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Resonate-and-Fire Neurons for Edge-Enhanced EEG Analysis

Neuromorphic approach for efficient processing of sensor data

Neuromorphic computing draws inspiration from biological neurons, which communicate over time via **sparse spikes**, resulting in **highly efficient processing**.

The efficiency and dynamics of these models makes them **well-suited for signal processing**, especially in real-time, edge computing use cases.

Spiking neuron models simulate this behaviour – typically for use in **Spiking Neural Networks (SNNs)** – capturing **richer temporal dynamics** than conventional artificial neurons.

Spiking Neuron Models

In biological neurons, the **membrane potential** U_{mem} refers to the voltage difference measured between the inside and outside of the neuron's cell membrane. This changes over time and in response to input stimuli; when this potential crosses a threshold ϑ, a spike is emitted.

For SNNs, this process is simplified, discretised, and made trainable. Different neuron models capture distinct aspects of biological spiking behaviour, trading off between **biological plausibility** and **computational efficiency**.

Below we describe three spiking neuron models: **1.** the most widely used model; **2.** a promising, but under-explored extension; **3.** our simplified adaptation, designed for SNNs and neuromorphic signal processing.

1. Leaky-Integrate-and-Fire (LIF)

$$U[t+1] = \overbrace{\beta U[t]}^{\text{decay}} + \overbrace{WX[t+1]}^{\text{input}} - \overbrace{S_{\text{out}}[t]\vartheta}^{\text{reset}}$$

$$S_{\text{out}}[t] = \begin{cases} 1, & \text{if } U[t] > \vartheta \\ 0, & \text{otherwise} \end{cases}, \quad \beta \in [0,1]$$

The default model; exponential decay.

- \+ Simple, only β, ϑ, Reset as hyperparameters;
- \+ Recursive formulation with weighted input, well-suited for training SNNs;
- \- Limited ability to encode signals.

2. Resonate-and-Fire (RF)

$$\frac{dU(t)}{dt} = (-b + i\omega)U(t) + I(t) \Rightarrow$$

$$U[t+1] = U[t] + \Delta t\big((-b + i\omega)U[t] + I[t]\big)$$

Extension of LIF; oscillatory convergence.

- \+ Extracts spectral information;
- \- Stiff ODE → difficult to train and initialise;
- \- Euler approx. can be unstable/inaccurate.

3. Simplified RF (SRF)

$$U[t+1] = \beta \cdot \rho \cdot U[t] + \overbrace{WX[t+1]}^{\text{input}} - \overbrace{S_{\text{out}}[t]\vartheta}^{\text{reset}}$$

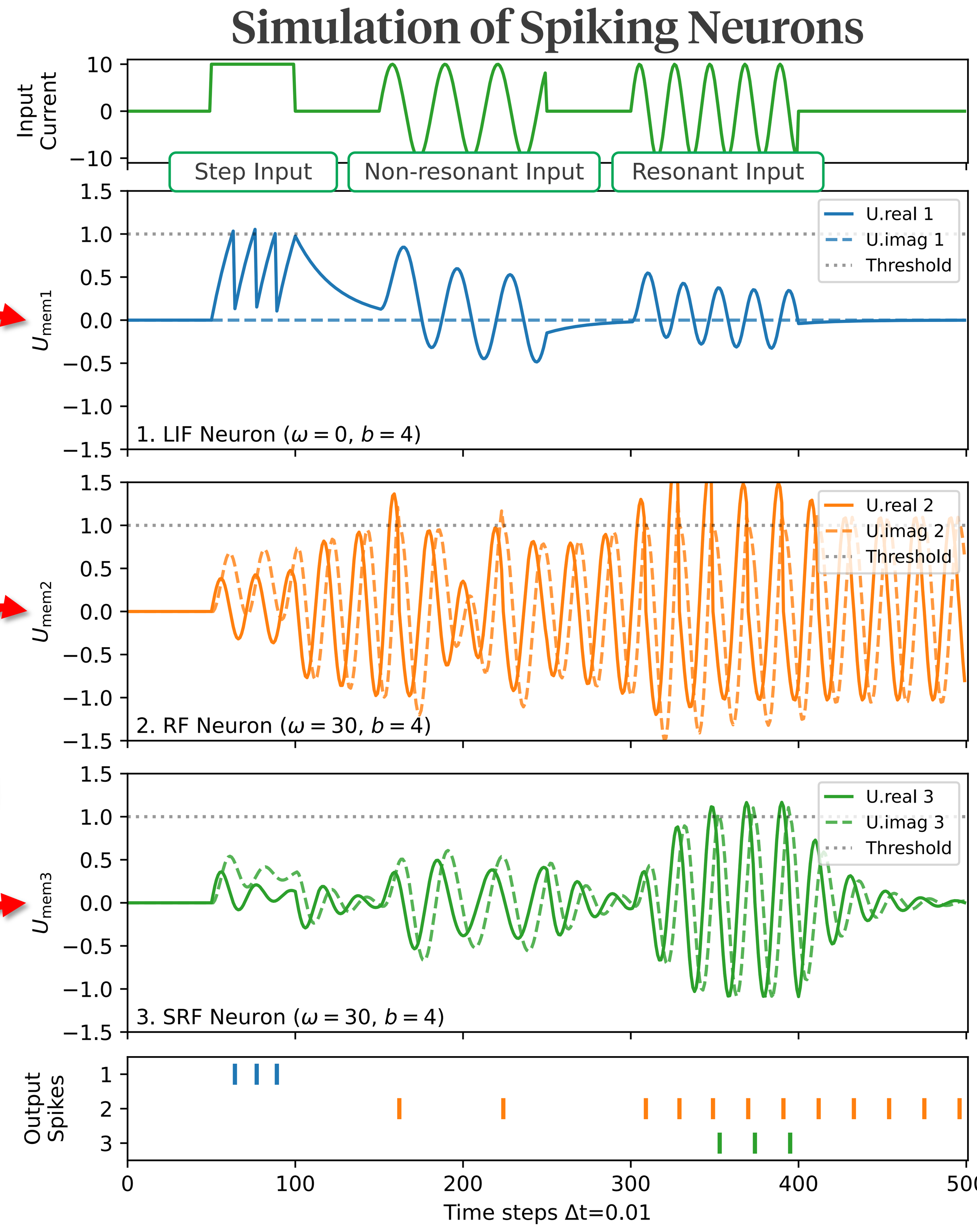
$$\beta = e^{-b\Delta t}\ (\text{decay}), \qquad \rho = e^{i\omega\Delta t}\ (\text{rotation}),$$

$$S_{\text{out}}[t] \begin{cases} 1, & \text{if } \Im(U[t]) > \vartheta \\ 0, & \text{otherwise} \end{cases}$$

Our reformulation of the RF neuron.

- \+ RF dynamics in intuitive *LIF-like* formulation;
- \+ Stable and accurate regardless of Δt size;
- \+ Precomputes β, ρ for efficiency;
- \+ ϑ derived from max. driven amplitude;
- ? More suitable for SNNs.

Simulation of Spiking Neurons

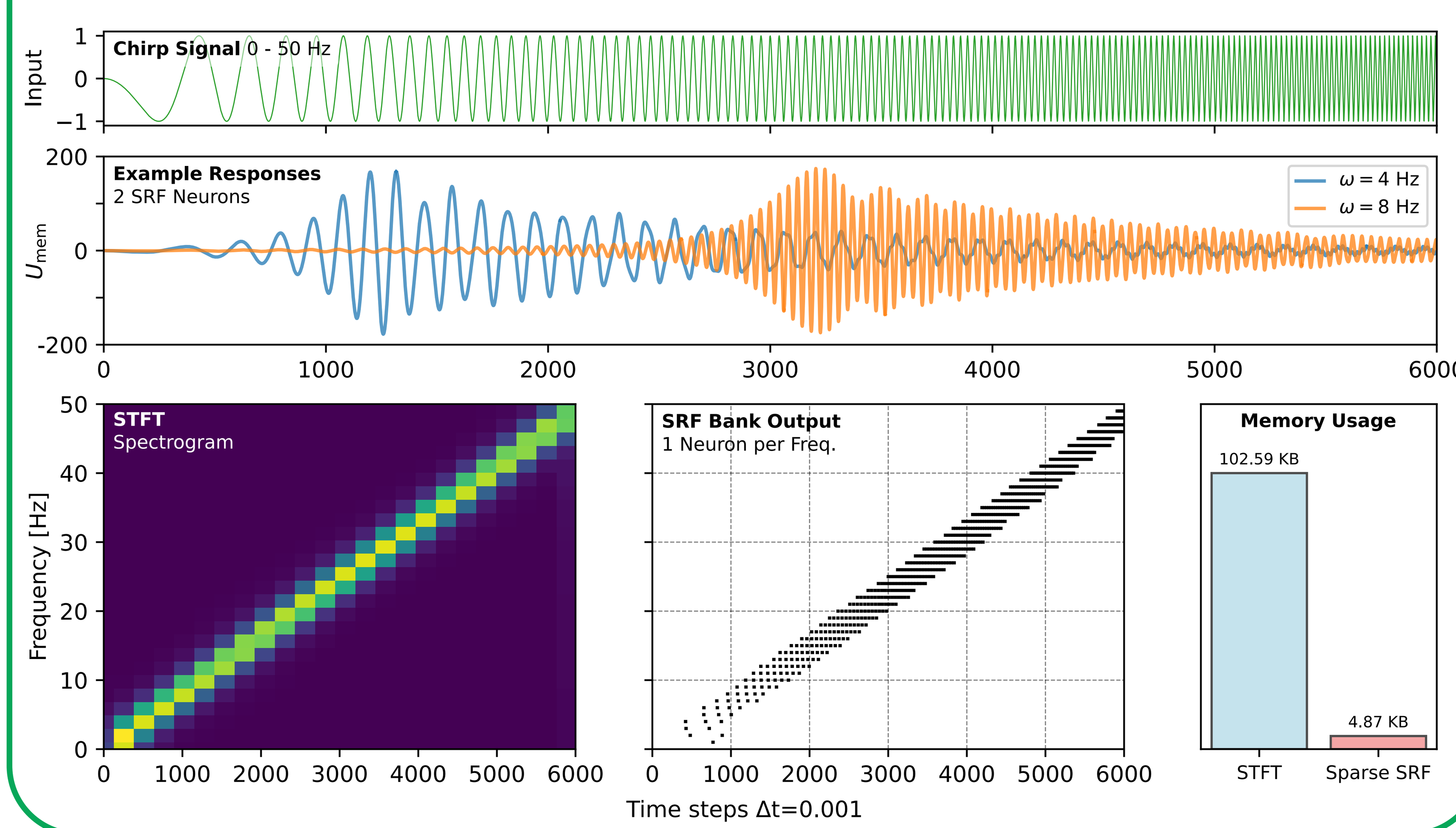


1) SRF Neurons for Spectral Analysis

Spectral decomposition is a ubiquitous step in time series analysis, e.g. for EEG, audio, etc. SRF neurons perform this in a spiking manner, reducing latency and memory use, since when unrolled, SRF dynamics mirror the Short-Time Fourier Transform (STFT) at a fixed frequency ω, using an exponential window:

$$U[t] = \sum_{n=0} X[t-n]\beta^n e^{i\omega n\Delta t} \approx \text{STFT}(t,\omega) = \sum_{n=0} X[t-n]w[n]e^{-i\omega n}$$

We analysed a chirp signal (0-50 Hz) using STFT and a bank of 50 SRF neurons. The results are roughly equivalent, but the output from the SRF bank offers greatly reduced latency and used memory (i.e., via sparse matrices). Importantly, the spiking STFT remains invertible, enabling reconstruction of the original signal [3].



2) EEG Data Feature Extraction

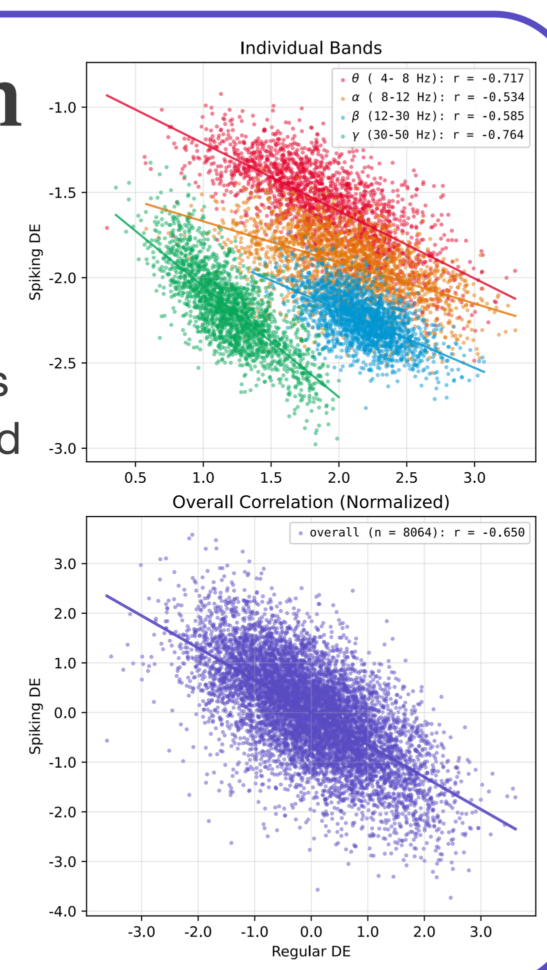
For a more practical experiment, we extracted the Differential Entropy (DE), a widely used feature for EEG emotion recognition data.

Standard DE_{norm} is computed after decomposing signals into fixed frequency bands. Spiking $\text{DE}_{\text{lognorm}}$ was instead derived from the *inter-spike intervals* in the output of a bank of SRF neurons.

Despite this indirect method, the spiking DE showed moderate to strong correlation with the standard DE ($r \approx -0.65$).

$$\text{DE}_{\text{norm}} = 0.5 \cdot \log(2\pi e\sigma^2), \quad (\text{normal dist.})$$

$$\text{DE}_{\text{lognorm}} = 0.5 \cdot \log(2\pi e\sigma^2) + \mu, \quad (\text{lognorm dist.})$$



Conclusion & Future Work

- Our Simplified Resonate-and-Fire neuron appropriately produces resonating dynamics, while its LIF-like formulation results in simplified initialisation and improved efficiency.
- Its unique spectral properties make it a promising foundation for more comprehensive neuromorphic signal processing pipelines.
- Next, we will integrate our new neurons into full SNNs – not only as spike encoder – to test for gains in trainability and performance.
- Planned studies include resonating SNNs for:
  - The benchmark N-TIDIGITS spiking audio dataset;
  - **Edge-enhanced EEG emotion recognition, building on prior SNN successes and as end-goal of the PhD project.**

Link to Github with Tutorial Notebooks and further reading