



同濟大學
TONGJI UNIVERSITY

光流控方向的一些理论

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1 Poincare sphere

It's a good method to describe the polarization of the light beam.

1.1 fundamental theory of polarized light

Assumption: The beam propagates along the z-direction, the amplitude is E_1 and E_2 in x,y-direction, respectively.

Under this assumption:

$$\begin{cases} E_x = E_1 \cos(kz - \omega t + \delta_1) \\ E_y = E_2 \cos(kz - \omega t + \delta_2) \end{cases}$$

We will use $\tau = kz - \omega t$ to simplify the proof.

$$\begin{cases} E_x = E_1 \cos(\tau + \delta_1) \\ E_y = E_2 \cos(\tau + \delta_2) \end{cases}$$

then:

$$\begin{aligned} \frac{E_x}{E_1} &= \cos \tau \cos \delta_1 - \sin \tau \sin \delta_1 \\ \frac{E_y}{E_2} &= \cos \tau \cos \delta_2 - \sin \tau \sin \delta_2 \end{aligned}$$

do these calculation:

$$\sin \delta_2 \cdot \frac{E_x}{E_1} - \sin \delta_1 \frac{E_y}{E_2} = \cos \tau (\sin \delta_2 \cos \delta_1 - \sin \delta_1 \cos \delta_2) \quad (1)$$

$$\cos \delta_2 \cdot \frac{E_x}{E_1} - \cos \delta_1 \frac{E_y}{E_2} = \sin \tau (\sin \delta_2 \cos \delta_1 - \sin \delta_1 \cos \delta_2) \quad (2)$$

Combining eq(1) and eq(2), we can find:

$$\begin{aligned} \left(\sin \delta_2 \cdot \frac{E_x}{E_1} - \sin \delta_1 \frac{E_y}{E_2} \right)^2 + \left(\cos \delta_2 \cdot \frac{E_x}{E_1} - \cos \delta_1 \frac{E_y}{E_2} \right)^2 &= \sin^2(\delta_1 - \delta_2) \\ \left(\frac{E_x}{E_1} \right)^2 - 2 \cos(\delta_1 - \delta_2) \cdot \frac{E_x E_y}{E_1 E_2} + \left(\frac{E_y}{E_2} \right)^2 &= \sin^2(\delta_1 - \delta_2) \end{aligned} \quad (3)$$

We can use $\delta = \delta_1 - \delta_2$ to simplify eq(3) (Although we can also use $\delta = \delta_2 - \delta_1$ to describe without changing the form of equation, it will keep a well and same style of later derivation.)

$$\left(\frac{E_x}{E_1}\right)^2 - 2\cos(\delta) \cdot \frac{E_x E_y}{E_1 E_2} + \left(\frac{E_y}{E_2}\right)^2 = \sin^2(\delta) \quad (4)$$

We can use elliptical discriminate equation to check:

$$\begin{vmatrix} \frac{1}{E_1^2} & -\frac{\cos \delta}{E_1 E_2} \\ -\frac{\cos \delta}{E_1 E_2} & \frac{1}{E_2^2} \end{vmatrix} = \frac{\sin^2 \delta}{E_1^2 E_2^2} \geq 0$$

1.2 Coordinates transformation of the elliptic

We can use a better coordinate system which is the major axis and minor axis to describe the elliptical light.

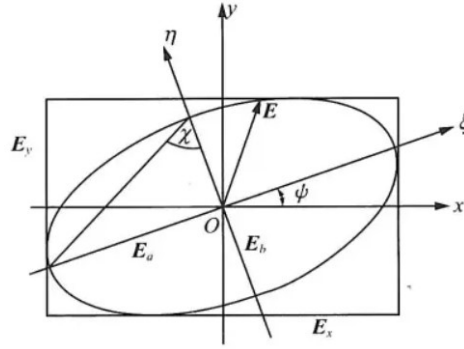


图 1: coordinates transformation - the derivation for a better method to describe the polarization

Suppose:

$$\begin{cases} E_\xi = E_a \cos(\tau + \varphi) \\ E_\eta = E_b \sin(\tau + \varphi) \end{cases}$$

Using the rotation matrix to describe:

$$\begin{bmatrix} E_\xi \\ E_\eta \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Expanding the equation:

$$\begin{cases} E_a (\cos \tau \cos \varphi - \sin \tau \sin \varphi) = E_1 (\cos \tau \cos \delta_1 - \sin \tau \sin \delta_1) \cos \psi + E_2 (\cos \tau \cos \delta_2 - \sin \tau \sin \delta_2) \sin \psi \\ E_b (\sin \tau \cos \varphi + \cos \tau \sin \varphi) = -E_1 (\cos \tau \cos \delta_1 - \sin \tau \sin \delta_1) \sin \psi + E_2 (\cos \tau \cos \delta_2 - \sin \tau \sin \delta_2) \cos \psi \end{cases}$$

Since τ ranges from $-\pi$ to π , $\cos \tau$ and $\sin \tau$ can be treated as independent variables.

$$E_a \cos \varphi = E_1 \cos \delta_1 \cos \psi + E_2 \cos \delta_2 \sin \psi \quad (5)$$

$$-E_a \sin \varphi = -E_1 \sin \delta_1 \cos \psi - E_2 \sin \delta_2 \sin \psi \quad (6)$$

$$E_b \cos \varphi = E_1 \sin \delta_1 \sin \psi - E_2 \sin \delta_2 \cos \psi \quad (7)$$

$$E_b \sin \varphi = -E_1 \cos \delta_1 \sin \psi + E_2 \cos \delta_2 \cos \psi \quad (8)$$

$(5)^2 + (6)^2 + (7)^2 + (8)^2$:

$$S_0 \equiv E_a^2 + E_b^2 = E_1^2 + E_2^2 \quad (9)$$

$(5) * (7) - (6) * (8)$:

$$E_a E_b = E_1 E_2 \sin(\delta_1 - \delta_2) = E_1 E_2 \sin \delta \quad (10)$$

$(5)/(6) + (7)/(8)$:

$$\begin{aligned} & \frac{E_1 \cos \delta_1 \cos \psi + E_2 \cos \delta_2 \sin \psi}{-E_1 \sin \delta_1 \cos \psi - E_2 \sin \delta_2 \sin \psi} + \frac{E_1 \sin \delta_1 \sin \psi - E_2 \sin \delta_2 \cos \psi}{-E_1 \cos \delta_1 \sin \psi + E_2 \cos \delta_2 \cos \psi} = 0 \\ & (E_1 \cos \delta_1 \cos \psi + E_2 \cos \delta_2 \sin \psi) \cdot (-E_1 \cos \delta_1 \sin \psi + E_2 \cos \delta_2 \cos \psi) \\ & + (E_1 \sin \delta_1 \sin \psi - E_2 \sin \delta_2 \cos \psi) \cdot (-E_1 \sin \delta_1 \cos \psi - E_2 \sin \delta_2 \sin \psi) = 0 \\ & 2E_1 E_2 \cos(\delta_1 - \delta_2) \cos 2\psi = (E_1^2 - E_2^2) \sin 2\psi \end{aligned}$$

$$2E_1 E_2 \cos(\delta_1 - \delta_2) \cos 2\psi = (E_1^2 - E_2^2) \sin 2\psi \quad (11)$$

Now, we define:

$$\frac{E_1}{E_2} = \tan \alpha \quad (12)$$

From eq(11), we can get:

$$\tan 2\psi = \frac{2E_1 E_2 \cos \delta}{E_1^2 - E_2^2} = \frac{2\frac{E_1}{E_2}}{(\frac{E_1}{E_2})^2 - 1} \cos \delta = \frac{2 \tan \alpha}{\tan^2 \alpha - 1} \cos \delta = -\tan 2\alpha \cos \delta \quad (13)$$

Another definition:

$$\frac{E_a}{E_b} = \tan \chi \quad (14)$$

(10)/(9):

$$\begin{aligned} \frac{E_a E_b}{E_a^2 + E_b^2} &= \frac{E_1 E_2 \sin \delta}{E_1^2 + E_2^2} \\ \frac{2 \tan \chi}{\tan^2 \chi + 1} &= \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \sin \delta \\ 2 \sin \chi \cos \chi &= 2 \sin \alpha \cos \alpha \sin \delta \\ \sin 2\chi &= \sin 2\alpha \sin \delta \end{aligned}$$

$$\sin 2\chi = \sin 2\alpha \sin \delta \quad (15)$$

(i) if $\delta = \delta_1 - \delta_2 = m\pi, m = 0, \pm 1, \pm 2, \dots$

$$\begin{aligned} \sin 2\chi &= 0 \\ \frac{2 \tan \chi}{1 + \tan^2 \chi} &= 0 \\ \tan \chi &= 0 \end{aligned}$$

Since $\tan \chi = 0$, we can get $E_a = 0$. This situation is called linear polarization.

(ii) if $\delta = \delta_1 - \delta_2 = \frac{\pi}{2}(2n + 1), n = 0, \pm 1, \pm 2, \dots$

The equation of polarized light will be:

$$\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 = 1$$

if we add another condition, $E_1 = E_2 = E$, the equation will be:

$$E_x^2 + E_y^2 = E^2$$

It is called circular polarization. The orthogonal linear component of this situation is:

$$\begin{cases} E_x = E \cos(\tau) \\ E_y = \pm E \cos(\tau - \frac{\pi}{2}) \end{cases}$$

An interesting trick: We can decompose a elliptical polarization to the combination of two

circular polarization.

$$\begin{cases} E_\xi = E_a \cos(\tau + \varphi) = (E_r + E_l) \cos(\tau + \varphi) \\ E_\eta = E_b \sin(\tau + \varphi) = (E_r - E_l) \cos(\tau + \varphi - \frac{\pi}{2}) \end{cases}$$

We can calculate these parameters easily:

$$\begin{cases} E_r = \frac{1}{2}(E_a + E_b) \\ E_l = \frac{1}{2}(E_a - E_b) \end{cases}$$

$$S_0 \equiv E_a^2 + E_b^2 = 2(E_r^2 + E_l^2)$$

1.3 Poincaré sphere

The coordinates of Poincaré sphere are:

$$\begin{cases} S_0 = I = E_a^2 + E_b^2 \\ S_1 = Q = S_0 \cos 2\chi \cos 2\psi \\ S_2 = U = S_0 \cos 2\chi \sin 2\psi \\ S_3 = V = S_0 \sin 2\chi \end{cases}$$

Attention: In fact, energy flux has a parameter: $\frac{1}{2}\epsilon$. But, it's just a easy method to describe the intensity of light. There just exists three independent variables:

$$\begin{aligned} S_0^2 &= S_1^2 + S_2^2 + S_3^2 \\ I^2 &= Q^2 + U^2 + V^2 \end{aligned}$$

We can simplify the Stokes parameters:

$$\begin{aligned} \sin 2\alpha &= \frac{2 \tan \alpha}{1 + \tan^2 \alpha} = \frac{2E_1 E_2}{E_1^2 + E_2^2} \\ \sin 2\chi &= \sin 2\alpha \cdot \sin \delta = \frac{2E_1 E_2}{E_1^2 + E_2^2} \sin \delta = \frac{2E_1 E_2}{I} \cdot \sin \delta \\ |\cos 2\chi| &= \sqrt{1 - \sin^2 2\chi} = \frac{1}{I} \cdot \sqrt{I^2 - (2E_1 E_2)^2 \sin^2 \delta} \\ \tan 2\psi &= -\tan 2\alpha \cdot \cos \delta = \frac{2E_1 E_2}{E_1^2 - E_2^2} \cdot \cos \delta \\ |\cos 2\psi| &= \frac{1}{\sqrt{1 + \tan^2 2\psi}} = \frac{|E_1^2 - E_2^2|}{\sqrt{I^2 - (2E_1 E_2)^2 \sin^2 \delta}} \\ |\sin 2\psi| &= \frac{|2E_1 E_2 \cos \delta|}{\sqrt{I^2 - (2E_1 E_2)^2 \sin^2 \delta}} \end{aligned}$$

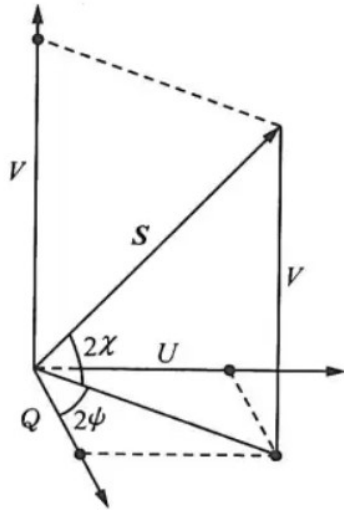


图 2: The definition of Stokes parameters. We use Q-U-V to replace right-hand orthogonal coordinates.

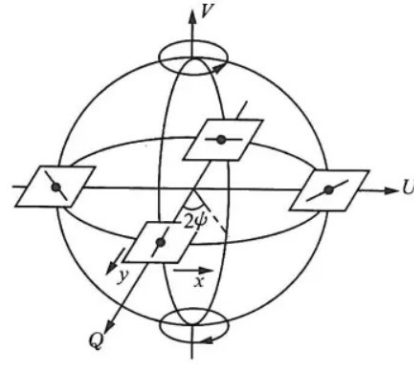


图 3: Upper hemisphere is all counterclockwise (when light propagates towards you) - also, it can be called left-handed polarized light.

So, we can change the Stokes parameters to:

$$\begin{cases} S_0 = I = E_a^2 + E_b^2 = E_1^2 + E_2^2 \\ S_1 = Q = E_1^2 - E_2^2 \\ S_2 = U = 2E_1E_2 \cos \delta \\ S_3 = V = 2E_1E_2 \sin \delta \end{cases}$$

(i) counterclockwise circular polarized light (left-handed polarized light):

$$E_1 = E_2 = E, \delta = \frac{\pi}{2}$$

$$S_0 = I = S$$

$$S_1 = Q = 0$$

$$S_2 = U = 0$$

$$S_3 = V = S$$

(ii) clockwise circular polarized light (right-handed polarized light):

$$E_1 = E_2, \delta = -\frac{\pi}{2}$$

$$S_0 = I = S$$

$$S_1 = Q = 0$$

$$S_2 = U = 0$$

$$S_3 = V = -S$$

(iii) linear polarized light:

$$E_b = E, E_a = 0, (just \tan \chi = \frac{E_a}{E_b} = 0)$$

$$S_0 = I = E^2 = S$$

$$S_1 = Q = I \cdot \cos 2\psi$$

$$S_2 = U = I \sin 2\psi$$

$$S_3 = V = 0$$

2-dimensional figure for Poincare sphere:

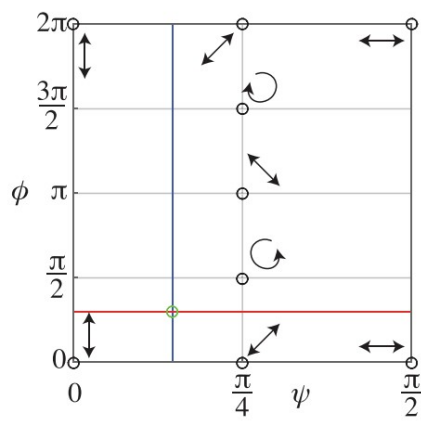


图 4: $\phi = 2\chi$

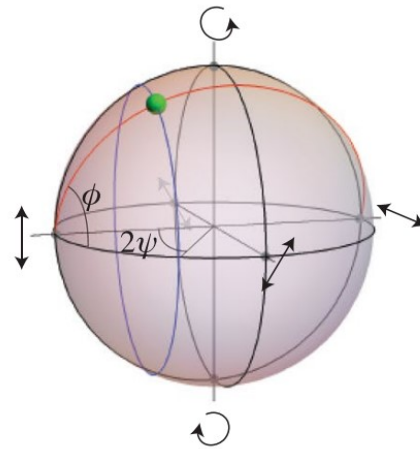


图 5: Here, ϕ is not discribed correct.