Kinematic Model of Ackerman Steering Vehicles

June 14, 2011

Relation between the Control Input and Speed of Wheels

Let:

- v denote the linear velocity of the vehicle;
- v_r the linear velocity of the right wheel;
- v_l the linear velocity of the left wheel;
- ω (omega) the angular velocity of the vehicle;
- ϕ (phi) the steering angle of the vehicle;
- L the distance between front and rear axles (or wheel base);
- l the distance between left and right wheels.

Figure 1 shows the approximate kinematic model of an Ackerman steering vehicle.

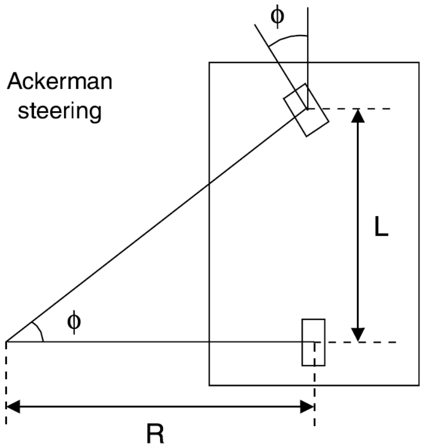

Figure 1: Approximate kinematic model of an Ackerman steering vehicle [SOT03].

The linear velocity of the right wheel, v_r, and the linear velocity of the left wheel, v_l, can be computed from the linear velocity of the vehicle, v, and the steering angle of the vehicle, ϕ, as follows. The linear velocity of the vehicle, v, is given by:

$$v = \frac{v_r + v_l}{2} \quad (1)$$

The angular velocity of the vehicle, ω, is given by:

$$\omega = \frac{v_r - v_l}{l} \quad (2)$$

The steering angle of the vehicle, ϕ, is given by:

$$\phi = \arctan\left(L\frac{\omega}{v} \right) \quad (3)$$

Isolating v_r in Equation 1:

$$v = \frac{v_r + v_l}{2} \rightarrow v_r + v_l = 2v \rightarrow v_r = 2v - v_l \quad (4)$$

Isolating v_r in Equation 2:

$$\omega = \frac{v_r - v_l}{l} \rightarrow v_r - v_l = \omega l \rightarrow v_r = \omega l + v_l \quad (5)$$

Isolating ω in Equation 3:

$$\phi = \arctan\left(L\frac{\omega}{v} \right) \rightarrow \tan(\phi) = L\frac{\omega}{v} \rightarrow \omega = \frac{v\tan(\phi)}{L} \quad (6)$$

Equating the Equations 4 and 5 and isolating v_l:

$$2v - v_l = \omega l + v_l \rightarrow 2v_l = 2v - \omega l \rightarrow v_l = \frac{2v - \omega l}{2} \quad (7)$$

Substituting the Equation 6 in Equation 7, we finally obtain the v_l from v and ϕ:

$$v_l = \frac{2v - \omega l}{2} \rightarrow v_l = \frac{2v - (\frac{v\tan(\phi)}{L})l}{2} \rightarrow v_l = v - \frac{lv\tan(\phi)}{2L} \quad (8)$$

Isolating v_r in Equation 1:

$$v = \frac{v_r + v_l}{2} \rightarrow v_r + v_l = 2v \rightarrow v_r = 2v - v_l \quad (9)$$

Substituting Equation 8 in Equation 9, we finally obtain v_r from v and ϕ:

$$v_r = 2v - v_l \rightarrow v_r = 2v - (v - \frac{lv\tan(\phi)}{2L}) \rightarrow v_r = v + \frac{lv\tan(\phi)}{2L} \quad (10)$$

The steering angle of the vehicle, ϕ, can be computed from the linear velocity of the right wheel, v_r, and the linear velocity of the left wheel, v_l, by substituting the Equations 1 and 2 in Equation 3:

$$\phi = \arctan\left(L\frac{\omega}{v}\right) \rightarrow \phi = \arctan\left(L\frac{\left(\frac{v_r - v_l}{l}\right)}{\left(\frac{v_r + v_l}{2}\right)}\right) \rightarrow \phi = \arctan\left(\frac{2L(v_r - v_l)}{l(v_r + v_l)}\right) \quad (11)$$

Dynamics of the Global Position and Orientation Variables

Let:

- (x, y) denote the pose of the vehicle in a global frame of coordinates;
- θ the orientation of the vehicle in a global frame of coordinates;

The dynamics of the global position and orientation variables, (x, y, θ), are given by:

$$\frac{dx}{dt} = v\cos(\theta) \quad (12)$$

$$\frac{dy}{dt} = v\mathrm{sen}(\theta) \quad (13)$$

$$\frac{d\theta}{dt} = v\frac{\tan(\phi)}{L} \quad (14)$$

References

[BON11] Philippe Bonnifait, Pascal Bouron, Paul Crubillé, Dominique Meizel. Data Fusion of Four ABS Sensor and GPS for an Enhanced Localization of Car-like Vehicles. International Conference on Robotics & Automation, pages 1597-1602, 2011.

[SOT03] Miguel Angel Sotelo. Lateral Control Strategy for Autonomous Steering of Ackerman-Like Vehicles. Robotics and Autonomous Systems 45:223-233, 2003.

[XIA] Jizhong Xiao. Mobile Robot Locomotion. Department of Electrical Engineering, City College of New York.