If we consider the score with the constant factor $\frac{1}{p}$ we have

$$\begin{aligned}\psi(W;\theta,\eta) &:= \frac{D(Y-g(0,X))}{p} - \frac{m(X)(1-D)(Y-g(0,X))}{p(1-m(x))} - \frac{D}{p}\theta \\ &= \psi_a(W;\eta)\theta + \psi_b(W;\eta)\end{aligned}$$

with $\eta = (g,m,p)$ and where the components of the linear score are

$$\begin{aligned}\psi_a(W;\eta) &= -\frac{D}{p}, \\ \psi_b(W;\eta) &= \frac{D(Y-g(0,X))}{p} - \frac{m(X)(1-D)(Y-g(0,X))}{p(1-m(X))}.\end{aligned}$$

If the score is linear in the parameter, we get for the asymptotic variance

$$\sigma^2 = J_0^{-2}\mathbb{E}(\psi^2(W;\theta_0,\eta_0)),$$

with $J_0 = \mathbb{E}(\psi_a(W;\eta_0))$. For the ATTE score we get

$$J_0 = \mathbb{E}(\psi_a(W;\eta_0)) = \mathbb{E}\left(-\frac{D}{p_0}\right) = -\frac{\mathbb{E}(D)}{p_0} = -\frac{p_0}{p_0} = -1.$$

Therefore, the variance simplifies to $\sigma^2 = \mathbb{E}(\psi^2(W;\theta_0,\eta_0))$. This is also mentioned in Chernozhukov et al. (2018) on page C35.

If we instead consider the score without the constant factor $\frac{1}{p}$ we get

$$\begin{aligned}\tilde{\psi}(W;\theta,\eta) &:= D(Y-g(0,X)) - \frac{m(X)(1-D)(Y-g(0,X))}{(1-m(x))} - D\theta \\ &= \tilde{\psi}_a(W;\eta)\theta + \tilde{\psi}_b(W;\eta)\end{aligned}$$

with $\eta = (g,m)$ and where the components of the linear score are

$$\begin{aligned}\psi_a(W;\eta) &= -D, \\ \psi_b(W;\eta) &= D(Y-g(0,X)) - \frac{m(X)(1-D)(Y-g(0,X))}{(1-m(X))}.\end{aligned}$$

The two score functions are related via $\psi(W;\theta,\eta) = \frac{1}{p}\tilde{\psi}(W;\theta,\eta)$. We further get

$$\tilde{J}_0 = \mathbb{E}(\tilde{\psi}_a(W;\eta_0)) = \mathbb{E}(-D) = -p_0.$$

For the variance we therefore get

$$\tilde{\sigma}^2 = \tilde{J}_0^{-2}\mathbb{E}(\tilde{\psi}^2(W;\theta_0,\eta_0)) = \frac{\mathbb{E}(p_0^2\psi^2(W;\theta_0,\eta_0))}{p_0^2} = \mathbb{E}(\psi^2(W;\theta_0,\eta_0)) = \sigma^2.$$

$\Longrightarrow$ So it basically does not matter whether one adds the constant factor $\frac{1}{p}$, but with the constant the formula of the variance simplifies to $\sigma^2 = \mathbb{E}(\psi^2(W;\theta_0,\eta_0))$.

References

Chernozhukov, V., Chetverikov, D., Demirer, M., Duflo, E., Hansen, C., Newey, W. and Robins, J. (2018), Double/debiased machine learning for treatment and structural parameters. The Econometrics Journal, 21: C1-C68.