


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# Sinusoid production



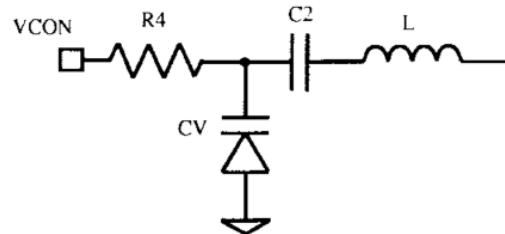
Ultrapure 1kHz Sine Wave Generator



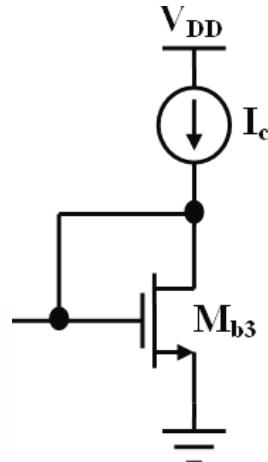
# Sinusoid production



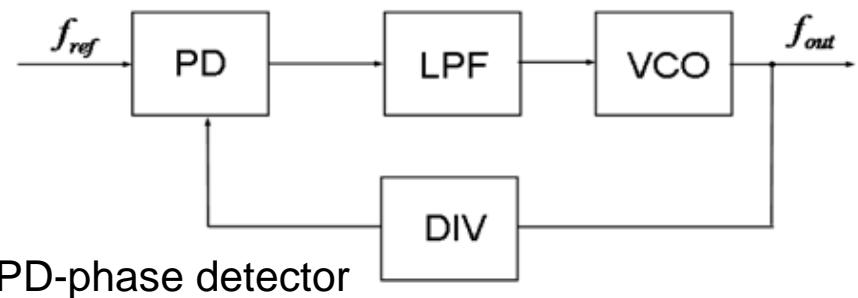
VCO-voltage controlled oscillator



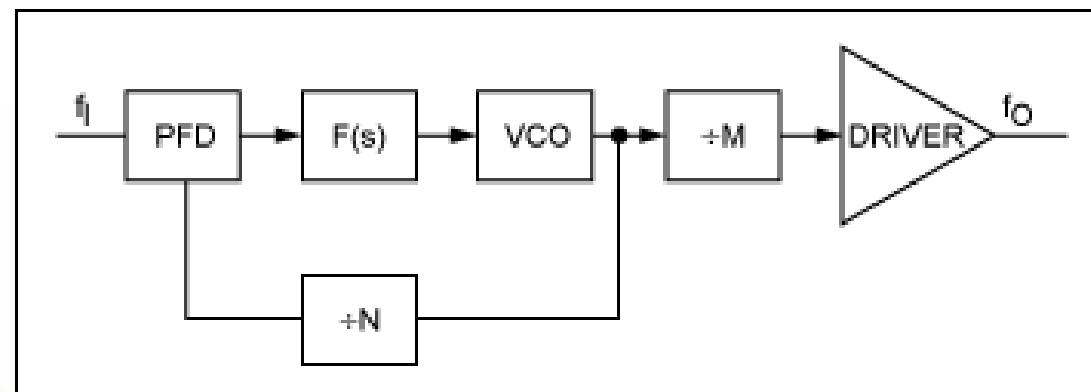
CCO-current controlled oscillator



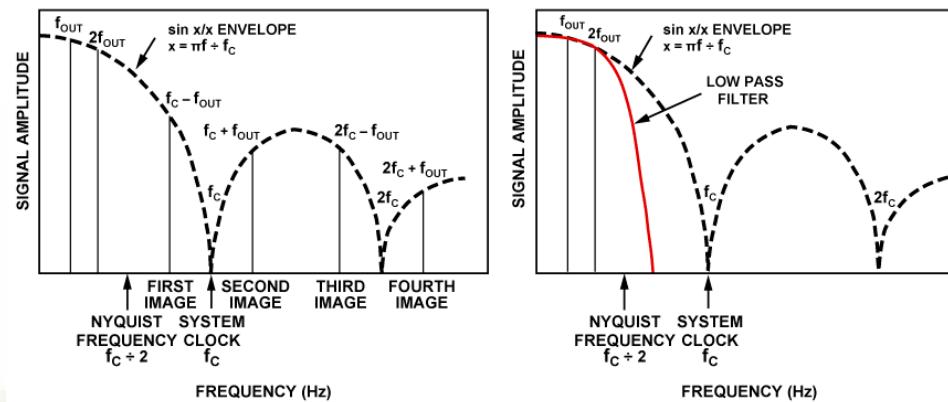
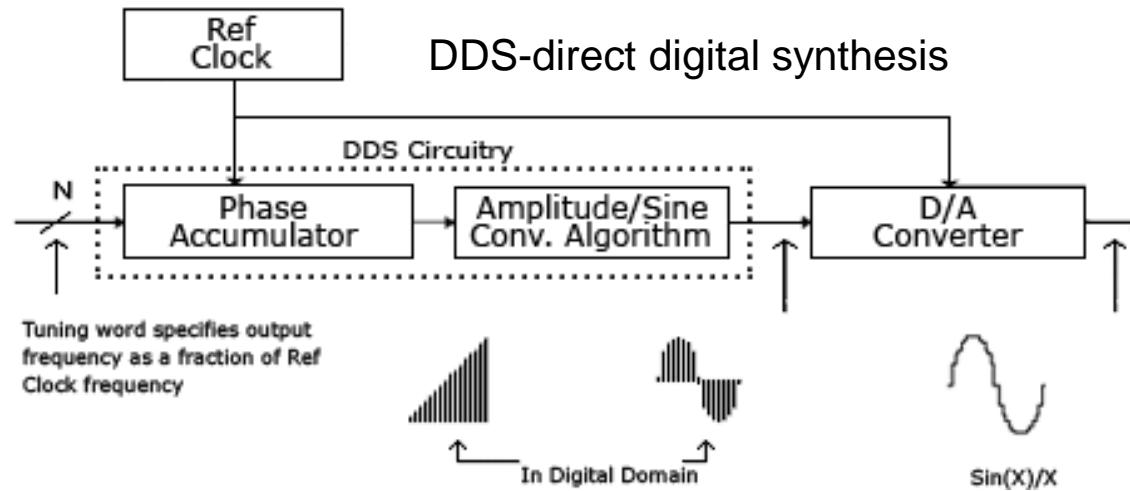
PLL-phase-locked loop



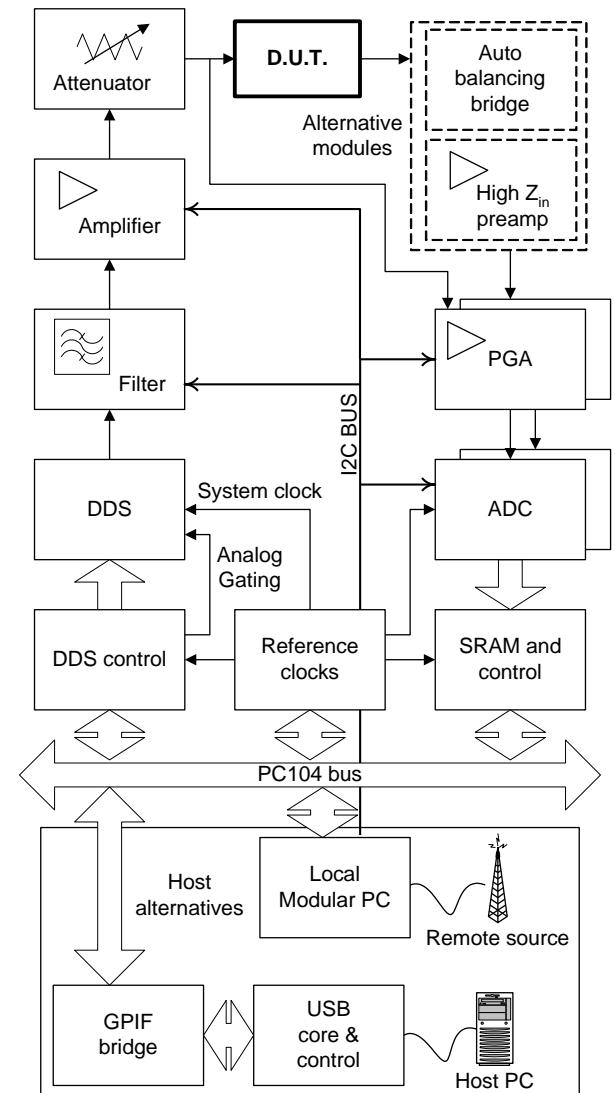
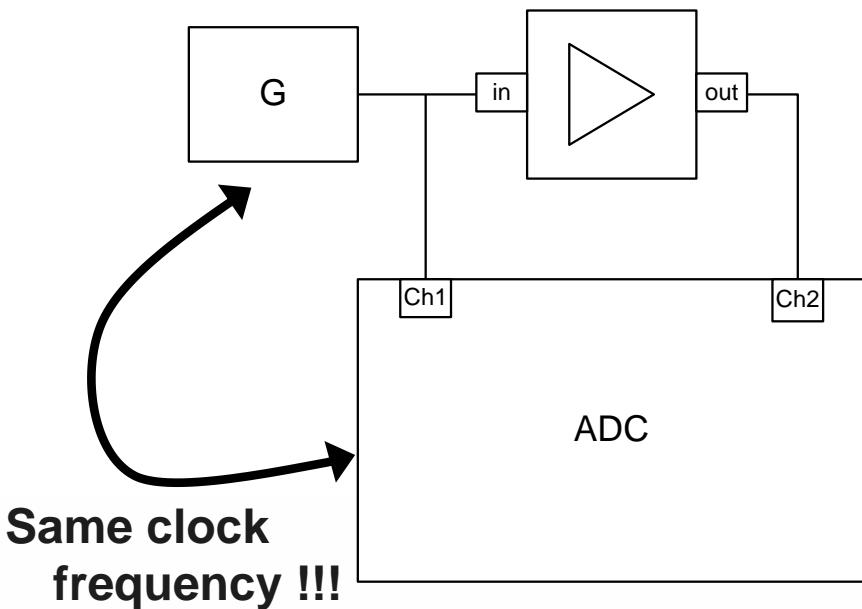
PD-phase detector



# Sinusoid production

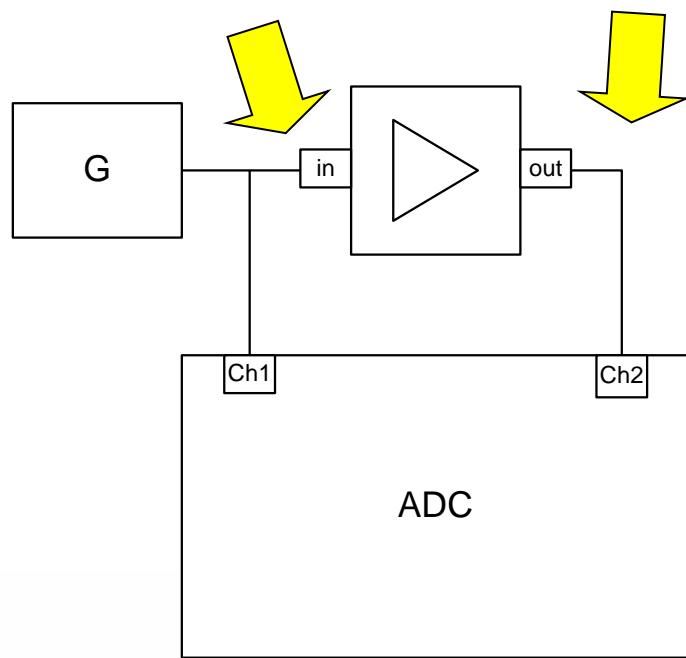


# Transmission AC response: SWC

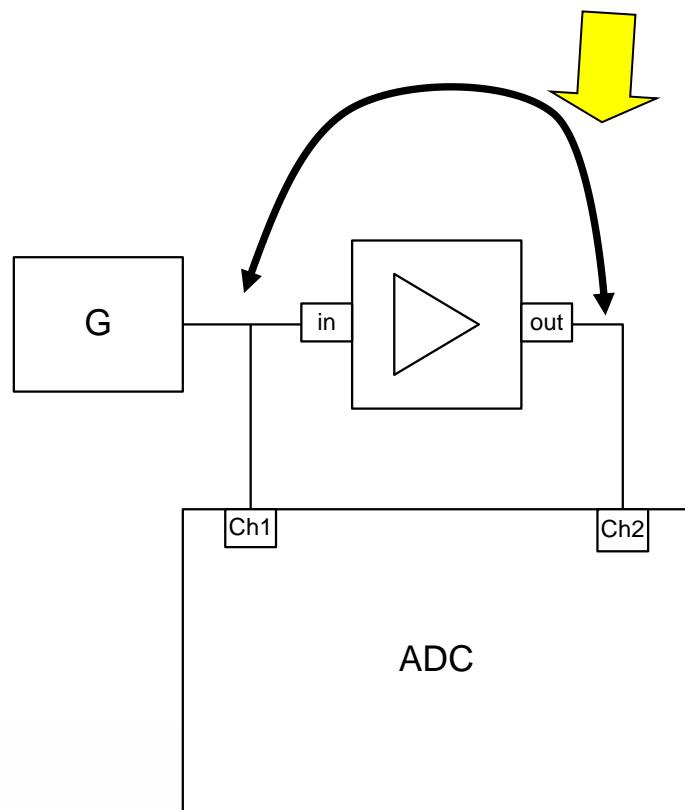


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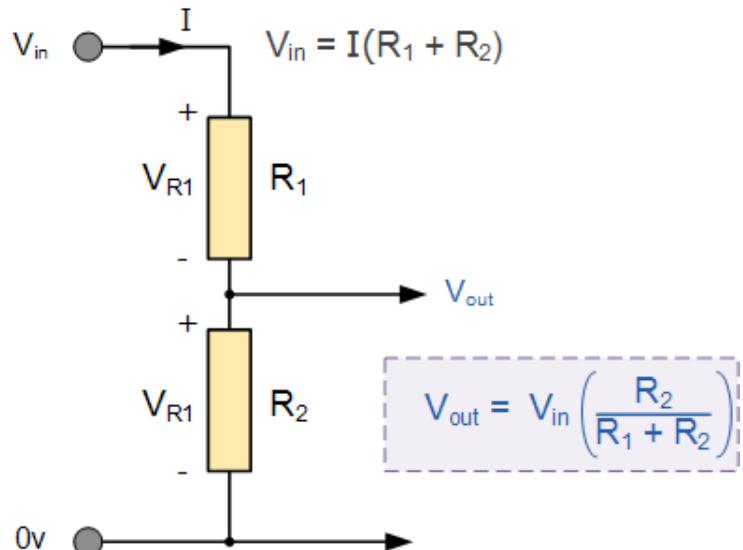
# Connections



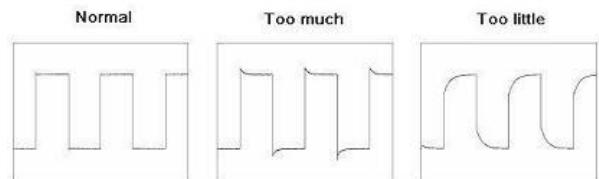
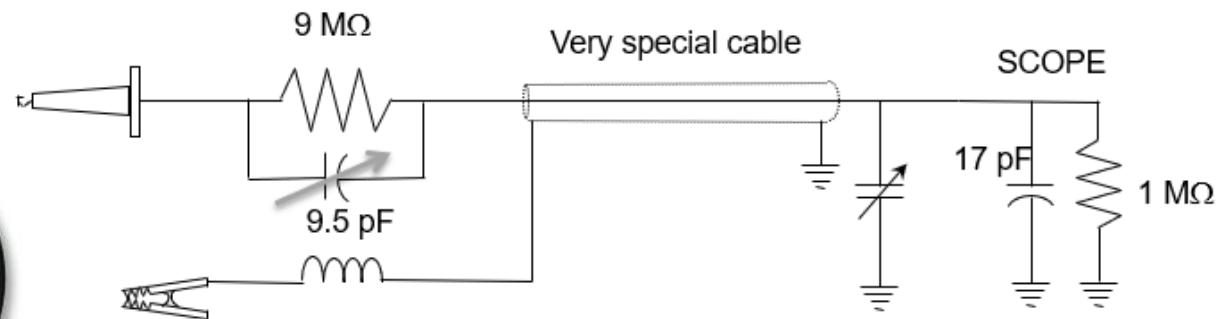
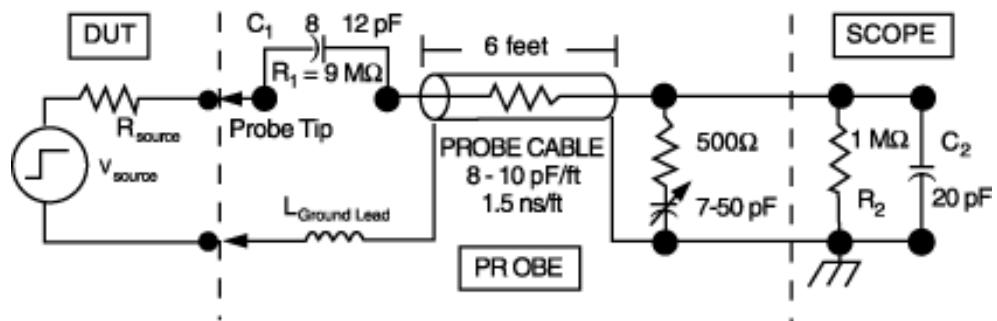
# Calibration



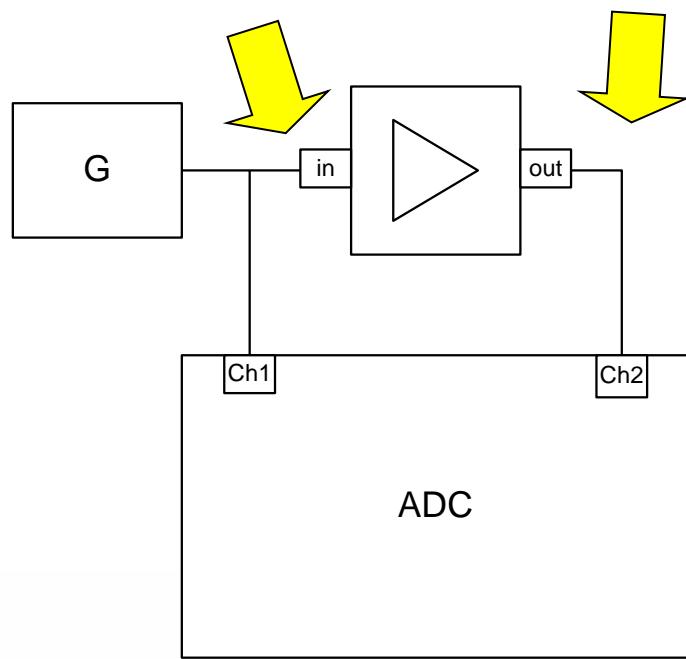
# Connections



## TYPICAL HIGH Z 10X PASSIVE PROBE MODEL

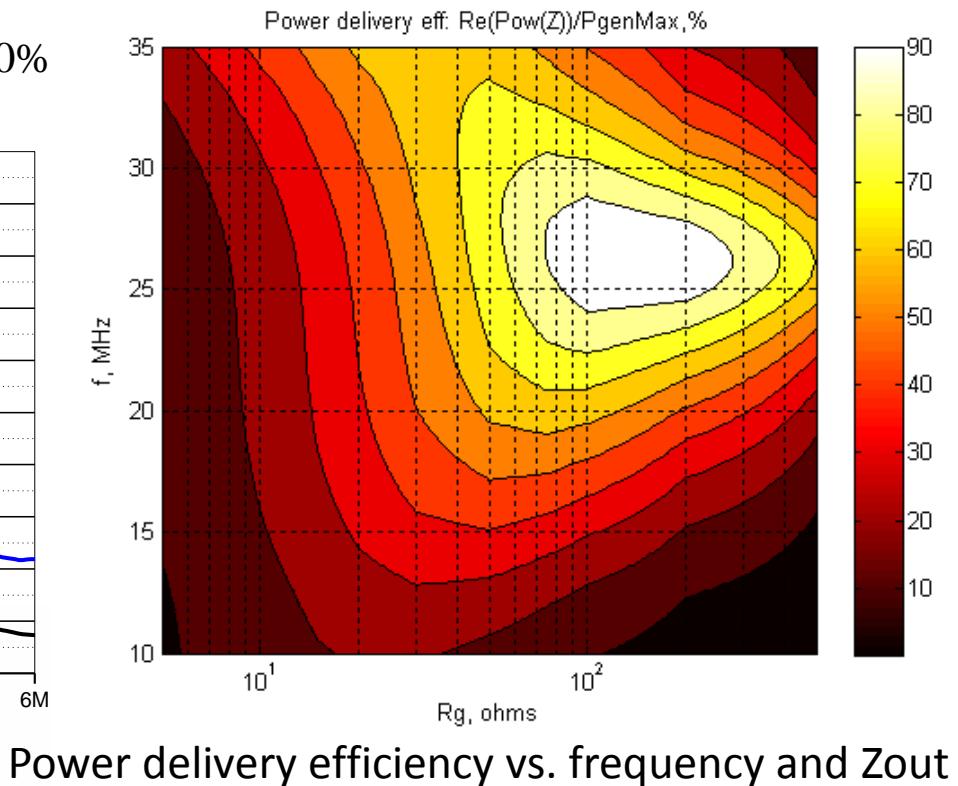
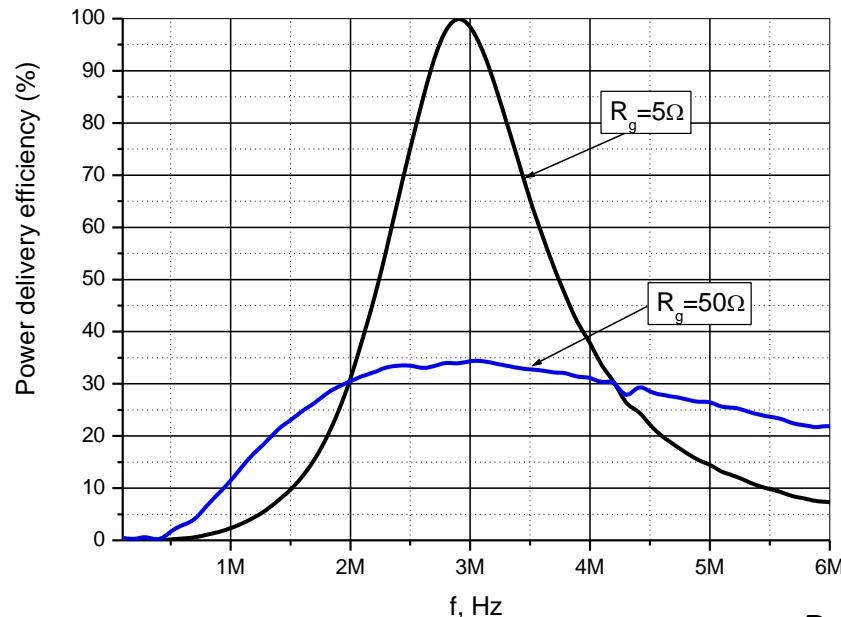


# Z matching



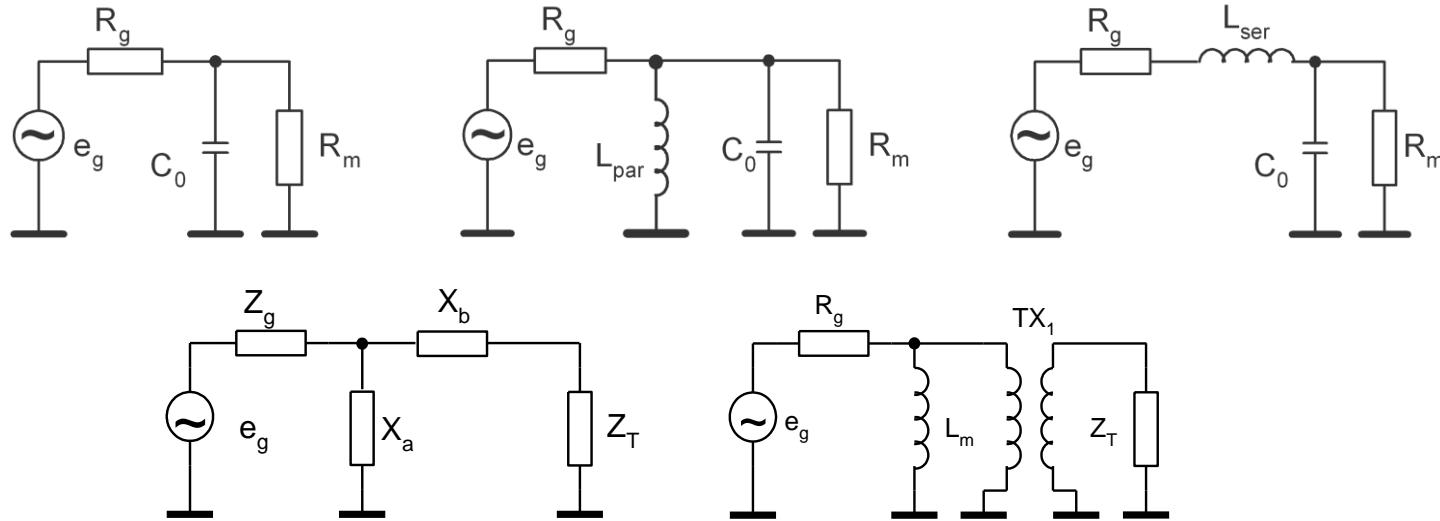
# Z matching

$$\eta = \frac{4R_g \operatorname{Re}(S_T)}{e_g^2} 100\% = \frac{4R_g Z_{in}}{(R_g + Z_{in})(R_g + Z_{in}^*)} 100\%$$



Power delivery efficiency vs. frequency and  $Z_{out}$

# Z matching

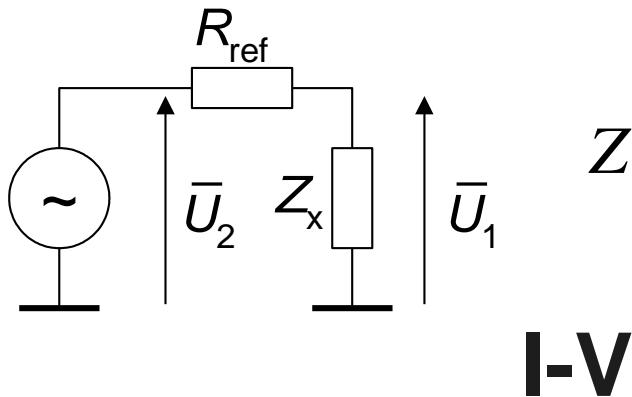


$$L_{par} = -\frac{X_{par}|_{\omega=\omega_s}}{\omega_s} \quad L_{ser} = \frac{X_T}{\omega_s} \quad X_a = -\frac{R_g^2 + X_g^2}{QR_g + X_g} \quad X_b = QR_T - X_T$$

$$Q = \pm \sqrt{\frac{R_g}{\text{Re}(Z_T)} \left[ 1 + \left( \frac{X_g}{R_g} \right)^2 \right] - 1}$$

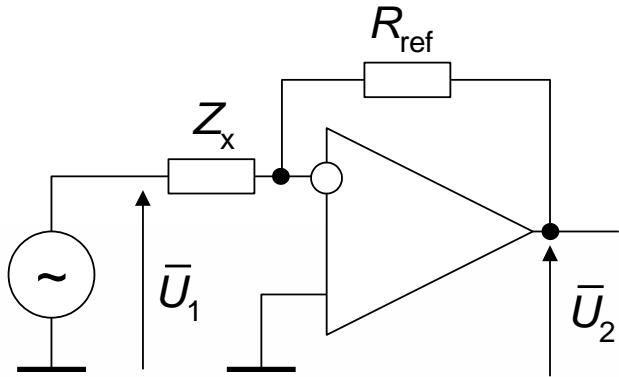
$$L_m = \frac{10R_g}{\omega_s} \quad n = \sqrt{\frac{|Z_T|}{R_g}}$$

# Z measurement



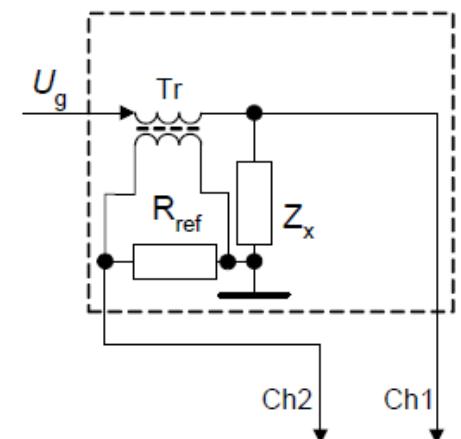
$$Z_x = \frac{\bar{U}_1}{\bar{U}_2 - \bar{U}_1} R_{\text{ref}}$$

I-V



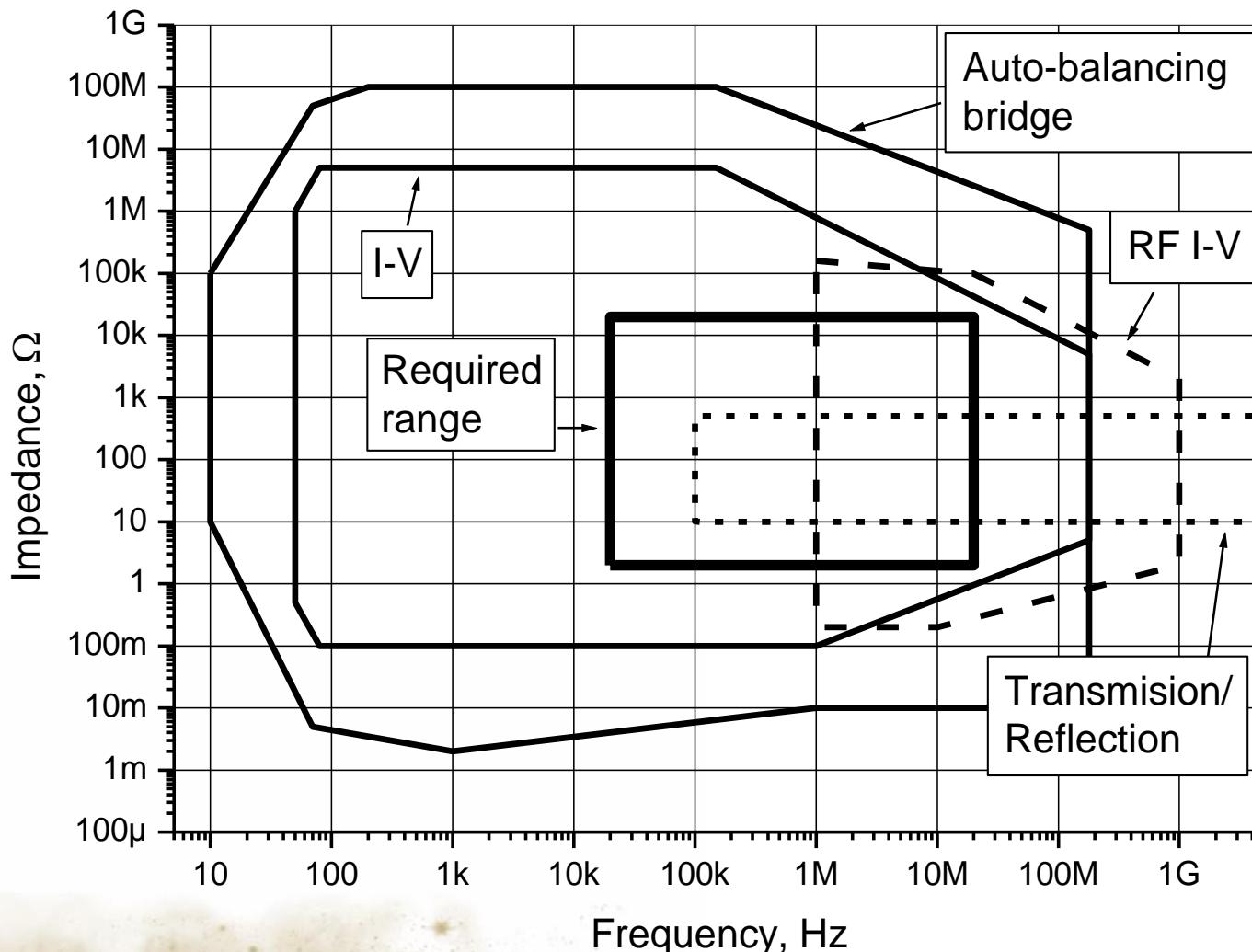
$$Z_x = \frac{\bar{U}_1}{\bar{U}_2} R_{\text{ref}}$$

ABB



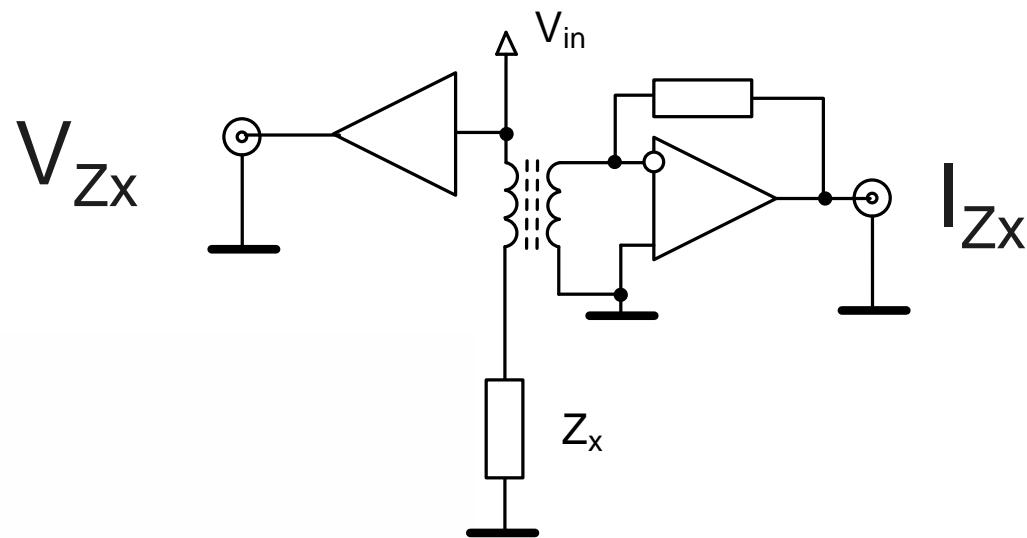
RF I-V

# Z measurement



# Z measurement: RF+ABB

From gen



# Z measurement: OSL



$$Z_{DUT} = \frac{Z_{xm} - Z_s}{1 - \frac{(Z_{xm} - Z_s)}{Z_o}}$$

$Z_{xm}$  – Z measured before compensation

$Z_s$  – Z measured @ short

$Z_o$  – Z measured @ open

$Z_{std}$  – Z measured @  $Z_{std}$  (known)

$$Z_{DUT} = Z_{std} \frac{(Z_o - Z_{std}) \cdot (Z_{xm} - Z_s)}{(Z_{std} - Z_s) \cdot (Z_o - Z_{xm})}$$

# $Z_{out}$ measurement

$$Z_x = \frac{(V_H - V_L) R_L R_H}{R_H V_L - R_L V_H}$$

$$V_i = \frac{(R_H - R_L) V_H V_L}{k(R_H V_L - R_L V_H)}$$

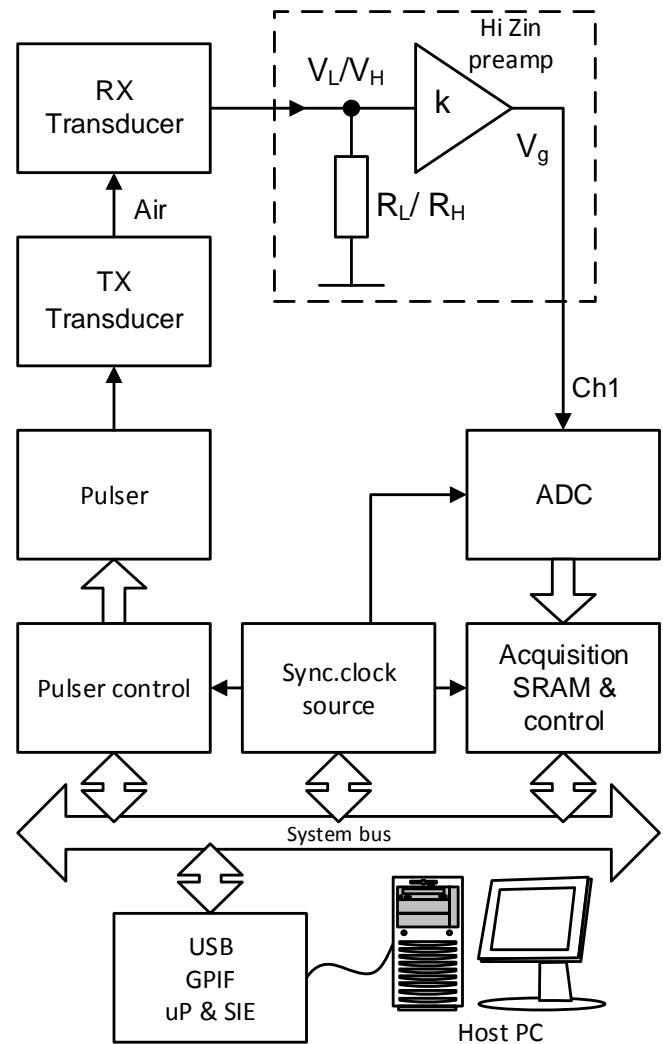
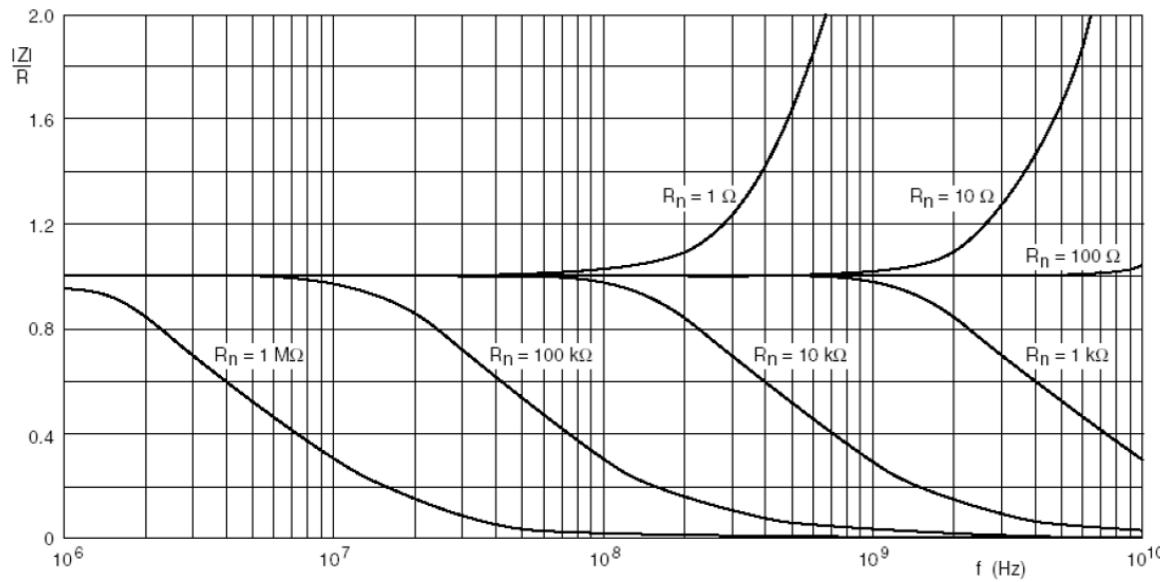


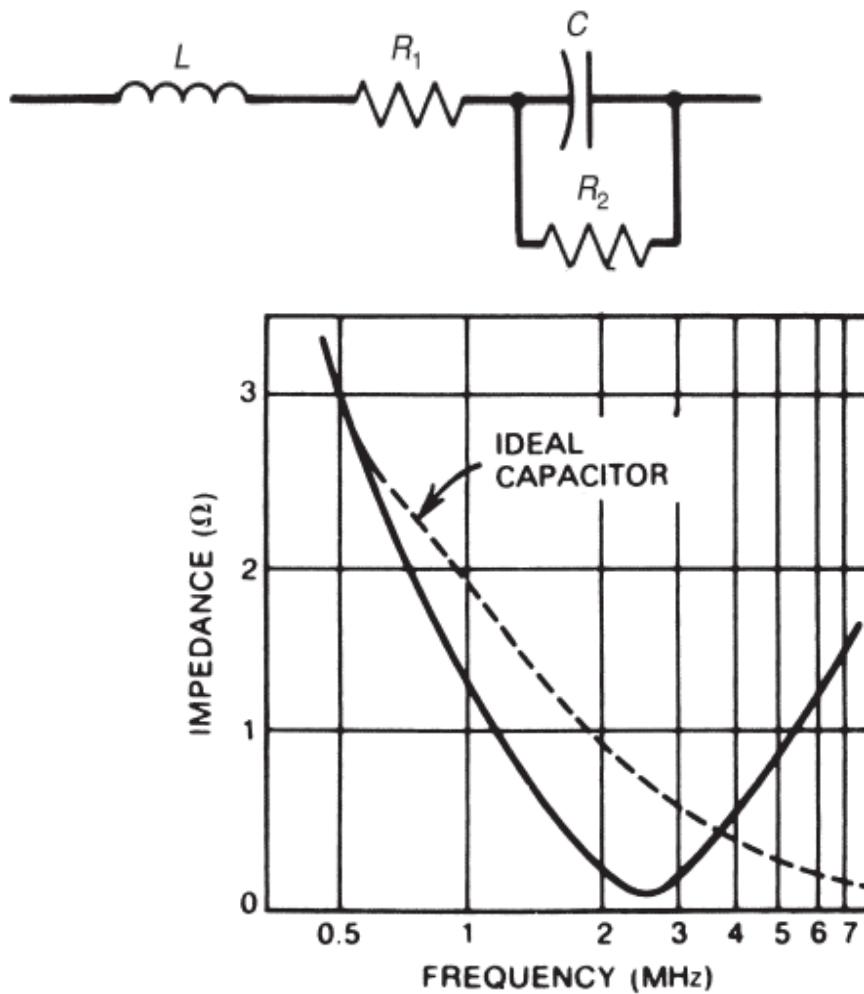


FIGURE 5-14. Equivalent circuit for a resistor.

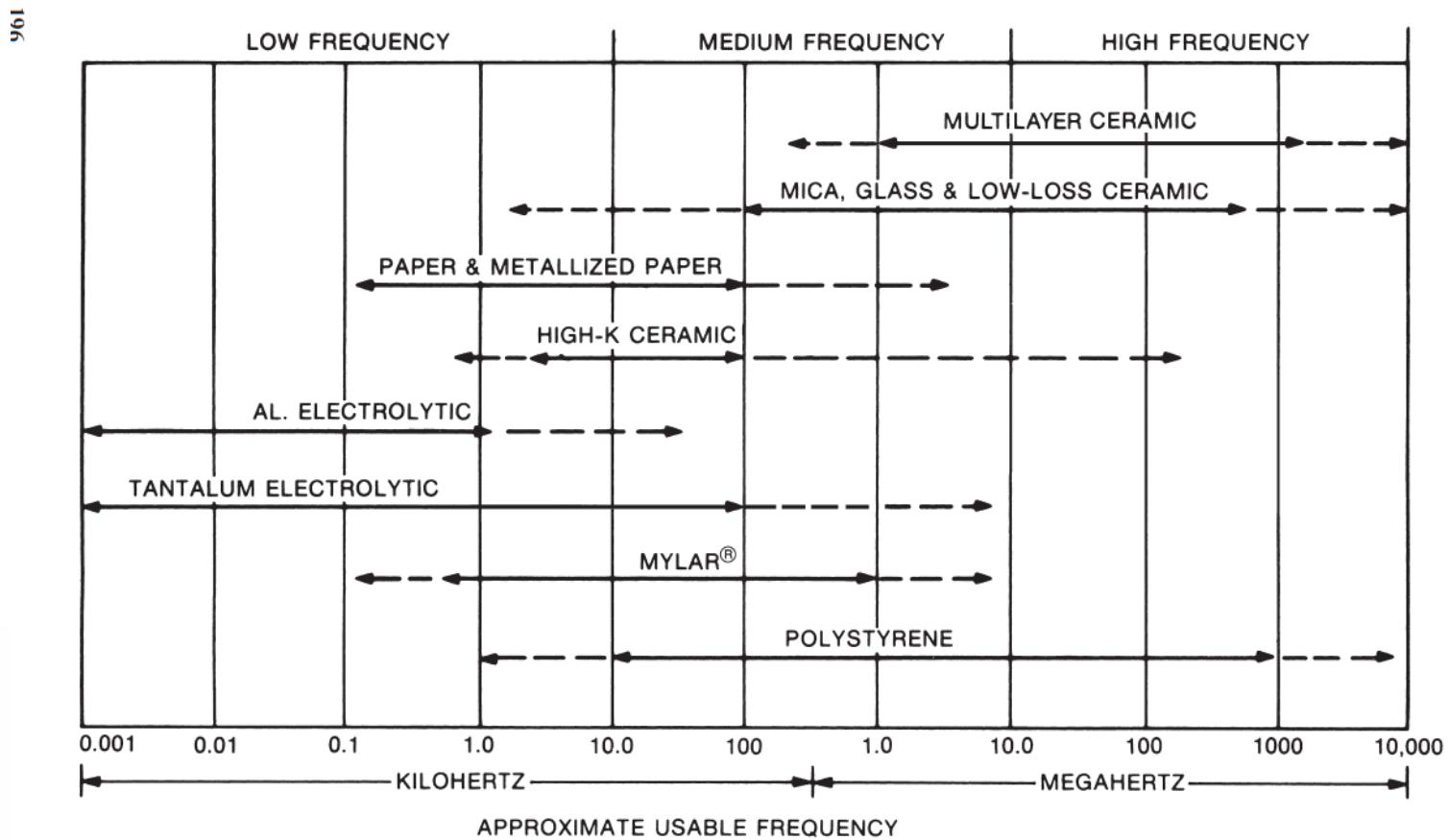
1206:0.05pF+2nH/ 0805:0.09pF+1nH/ 0603: 0.05pF+0.4nH



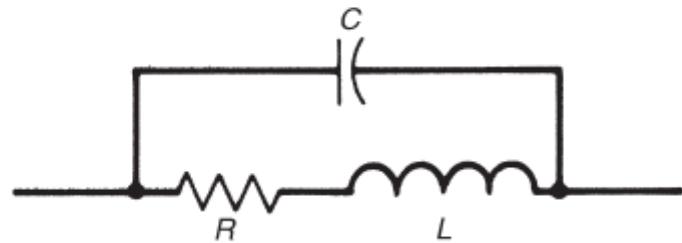
RESISTORS	CARBON FILM	METAL FILM	THICK FILM	METAL FOIL	CARBON COMPOSITION	WIREWOUND	POWER METAL STRIP
Resistance Value	10 Ω to 22 MΩ	0.22 Ω to 22 MΩ	1 Ω to 100 MΩ	2 mΩ to 1 MΩ	1 Ω to 20 MΩ	0.1 Ω to 300 kΩ	0.1 mΩ to 1.0 Ω
Tolerance [%]	± 2 to ± 10	± 0.1 to ± 2	± 1 to ± 5	± 0.005 to ± 5	± 5 to ± 20	± 0.1 to ± 10	± 0.5 to ± 1
Temperature Coefficient [ppm/K]	- 200 to - 1500	± 5 to ± 50	± 50 to ± 200	± 2 to ± 50	- 200 to - 1500	± 1 to ± 200	± 30 to ± 250
Maximum Operating Temperature [°C]	+ 155	+ 155	+ 155	+ 150	+ 150	+ 400	+ 275
Rated Dissipation $P_{70}$ [W]	0.25 to 2	0.063 to 1	0.063 to 0.25	0.25 to 10	0.25 to 1	0.25 to 100	0.1 to 5
Stability at $P_{70}$ (1000 h) ΔR/R [%]	± 0.8 to ± 3	± 0.15 to ± 0.5	± 1 to ± 3	± 0.05	+ 4/- 6 (typical - 3)	± 1 to ± 10	± 1 to ± 2
Operating Voltage $U_{max.}$ [V]	200 to 1000	50 to 500	50 to 200	200 to 500	150 to 350	25 to 1000	$\sqrt{P_{70} \times R}$
Current Noise [ $\mu$ V/V]	< 1	< 0.1	< 10	< 0.025	2 to 6	negligible	negligible
Non-linearity $A_3$ [dB]	> 100	> 110	> 50	negligible	~ 60	negligible	negligible

**C**

**FIGURE 5-2.** Effect of frequency on the impedance of a  $0.1-\mu\text{F}$  paper capacitor.



**FIGURE 5-3.** Approximate usable frequency ranges for various types of capacitors.



**FIGURE 5-9.** Equivalent circuit for an inductor.

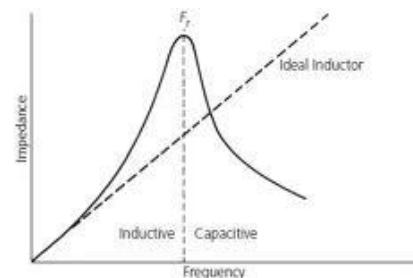
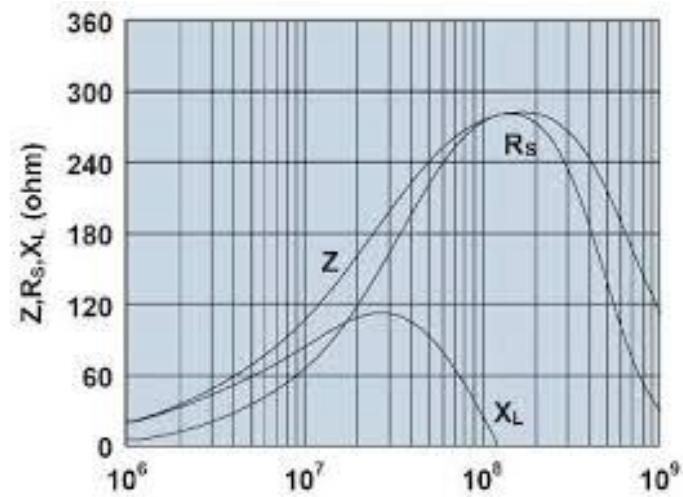
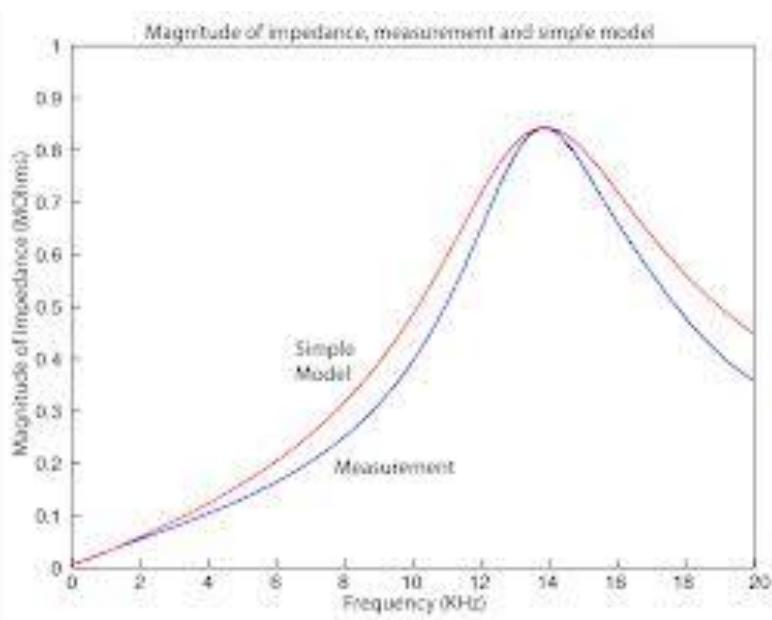
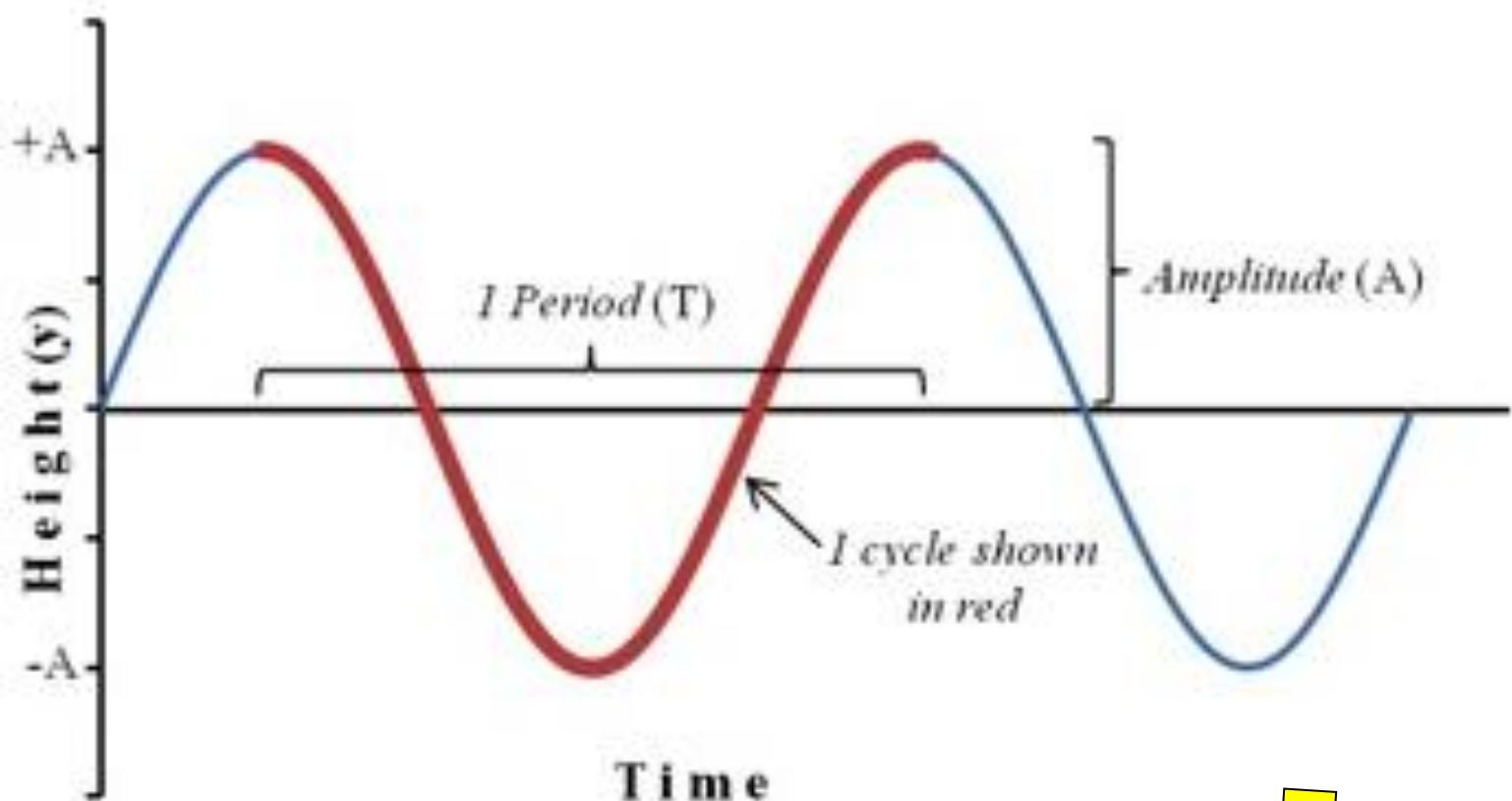


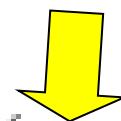
FIG. 1-16. Impedance characteristic vs. frequency for a practical and an ideal inductor.



# Frequency estimation



$$x(t) = A \sin(\omega t + \phi)$$

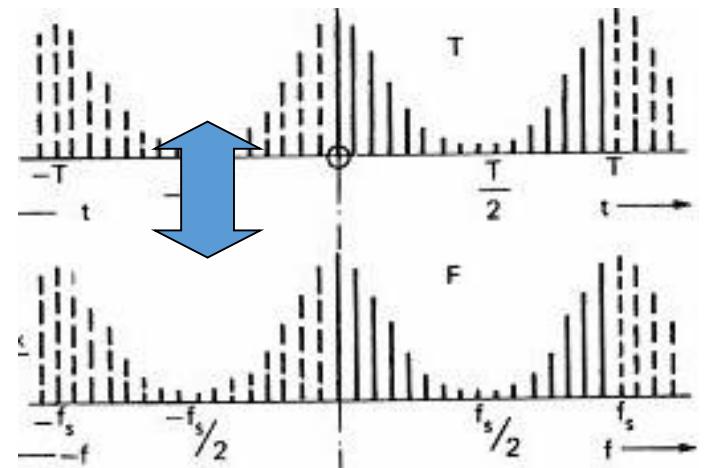
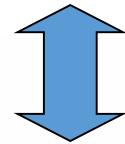


# DFT

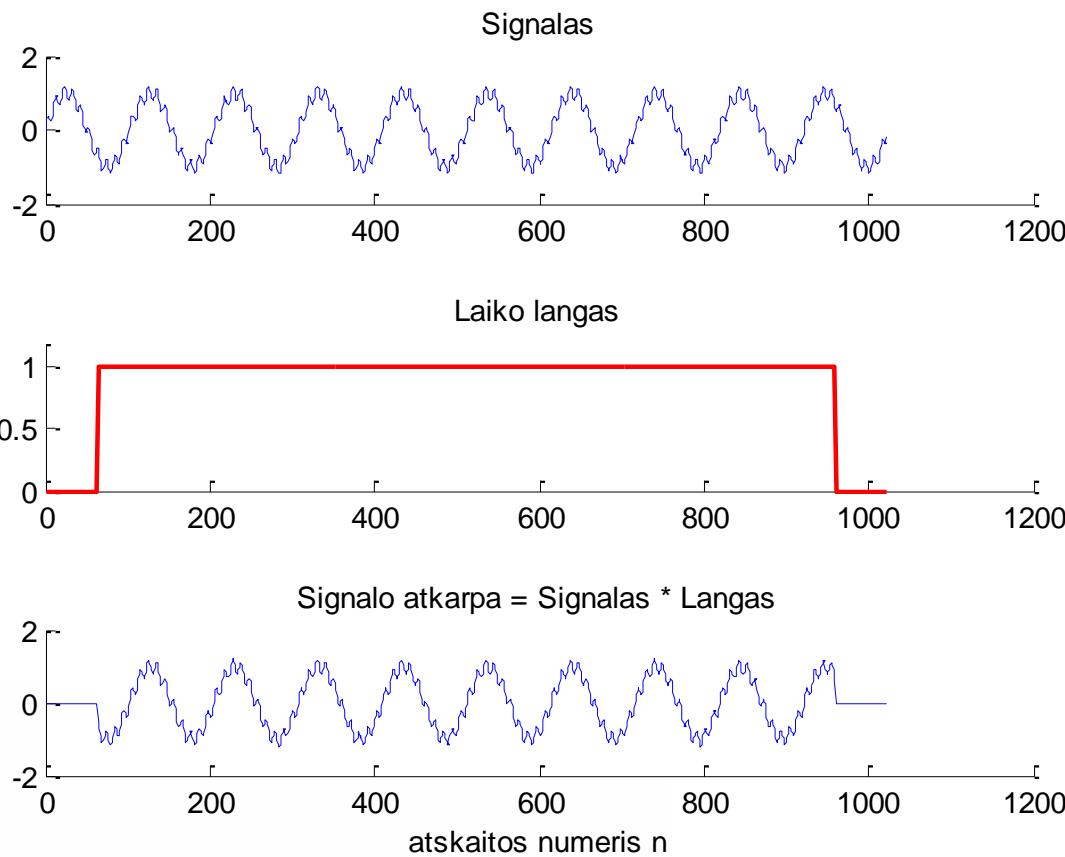
Discrete time AND frequency signal is presented only at discrete time AND frequency samples. Such signal is periodic both in time AND frequency domain. Briefly: such signal is both discrete and periodic;

**DFT**-Discrete Fourier transform is used to transforms the siignal between time and frequency axes. **FFT** is the **separate DFT case** where signal length N is log2 (4,16,32,64,128,256,512,1024 etc.)

$$S_k = \sum_{n=0}^{N-1} s_n e^{-j2\pi nk/N} = S_{k+N}$$
$$s_n = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{j2\pi nk/N} = s_{n-N}$$

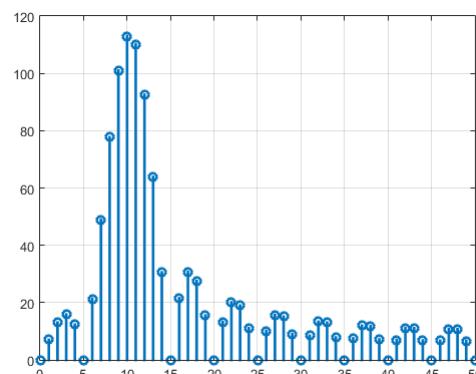
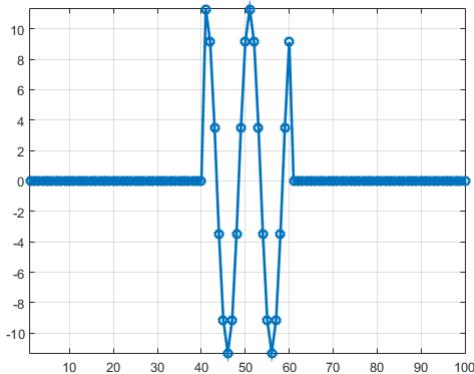
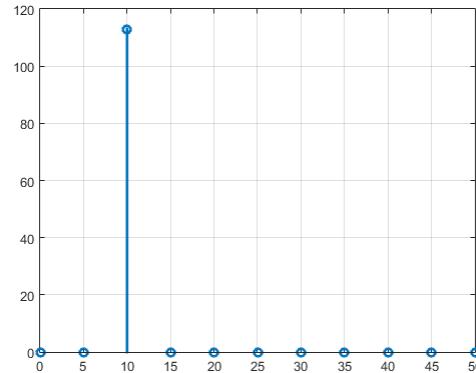
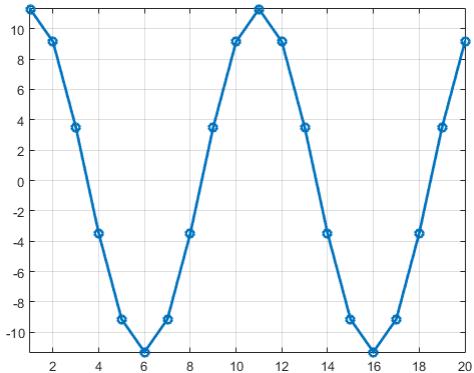


# DFT frequency estimation



Widowing/gating can not be avoided

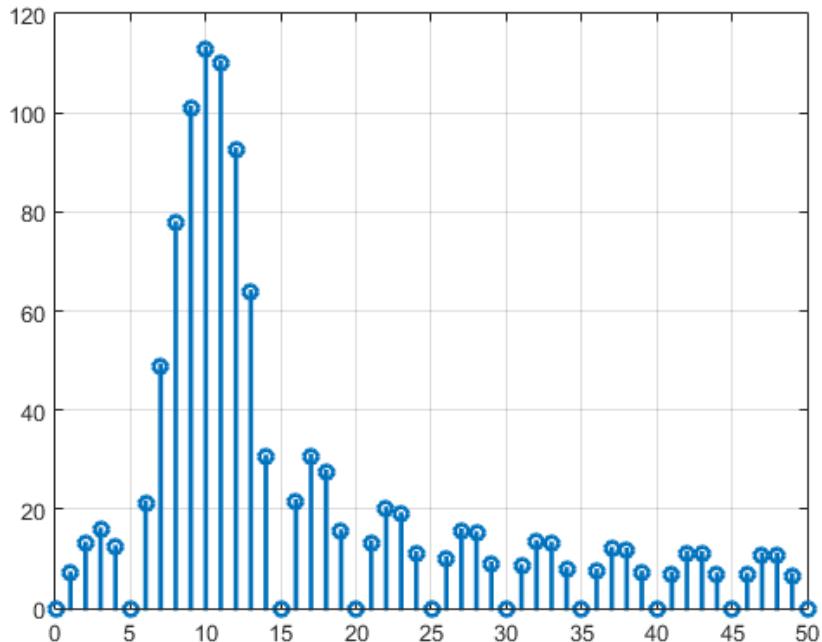
# DFT frequency estimation



Record length matches period only in rare occasions

Result of windowing + wrong record length

# DFT frequency estimation



DFT frequencies:

$$f = \begin{cases} -f_N, & n = 1 \\ -2f_N \dots -\Delta f, & n = 2 \dots N/2 \\ 0, & n = N/2 + 1 \\ \Delta f \dots f_N, & n = (N/2 + 2) \dots N \end{cases}$$

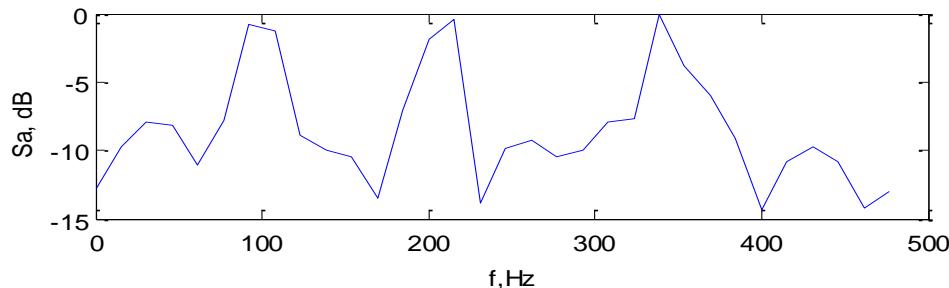
Actual frequency may be not on grid:

$$\Delta f = f_s / N$$

Zero-padding does reduce  $\Delta f$ , because  $N$  is increased

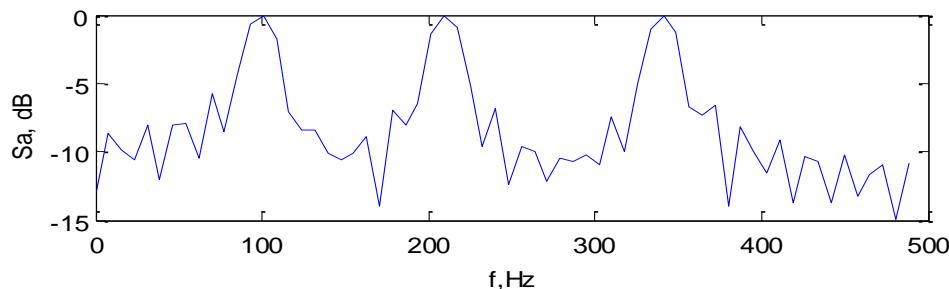
# DFT frequency estimation

N=64

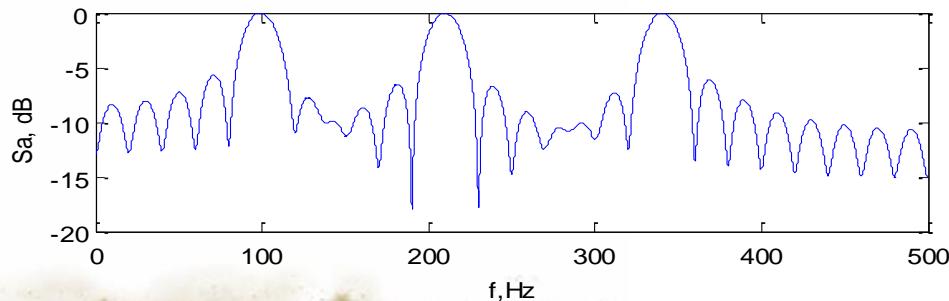


Course frequency grid

N=128



N = 16384

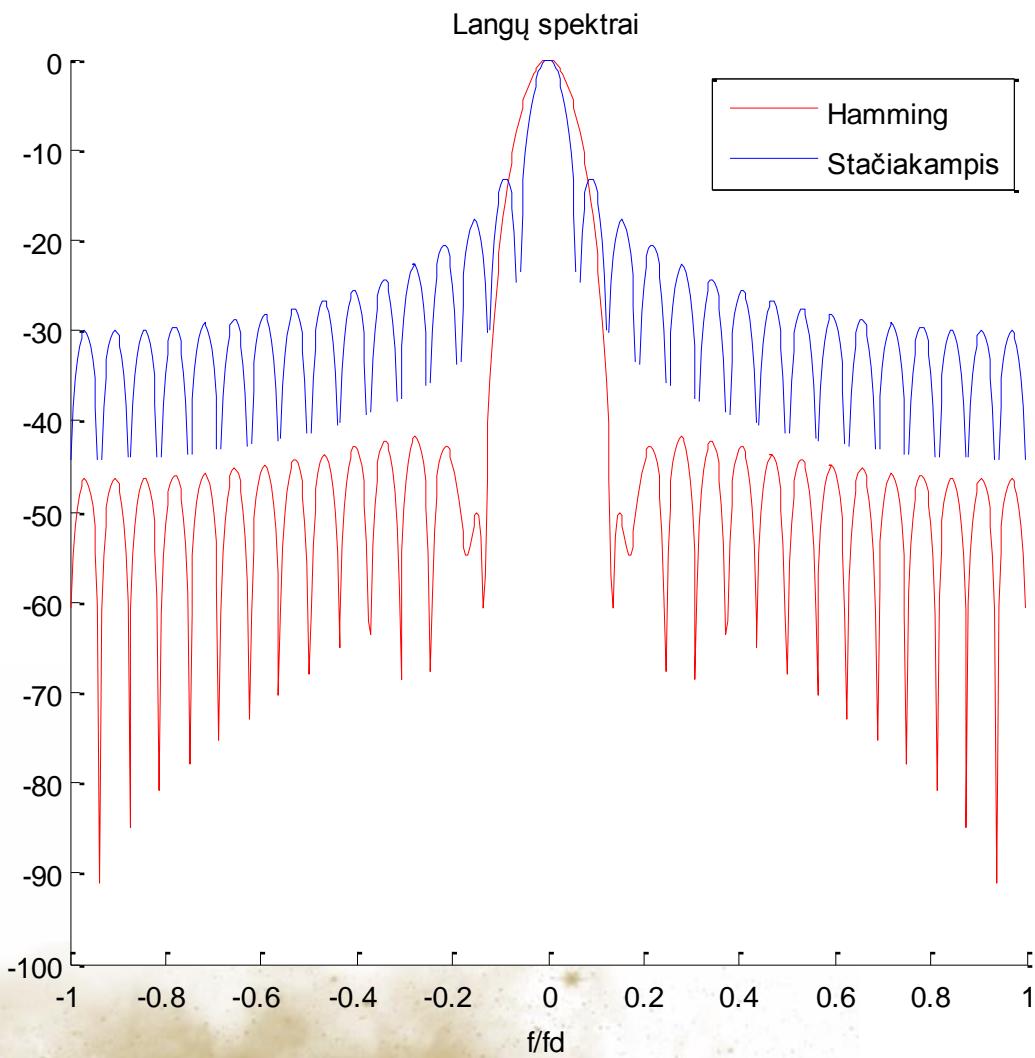


Denser grid

Note that shape of the spectral peaks remains the same

Zero-padding does not improve resolution (sharpness of the peak)

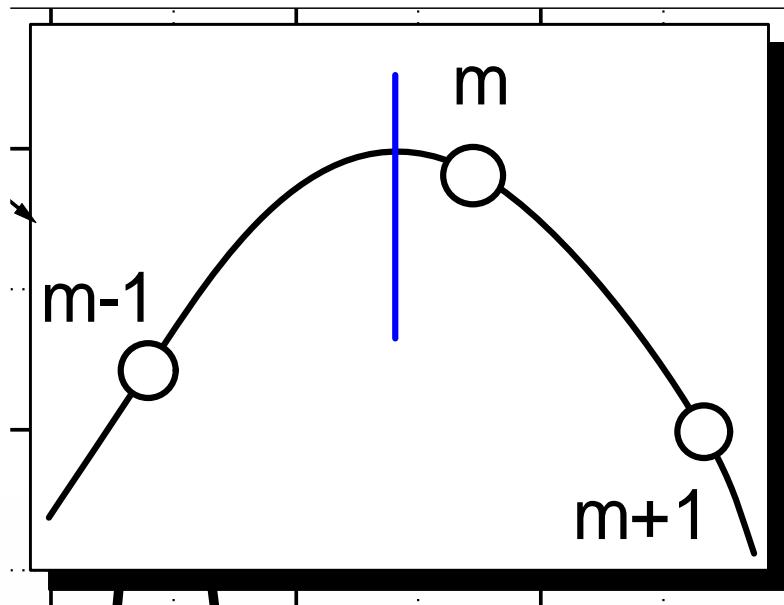
# DFT frequency estimation



Other window types only worsen situation

# DFT frequency estimation

Interpolation helps for single frequency



$$\Delta f_{corr_p} = \frac{x_{m-1} - x_{m+1}}{2(x_{m-1} - 2x_m + x_{m+1})} \cdot \Delta f$$

$$\Delta f_{corr_G} = \frac{\ln(x_{m+1}) - \ln(x_{m-1})}{(4\ln(x_m) - 2\ln(x_{m-1}) - 2\ln(x_{m+1}))} \cdot \Delta f$$

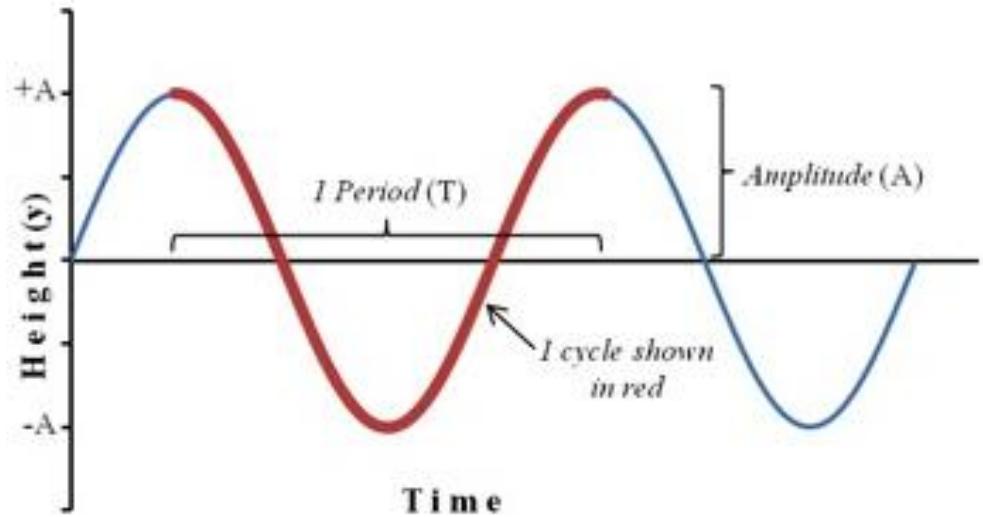
$$\Delta f_{corr_{\cos}} = -\frac{\theta}{s\omega_0} \cdot \Delta f \quad \omega_0 = \arccos\left(\frac{x_{m-1} + x_{m+1}}{2x_m}\right) \quad \theta = \arctan\left(\frac{x_{m-1} - x_{m+1}}{2x_m \sin \omega_0}\right)$$

# Sine parameters estimation: 4 parameters fit



Probing frequency is unknown:  
4 parameters fit:  
 $V_m$ ,  $V_0$ , phase, frequency

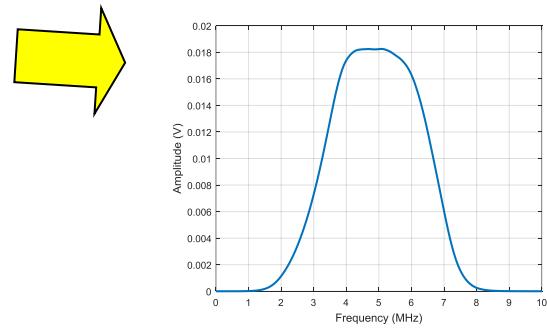
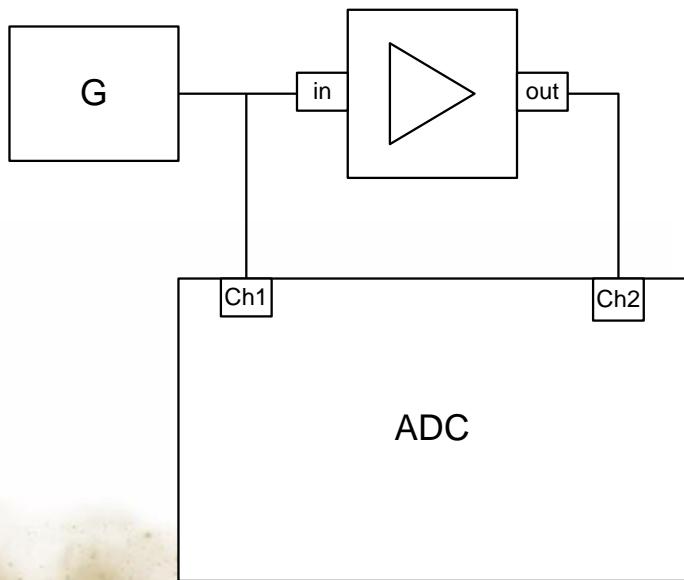
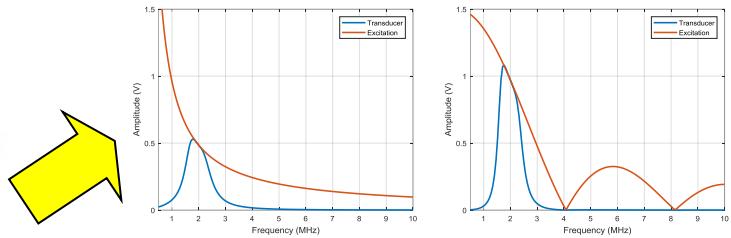
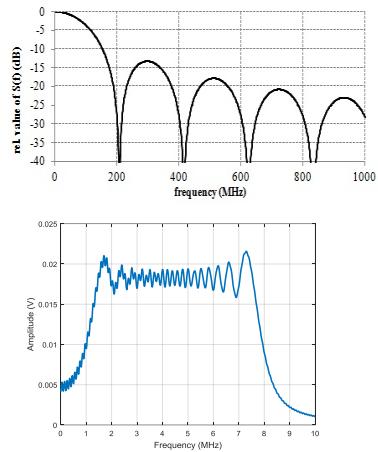
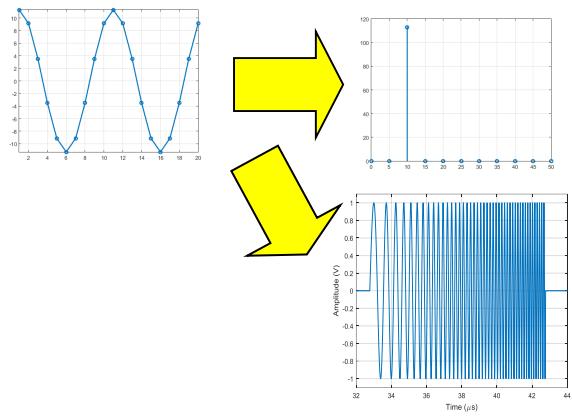
$$x(t) = A \sin(\omega t + \phi)$$



$$r_n = y_n - A \cos(2\pi f t_n) - B \sin(2\pi f t_n) - C$$

<https://se.mathworks.com/matlabcentral/fileexchange/23214-four-parameter-sinefit>

# AC measurement using broadband signal



$$s_{in}(t)$$

$$s_{out}(t)$$

DFT

DFT

$$S_{in}(f)$$

$$S_{out}(f)$$

$$= T(f)$$

# End: AC measurements



## Conclusions:

- Harmonic signal parameters (amplitude, phase, frequency) estimation using DFT will not produce reliable results unless signal closely matches Fourier bin frequency
- Bin frequency depends on sampling frequency and record length: not all frequencies can satisfy bit match
- High accuracy, automated AC measurements can be carried out using SWC (derivative of TDFT)
- Same clock frequency should be used both for probing sine production and ADC (this is why measurement equipment has 10MHz sync signal)
- Measurements using SWC are accurate, but time-consuming. Broadband signals can be used for AC estimation via DFT.
- Proper use of windows is essential in such case

**END**



Thank you for your attention