Medial Axis analysis of Valles Marineris

Dan James

Abstract

The Medial Axis of the Main Canyon of Valles Marineris is determined geometrically with maximally inscribed discs aligned with the boundaries or rims of the Main Canyon. Inscribed discs are placed at evenly spaced longitude intervals and, using the radius function, the locus of the centre of all discs is determined, together with disc centre co-ordinates. These centre co-ordinates result in arrays of x, y co-ordinates which are curve fitted to a Sinusoidal function and residuals appropriate for nonlinear regression are evaluated using the R-squared value (R^2) and the Root Mean Squared Error (RMSE). This evaluation demonstrates that a Sinusoidal Curve closely fits to the co-ordinate data.

Introduction

There is an extensive literature on the morphology of Canyons of Valles Marineris[1,2,3,4], a feature which at over 4,000km in length, comprises almost a quarter of the Martian Equatorial region. Here a novel approach is taken to extend the known morphometrics of Valles Marineris (VM)[5,6], by establishing the Medial Axis of the Main Canyon of VM, with the principal aim of the research to determine if any mathematical relationship exists between points on the locus.

Data Collection

The Medial Axis or Voronoi Skeleton of a polygon[7,8,9], is the set of all the points with 2 or more closest points on the polygon's boundary and creates a locus of the centre of all the maximally inscribed discs. A Medial Axis Transform (MAT) is particularly effective in determining the central axis for irregular shapes such as Canyons which may have varying widths and asymmetric boundaries.

Using USGS base map data[10], and positive East reporting, a map area from 270° East to 320° East and 4° North to -20° North was selected. This area encompasses most of the VM extent. The 50° of longitude provided a convenient range for establishing, with the radius function, the centres of 50 maximally inscribed discs that extend to the North and South boundaries of the Main Canyon of VM (see Figs 1 & 2):

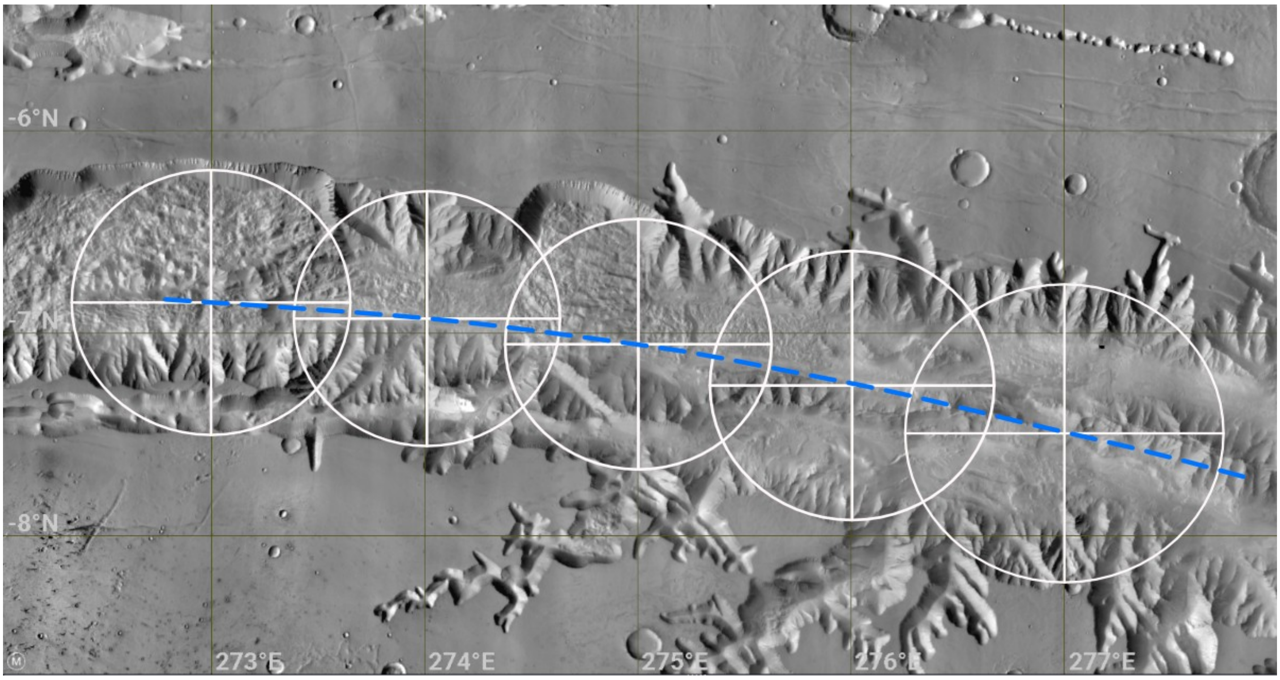


Fig 1: A section of Ius Chasma (at the western extreme of Valles Marineris), showing maximally inscribed discs aligned to the Canyon boundaries, with disc centres enabling a locus (blue dotted line) which represents a segment of the Medial Axis of VM.

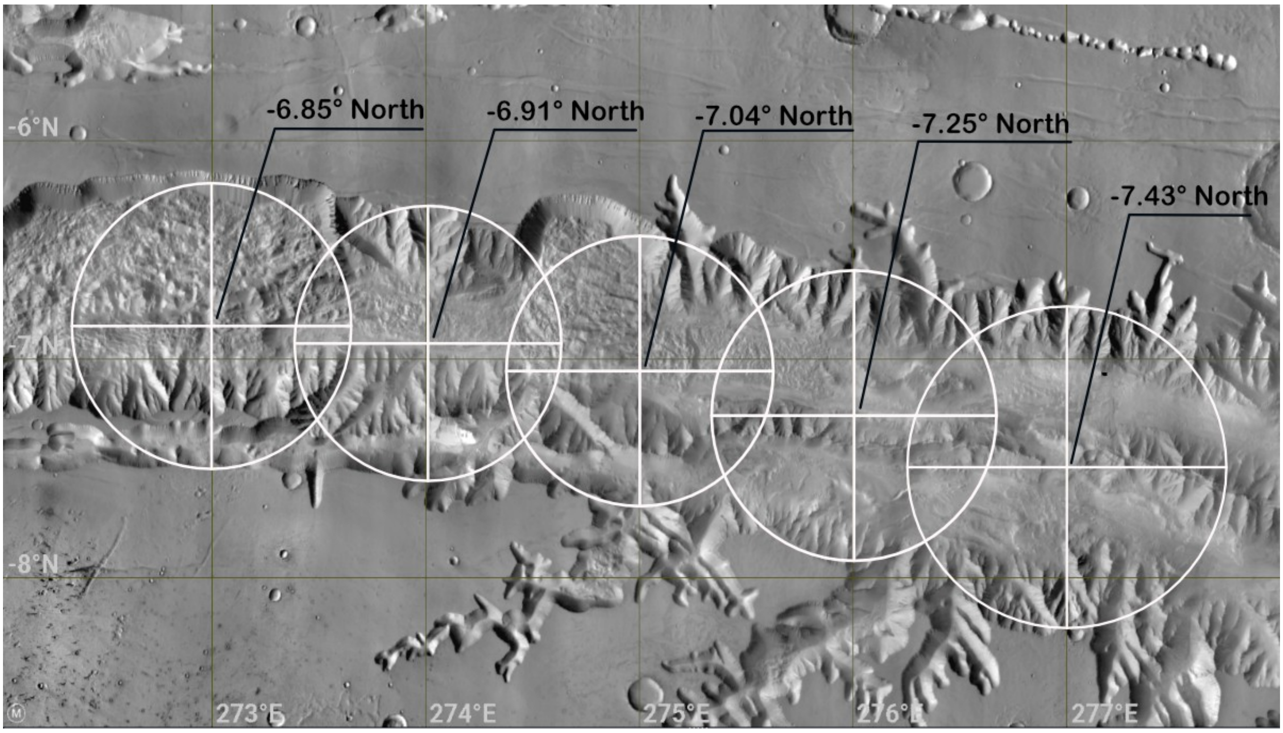


Fig 2: A section of Ius Chasma (at the western extreme of Valles Marineris), showing maximally inscribed discs aligned to the Canyon boundaries, with disc centres enabling precise y co-ordinates.

The MAT procedure resulted in a set of 50 maximally inscribed discs for the whole length of VM (from 270° East to 320° East), to produce the following y-co-ordinates array:

y-data
-6.76, -6.76, -6.85, -6.91, -7.04, -7.25, -7.43, -7.65, -7.83, -8.08, -8.29, -8.57, -8.81, -9.09, -9.39, -9.7, -10, -10.31, -10.58, -10.92, -11.23, -11.53, -11.87, -12.14, -12.45, -12.72, -13.03, -13.27, -13.49, -13.73, -13.85, -14.1, -14.28, -14.44, -14.59, -14.68, -14.74, -14.77, -14.8, -14.8, -14.77, -14.74, -14.62, -14.5, -14.44, -14.25, -14.13, -13.85, -13.67, -13.55

For convenience in handling (eliminating the '-'), the data points were shifted 20 points to produce the following array of positive numbers:

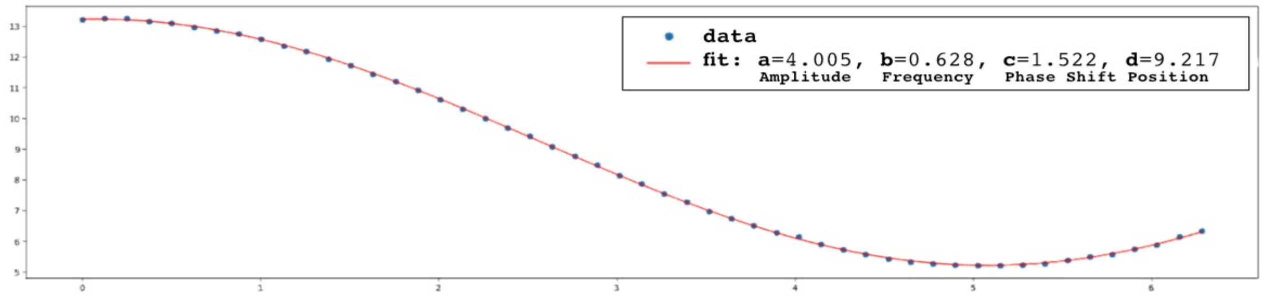
y-data (shifted 20 points)
13.24, 13.24, 13.15, 13.09, 12.96, 12.75, 12.57, 12.35, 12.17, 11.92, 11.71, 11.43, 11.19, 10.91, 10.61, 10.3, 10, 9.69, 9.42, 9.08, 8.77, 8.47, 8.13, 7.86, 7.55, 7.28, 6.97, 6.73, 6.51, 6.27, 6.15, 5.9, 5.72, 5.56, 5.41, 5.32, 5.26, 5.23, 5.2, 5.2, 5.23, 5.26, 5.38, 5.5, 5.56, 5.75, 5.87, 6.15, 6.33, 6.45

Curve Fitting

An Excel charting tool was used to display, the y co-ordinates in a scatter plot which indicated that a Sinusoidal Curve was a good fit. This was further confirmed by importing the y co-ordinates into Python software with the Matplotlib extension, where code was written to fit the y co-ordinates to a curve and determine, with non-linear regression techniques, the Root Mean Square Error (RMSE) and R^2 value[11]. The resulting Sinusoidal Curve was calculated as:

$$y = 4.005 * \sin(0.628 * x + 1.522) + 9.217$$

The RSME for the data points was calculated as 0.025, with an R^2 value of 0.957. As shown on the following chart:



Root Mean Squared Error: 0.025

$y = 4.005 * \sin(0.628 * x + 1.522) + 9.217$

R^2 = .957

Fig 3: Chart showing plot for Medial Axis data points of VM fitted against a Sinusoidal Curve[12]

Interpretation

A Root Mean Squared Error (RMSE) of 0.025 for the above data is extremely small given that the y-values range from 13.24 to 6.45. The residuals (or differences) between the observed and predicted values indicate that the sinusoidal model fits the data well.

The R^2 value or coefficient of determination measures the proportion of variance for a dependent variable and the determined R^2 value of 0.957 suggests that 95.7% of the y-data in the above can also be explained best by the sinusoidal model.

Reproducibility

The MAT data collection techniques outlined are easily reproducible and highly verifiable. However, although MAT is a proven morphometric tool for analysing and computing with shape, any changes in an object's boundary may vary the connection information[13,14]. These variations may be particularly apparent in the Ius Chasma at the western extreme of VM, where the irregular and punctuated Canyon rims make disc radius problematic.

Any slight variations in the placement or radius of the discs may affect either the RMSE or R^2 values for the derived Sinusoidal Curve. In practice it is proposed that any such variations will not substantially alter the fit to a sinusoidal curve.

Discussion

Though the MAT data confirms a sinusoidal curve, whether or not such a curve can be used to inform a formation process for VM is an open question and a good subject for further research. Conceivably the observation of a sinusoidal curve might disfavour uniformitarian hypotheses, because Sinusoidal Curves or Sine Waves, which are ubiquitous in nature, are invariably associated with some aspect of a dynamic process.

Conclusion

This paper has demonstrated a method to determine the Medial Axis of the Main Canyon of Valles Marineris and provided a geometric proof that the derived Axis conforms to a Sinusoidal Curve.

References/Notes

1. Richard A. Schultz (1998) ‘Multiple-process origin of Valles Marineris basins and troughs, Mars’ *Planetary and Space Science*. Volume 46, Issues 6–7, June–July 1998, Pages 827-829, 831-834 https://doi.org/10.1016/S0032-0633(98)00030-0

2. Jeffrey C. Andrews-Hanna (2011) ‘The formation of Valles Marineris: 1. Tectonic architecture and the relative roles of extension and subsidence’ J*ournal of Geophysical Research Atmospheres* Volume 46, Issues 6–7, June–July 1998, Pages 827-829, 831-834 https://doi.org/10.1016/S0032-0633(98)00030-0

3. E. Hauber, Matthias Grott (2009) ‘Martian rifts: Structural geology and geophysics’ *Earth and Planetary Sciences Letters* 294:393-410, https://doi.org:10.1016/j.epsl.2009.11.005

4. Giovanni Leone (2014) ‘A network of lava tubes as the origin of Labyrinthus Noctis and Valles Marineris on Mars’ *Journal of Volcanology and Geothermal Research* Volume 277,https://doi.org/10.1016/j.jvolgeores.2014.01.011

5. Le Deit, L., O. Bourgeois, D. Mege, E. Hauber, S. Le Mouelic, M. Masse, R. Jaumann, and J.-P. Bibring (2010), ‘Morphology, stratigraphy, and mineralogical composition of a layered formation covering the plateaus around Valles Marineris, Mars: Implications for its geological history’, *Icarus*, 208, 684–703, doi:10.1016/j.icarus.2010.03.012.

6. Davis, B. J., and J. C. Andrews-Hanna (2011), ‘Flexural response to sediment erosion and unloading at Valles Marineris, Mars,’ *Proc. Lunar Planet. Sci. Conf.*, 42nd, 2557.

7. Harry Blum, (1967). "A transformation for extracting new descriptors of shape". In Wathen-Dunn, Weiant (ed.). *Models for the Perception of Speech and Visual Form* (PDF). Cambridge, Massachusetts: MIT Press. pp. 362–380.

8. Harry Blum, Roger Nagel (1978). "Shape description using weighted symmetric axis features". *Pattern Recognition* Volume 10, Issue 3 1978 Pages 167-180

9. Rolf Klein, (1989). "Abstract Voronoi diagrams and their applications". *Computational Geometry and its Applications Lecture Notes in Computer Science*. Vol.333. Springer. pp.148–157. doi:10.1007/3-540-50335-8_31.ISBN.

10. The United States Geological Survey (USGS), has been instrumental in the development and maintenance of the Mars global geospatial reference system which provides a standardized framework for Mars data, enabling the scientific community to locate, measure, and describe the spatial relationships among Martian geographic features with enhanced precision. Uniformity in spatial referencing is paramount for effective and consistent global and regional analyses of Mars. This data amalgamation, coupled with rigorous calibration and validation procedures, ensures the production of high-quality and reliable cartographic products. USGS has built its Mars cartography using data from several Mars orbiting missions, in particular: HiRISE (High Resolution Imaging Science Experiment) and HRSC (High Resolution Stereo Camera) are imaging systems onboard the Mars Reconnaissance Orbiter (MRO) and Mars Express Orbiter, respectively, that capture high-resolution images of the Martian surface. Both imaging systems take into account the spherical distortion of Mars when processing their data. To compensate for spherical distortion, both HiRISE and HRSC rely on a combination of techniques, including:

1. **Pre-processing**: The raw images captured by HiRISE and HRSC are initially pre-processed using geometric and radiometric corrections. This includes correcting for the effects of camera and spacecraft motion, as well as compensating for the differences in illumination conditions and atmospheric effects. This step helps ensure that the images are ready for further processing to create accurate representations of the Martian surface.
2. **Projection**: To correct for the spherical distortion, images are projected onto a reference ellipsoid that approximates the shape of Mars. This projection process essentially "flattens" the images onto a two-dimensional surface, allowing for more accurate analysis and interpretation of the surface features.
3. **Digital Terrain Models** (DTMs): Both HiRISE and HRSC generate DTMs from their stereo image pairs. DTMs are essentially 3D models of the surface, created by combining the images taken from

different angles. These models provide elevation data for each pixel in the image, which is crucial for correcting spherical distortion and enabling accurate representations of the terrain.

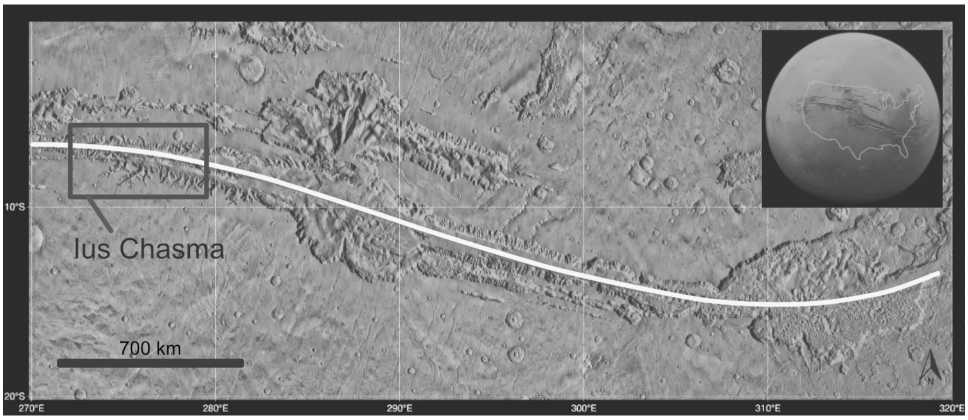
4. **Co-registration and Mosaicking**: To create a seamless mosaic of the Martian surface, individual images must be co-registered and adjusted to account for the spherical distortion.

By using these techniques, HiRISE and HRSC images can be corrected for spherical distortion, ensuring that the data they provide is accurate and useful for the scientific community's study of Mars.
In summary the USGS base maps used in this paper are highly accurate in terms of spatial co-ordinates and the elimination of distortion.

https://www.usgs.gov/special-topics/planetary-geologic-mapping/mapping-and-gis-resources

11. Python Code written to fit data to curve: **https://github.com/DoubleCurvature/Curve-fit**

12. Graphic showing overlay of Medial Axis Curve in the Main Canyon of Valles Marineris:



13. Robert A, Katz, Stephen M. Pizer, (2003). "Untangling the Blum Medial Axis Transform". *International Journal of Computer Vision* 55 (2): 139-153 doi:10.1023/A:1026183017197

14. Hao Xia, Paul Tucker, (2011). ‘Fast equal and biased distance fields for medial axis transform with meshing in mind’ *Applied Mathematical Modelling. Volume 35, Issue 12,* December 2011, Pages 5804-5819 https://doi.org/10.1016/j.apm.2011.05.001