DoubleML - Sensitivity Analysis

DoubleML Trainings

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Motivation: Sensitivity Analysis

Motivation: DoubleML Workflow

0. Problem Formulation
1. Data-Backend
2. Causal Model
3. ML Methods
4. DML Specification
5. Estimation
6. Inference

7. Sensitivity Analysis

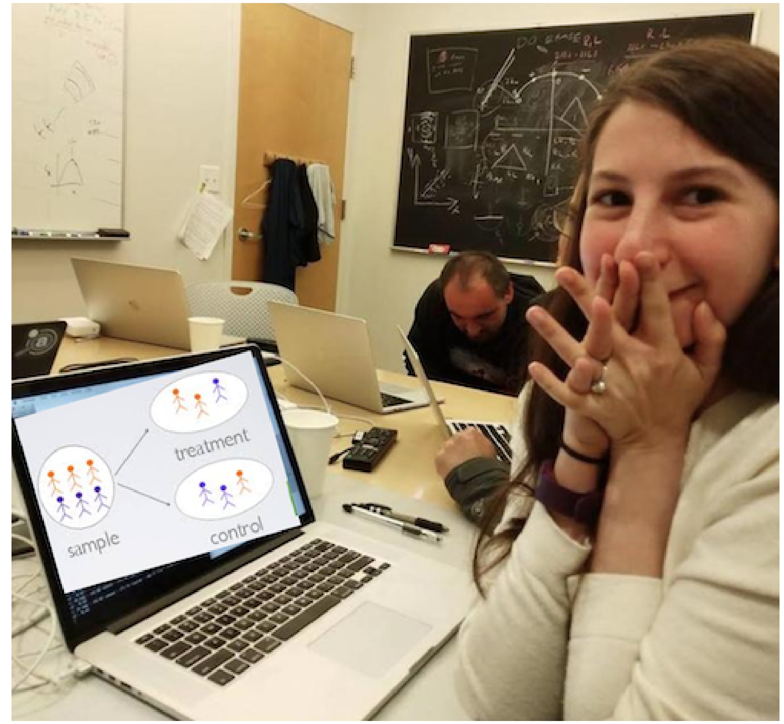
- Earlier we introduced 6 steps of the **DoubleML workflow**.
- Actually, there is a **7th step** that is often overlooked: **Sensitivity Analysis**

Motivation

- Whenever we have (properly collected) experimental data, we have good reason to believe that the **independence** assumption holds, i.e., the treatment assignment is independent of the potential outcomes

$$Y(d) \perp D$$

- However, what about causal evidence from **observational data**? 😱



"A well designed RCT is the dream of any scientist",
Source: Chapter 2 of Facure and Germano (2021)

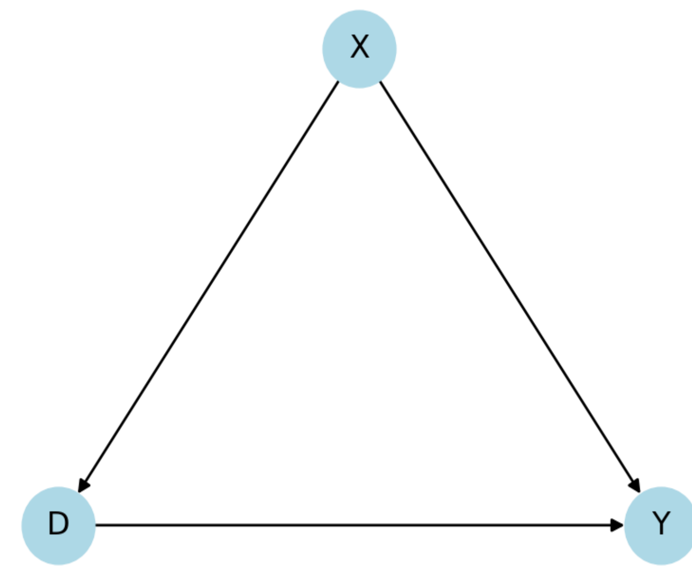
Motivation

- Observational studies are often based on the assumption of **conditional independence**[1]
- Conditional on pre-treatment confounders X, the treatment is *as good as randomly assigned*

$$Y(d) \perp D \mid X$$

- In general, the **independence** assumption is not testable!

▶ Code



DAG: Causal Effect of D on Y, with observed confounders X.

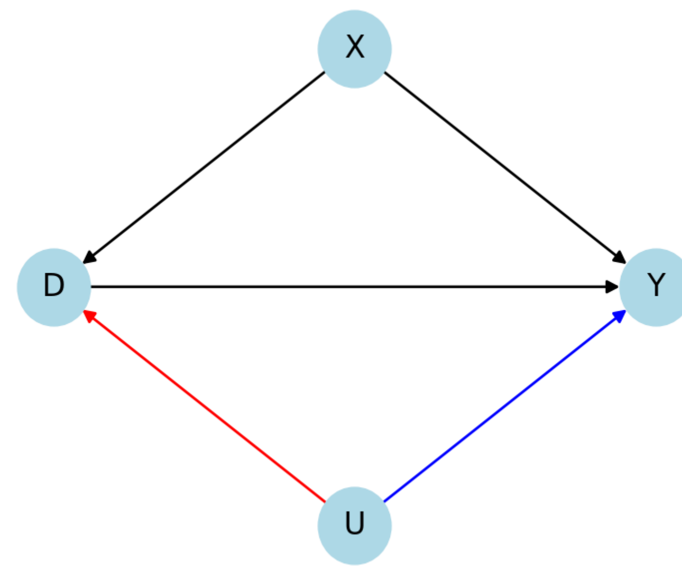
1. Also known as *conditional exogeneity*, *unconfoundedness* or *ignorability*

Motivation

- What if the conditional independence assumption is **violated**?
- Unobserved confounders U would introduce an *omitted variable bias/ selection-into-treatment bias*
- Key questions:

1. *How strong would a confounding relationship need to be in order to change the conclusions of our analysis?*
2. *Would such a confounding relationship be plausibly present in our data?*

▶ Code



DAG: Causal Effect of D on Y, with observed confounders X and unobserved confounders U.

Outlook: Sensitivity Analysis

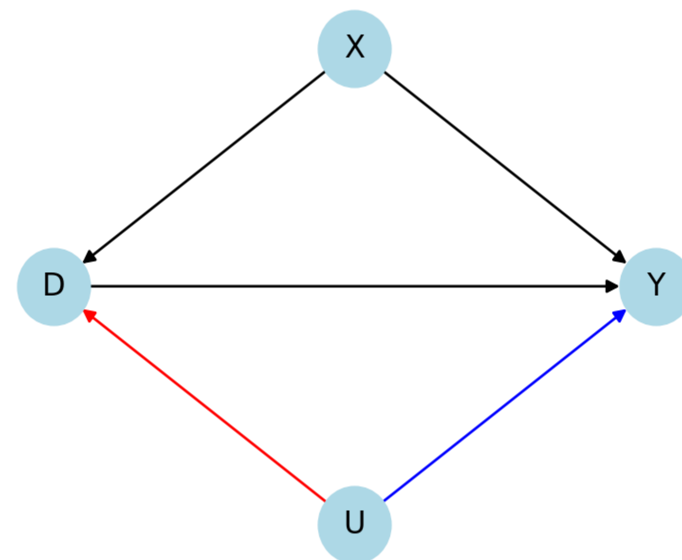
1. *How strong would a confounding relationship need to be in order to change the conclusions of our analysis?*

- Model the strength of the confounding relationships in terms of some **sensitivity parameters**, i.e.,
  - $U \rightarrow D$ and
  - $U \rightarrow Y$

2. *Would such a confounding relationship be plausibly present in our data?*

- Benchmarking framework

▶ Code



DAG: Causal Effect of D on Y, with observed confounders X and unobserved confounders U.

Example: Linear Regression Model

Consider a linear regression model with observed and unobserved confounders X and U, respectively

$$
\begin{aligned}
Y &= \theta_0 D + \beta X + \gamma_1 U + \epsilon, \\
D &= \delta X + \gamma_2 U + \nu,
\end{aligned}
$$

with ϵ and ν being uncorrelated error terms and θ_0 corresponds to the average treatment effect of the (continuous) treatment D on Y.

- *Short* parameter $\theta_{0,s}$ - corresponds to model using data $\{Y, D, X\}$ (*short* model)[1]
- *Long* parameter θ_0 - corresponds to model using data $\{Y, D, X, U\}$ (*long* model)

$\Rightarrow$ Omitted variable bias:

$$\theta_{0,s} - \theta_0$$

1. Reference: Chernozhukov et al. (2022)

Example: Linear Regression Model

$$
\begin{aligned}
Y &= \theta D + \beta X + \gamma_1 U + \epsilon, \\
D &= \delta X + \gamma_2 U + \nu,
\end{aligned}
$$

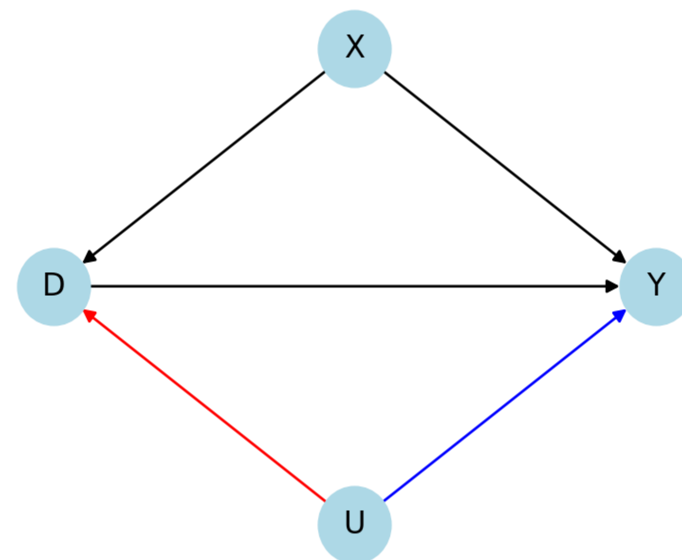
- Simple idea for sensitivity analysis:
  - Parametrically model / simulate the unobserved confounder U and derive sensitivity bounds according to different values of γ_1 and γ_2
  - ⚡ Problem: Sensitivity results will depend on the parametric model for U, which might be too simplistic, for example,
    - Binary or continuous U?
    - Single or multiple confounders U?
    - Correlation of U with X?
- Alternative: Use R^2-based sensitivity parameters (Cinelli and Hazlett 2020)

Example: Linear Regression Model

Sensitivity parameters in Cinelli and Hazlett (2020)

▶ Code

- $U \rightarrow D$: Share of residual variation of D explained by omitted confounder(s) U, after taking X into account, $R^2_{D\sim U|X}$
- $U \rightarrow Y$: Share of residual variation of Y explained by U, after taking X and D into account, $R^2_{Y\sim U|D,X}$

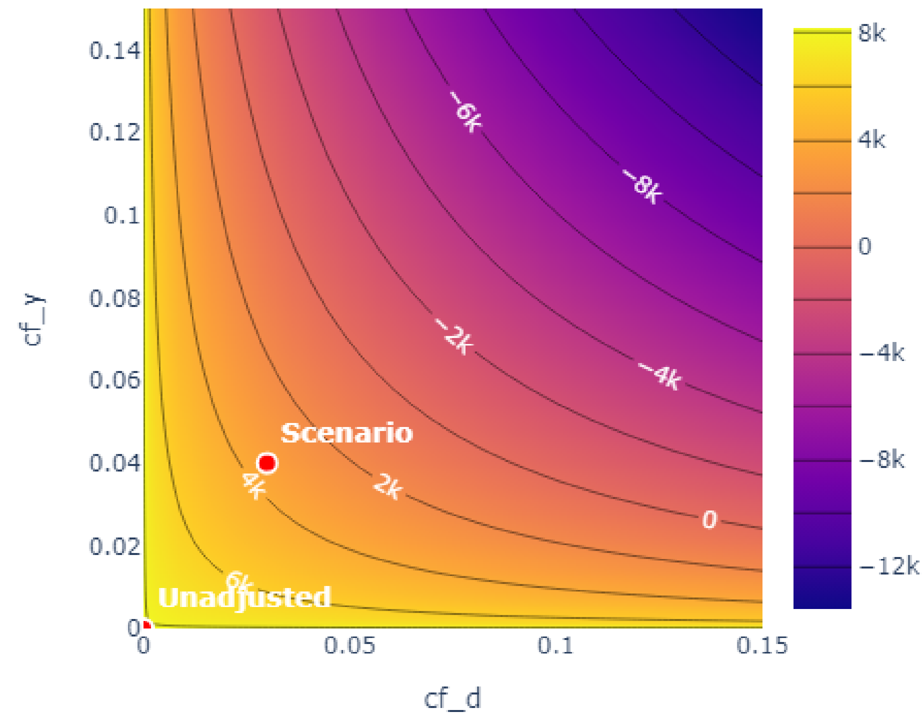


DAG: Causal Effect of D on Y, with observed confounders X and unobserved confounders U.

Example: Linear Regression Model

Results in Cinelli and Hazlett (2020)

- Bounds for the omitted variable bias $\theta_{0,s} - \theta_0$ and confidence intervals as based on values for these sensitivity parameters
- Various measures to be reported:
  - Robustness value
  - Extreme scenarios
- Visualization:
  - Contour plots (Imbens 2003)



Example contour plot, Source DoubleML User Guide

Example: Linear Regression Model

- The approach by Cinelli and Hazlett (2020) can be used to derive sensitivity bounds depending on various specifications of the sensitivity parameters
- However, how do we know if the values for these parameters are **plausible**?
- Cinelli and Hazlett (2020) also develop a formal **benchmarking framework** to relate the values of the sensitivity parameters to observed confounders
- **Idea**:
  - Mimic omitting an important observed (benchmark) confounder and re-compute the values of the sensitivity parameters
  - Use domain expertise to assess plausibility of the confounding scenarios

Example: Linear Regression Model

Benchmarking example

- Assume we know that X_1 is a very important predictor for Y and D, then we can calculate the benchmark values for the sensitivity parameters, i.e.,
  - $X_1 \to D$: Share of residual variation of D explained by the omitted confounder(s) X_1, after taking X_{-1} into account,[1] $R^2_{D\sim X_1|X_{-1}}$
  - $X_1 \to Y$: Share of residual variation of Y explained by X_1, after taking X_{-1} and D into account, $R^2_{Y\sim X_1|D,X_{-1}}$
- Given the values for the benchmarking variable, we can judge whether critical values of the sensitivity parameters are plausible or not

1. X_{-1} denotes the set of all predictors except X_1

Sensitivity Analysis for Causal ML

Sensitivity Analysis for Causal ML

- The framework of Cinelli and Hazlett (2020) is very intuitive and powerful for the linear regression model
- Here, linearity helps to endow the sensitivity parameters with an intuitive interpretation (partial R^2)
- However, the framework itself does not directly expand to non-linear models, such as the interactive regression model

Sensitivity Analysis for Causal ML

- Chernozhukov et al. (2022) propose a generalization of the sensitivity analysis framework to non-linear models and is suitable for ML-based estimation
- We do not go into the formal details[1] as the approach is technically evolved
- We sketch the main ideas and demonstrate the implementation in DoubleML with an example

1. See the summary in the DoubleML User Guide and the paper by Chernozhukov et al. (2022) for details.

Sensitivity Analysis for Causal ML

- The sensitivity parameters in Cinelli and Hazlett (2020) are formulated in terms of partial R^2 measures which apply to *linear relationships*
- However, we might want to model **non-linear relationships**
- Example: *Partially* linear regression model

$$
\begin{aligned}
Y &= D\theta_0 + g_0(X) + \zeta, & \mathbb{E}(\zeta|D, X) &= 0, \\
D &= m_0(X) + V, & \mathbb{E}(V|X) &= 0,
\end{aligned}
$$

with nonlinear functions g_0 and m_0.

- Moreover, we would like to apply sensitivity analysis for
  - Other **causal models**, such as the interactive regression model
  - **ML-based** estimation

Sensitivity Analysis for Causal ML

Brief summary: Chernozhukov et al. (2022)

- Generalization of the ideas in Cinelli and Hazlett (2020) to a broad class of causal models, including
  - Partially linear regression
  - Interactive regression model
  - Difference-in-Differences
- The approach is based on the so-called *Riesz-Fréchet representation*, which is related to the orthogonal score of a causal model (*debiasing*)
- Sensitivity parameters are defined in terms of **nonparametric** partial R^2

Sensitivity Analysis for Causal ML

Brief summary: Chernozhukov et al. (2022)

Advantages

- Various causal models (including non-separable models, like IRM)
- ML-based estimation
- Non-linear confounding relationships

Limitations

- Technical complexity
- Generalization comes at costs of interpretability (R^2?)

Sensitivity Analysis in DoubleML

Sensitivity Analysis in DoubleML

- Let's complete the 7th step of the DoubleML workflow example[1]
- We obtained the following results for the ATE (IRM)

```
dml_irm_rf.summary.round(3)
```

	coef	std err	t	P>|t|	2.5 %	97.5 %
e401	8121.565	1106.553	7.34	0.0	5952.76	10290.369

- Now, we wonder how robust these effects are with respect to unobserved confounding

1. You can find an online version here

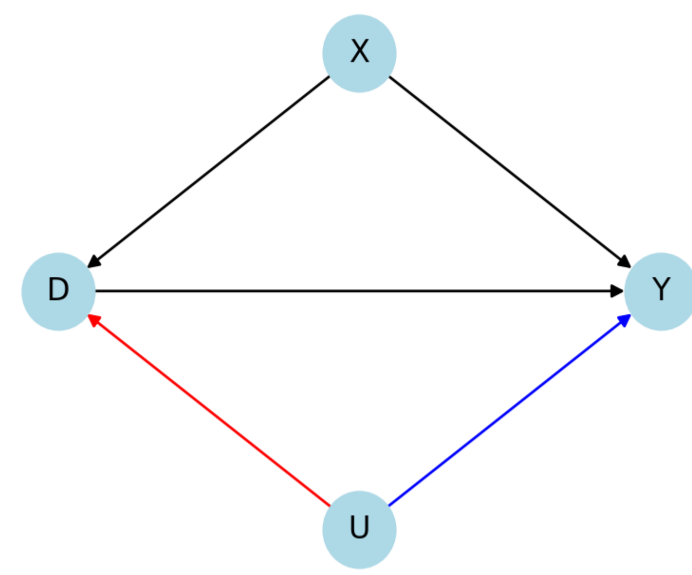
Sensitivity Analysis in DoubleML

- At given values for the sensitivity parameters `cf_d` and `cf_y`, we can compute bounds for
  - The parameter θ_0 and
  - $(1-\alpha)$ confidence intervals
- The interpretation of the sensitivity parameters depends on the causal model

PLR

- $U \rightarrow D$: Partial nonparametric R^2 of U with D, given X
- $U \rightarrow Y$: Partial nonparametric R^2 of U with Y, given D and X

▶ Code



DAG: Causal Effect of D on Y, with observed confounders X and unobserved confounders U.

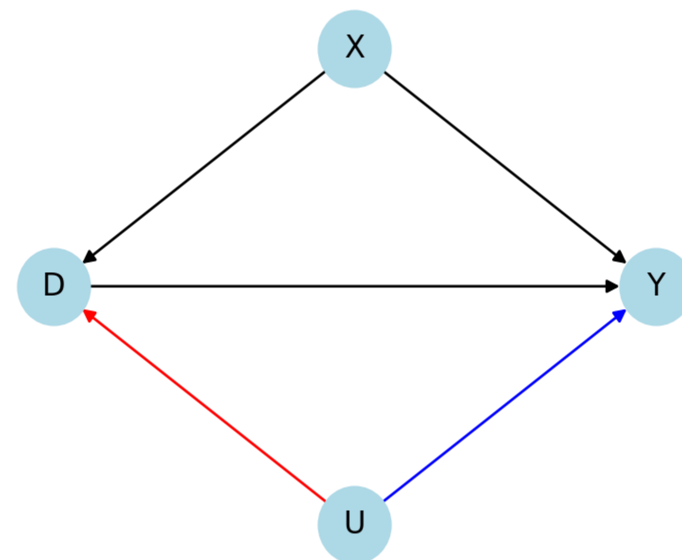
Sensitivity Analysis in DoubleML

- At given values for the sensitivity parameters `cf_d` and `cf_y`, we can compute bounds for
  - The parameter θ_0 and
  - $(1-\alpha)$ confidence intervals
- The interpretation of the sensitivity parameters depends on the causal model

IRM

- $U \rightarrow D$: Average gain in quality to predict D by using U in addition to X (relative)
- $U \rightarrow Y$: Partial nonparametric R^2 of U with Y, given D and X

▶ Code



DAG: Causal Effect of D on Y, with observed confounders X and unobserved confounders U.

Sensitivity Analysis in DoubleML

- Sensitivity analysis as of Chernozhukov et al. (2022) is implemententted in the method `sensitivity_analysis()`

```
dml_irm_rf.sensitivity_analysis()
print(dml_irm_rf.sensitivity_summary)
```

```
================== Sensitivity Analysis ==================

------------------ Scenario          ------------------
Significance Level: level=0.95
Sensitivity parameters: cf_y=0.03; cf_d=0.03, rho=1.0

------------------ Bounds with CI    ------------------
        CI lower  theta lower        theta   theta upper
CI upper
e401  2145.47168  4033.019801  8121.564764  12210.109726
14101.927365

------------------ Robustness Values ------------------
      H_0    RV (%)   RVa (%)
e401  0.0  5.870442  4.478475
```

- Robustness value RV: Minimum strength of the confounding relationship that would lead to an adjustment of the parameter bounds such that they include 0[1]

  - A confounding relationship with `cf_d`$=$`cf_y`$=5.87\%$ would suffice to set the lower bound for the ATE to 0

1. It is also possible to choose another hypothesized value $\theta \neq 0$

Sensitivity Analysis in DoubleML

- Sensitivity analysis as of Chernozhukov et al. (2022) is implementented in the method `sensitivity_analysis()`

```
dml_irm_rf.sensitivity_analysis()
print(dml_irm_rf.sensitivity_summary)
```

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================== Sensitivity Analysis ==================

------------------ Scenario          ------------------
Significance Level: level=0.95
Sensitivity parameters: cf_y=0.03; cf_d=0.03, rho=1.0

------------------ Bounds with CI    ------------------
        CI lower  theta lower        theta   theta upper
CI upper
e401   2145.47168  4033.019801  8121.564764  12210.109726
14101.927365

------------------ Robustness Values ------------------
      H_0     RV (%)    RVa (%)
e401  0.0   5.870442   4.478475
```

- Robustness value RVa: Minimum strength of the confounding relationship that would lead to an adjustment of the (1-α) confidence interval bounds such that they include the observed parameter estimate

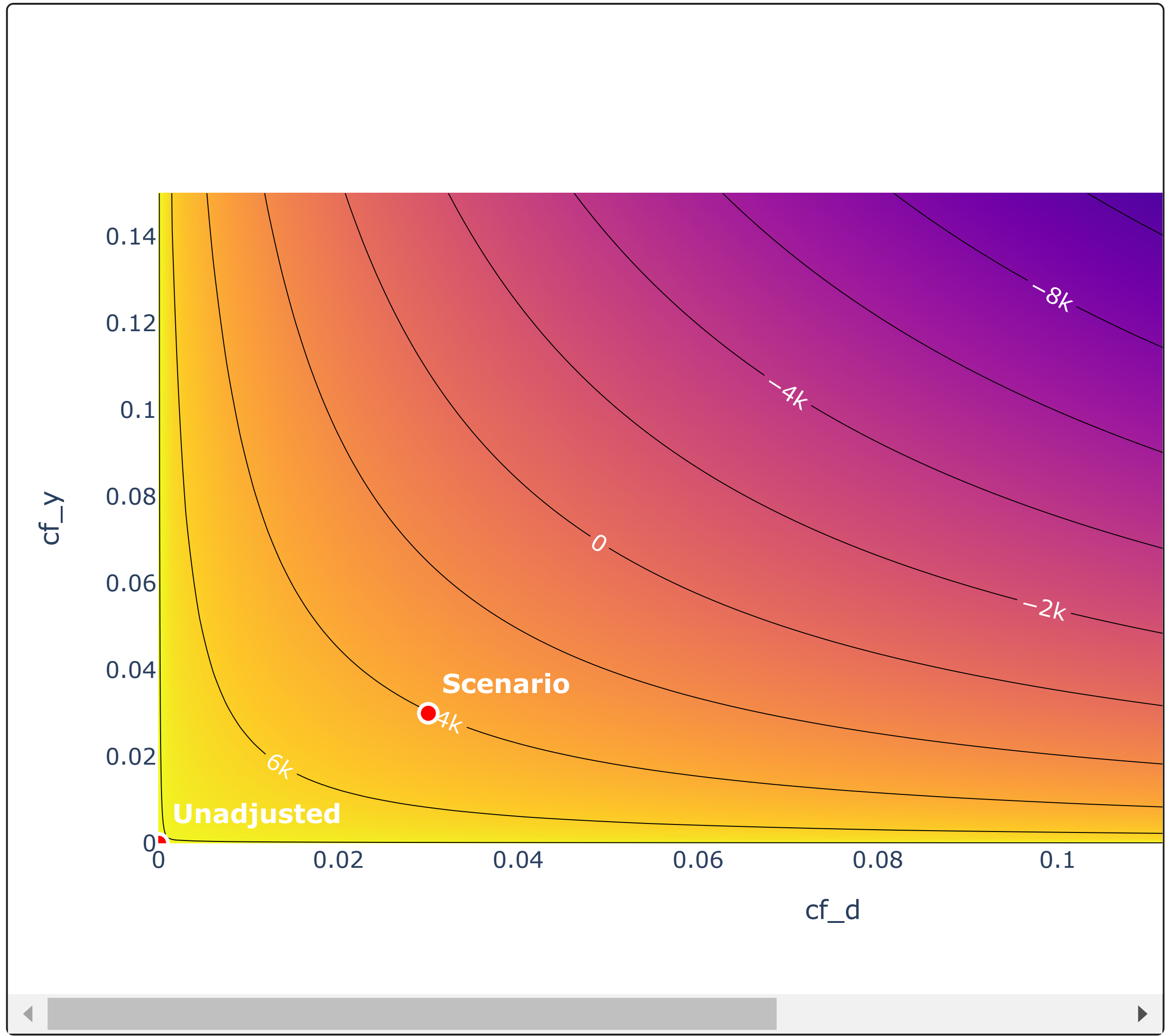
  - A confounding relationship with `cf_d`$=$`cf_y`$=4.48\%$ would suffice to let the lower bound for the 95% confidence interval include 0 (=render effect non-significant)

Sensitivity Analysis in DoubleML

Visualization

- Contour plots make it possible to visualize many different sensitivity scenarios at once
- Each contour line indicates the combinations of `cf_d` and `cf_y` that lead to the same level of bias

```
dml_irm_rf.sensitivity_plot()
```



Sensitivity Analysis in DoubleML

- So far, we considered how we can perform sensitivity analysis for a given set of parameter values
- However, we might wonder how we can **choose** plausible values for the sensitivity parameters
- There are basically two ways to do this:
    1. **Domain expertise**: Use domain expertise to postulate certain values for the confounding scenarios
    2. **Benchmarking**: Mimic omitting an important observed (benchmark) confounder and re-compute the values of the sensitivity parameters

Sensitivity Analysis in DoubleML

Benchmarking

- We consider the variable `inc` which measures individuals' income
- For more details, see Chapter 6 in Chernozhukov et al. (2022) as well as the notebook in the DoubleML Example Gallery

```
benchmark_inc = dml_irm_rf.sensitivity_benchmark(benchmarking_set=["inc"])
print(benchmark_inc)
```

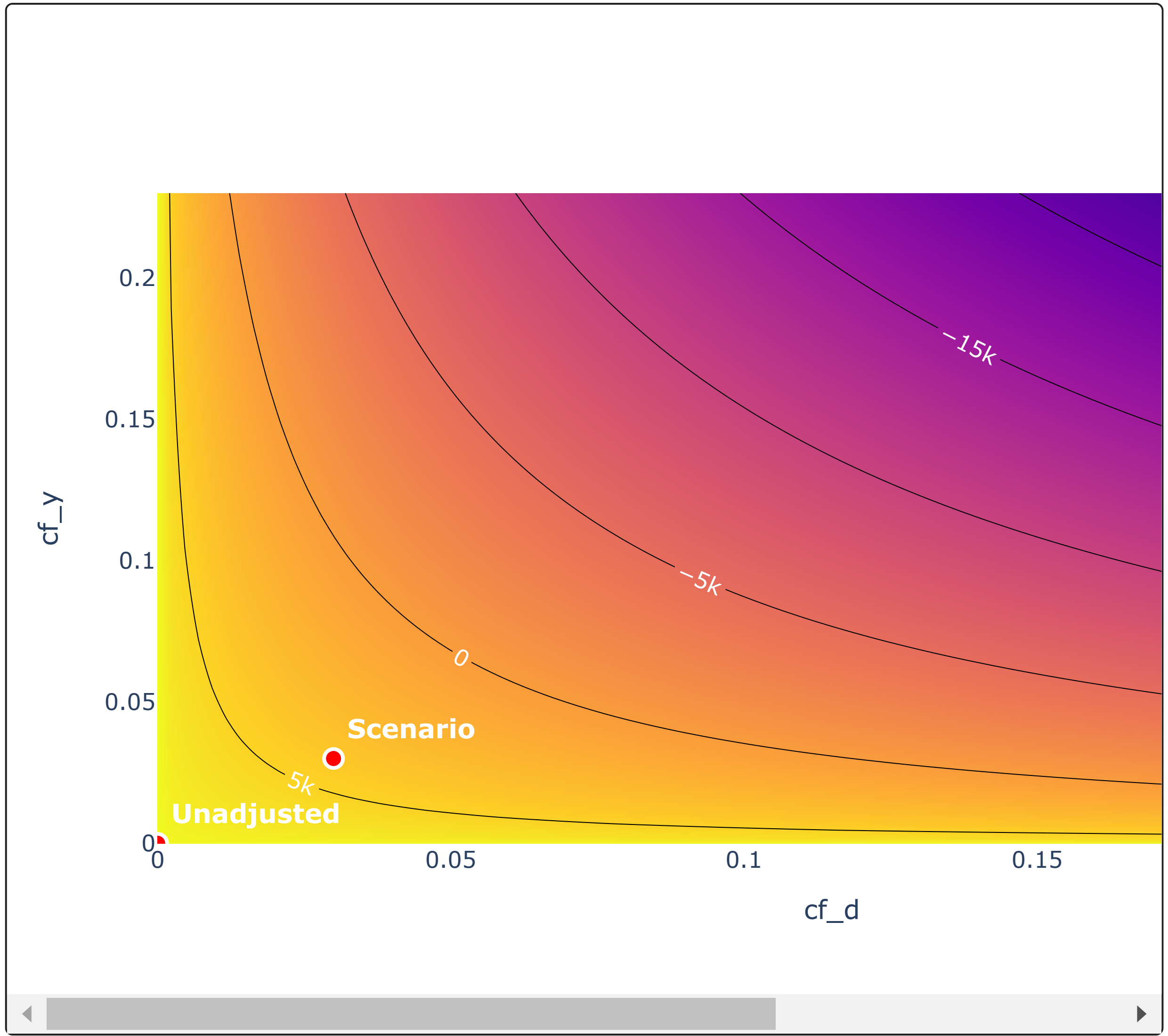
```
          cf_y      cf_d       rho  delta_theta
e401  0.145986  0.175333  0.211203  4183.495564
```

Sensitivity Analysis in DoubleML

Benchmarking

- Let's add this scenario to the contour plot

```
benchmark_dict = {"cf_y" : [benchmar
                  "cf_d" : [benchmar
                  "name" : ["inc"]}
dml_irm_rf.sensitivity_plot(benchmar
```



Sensitivity Analysis in DoubleML

Degree of adversity

- When specifying the main confounding scenario in the `sensitivity_analysis()` call, there is a parameter called ρ (*degree of adversity*)
- Informally speaking, $\rho \in [-1, 1]$ measures the **correlation** between the deviations that are created by the confounder(s) in terms of the relationships
  - $U \rightarrow D$ and
  - $U \rightarrow Y$
- Intuitively, if the variations in D and Y, which can be explained by the omitted variable U, are uncorrelated, the resulting bias would be 0

Sensitivity Analysis in DoubleML

Degree of adversity

- ρ operates as a scaling factor in the omitted variable bias formula
- Results are most conservative results with $\rho = 1$ (default)
- We can calibrate ρ during the empirical benchmarking procedure
- Without further modification, the scenarios added to the contour plot are conservative (i.e., based on $\rho = 1$)

Sensitivity Analysis in DoubleML

Benchmarking

- We can use the benchmarking scenario as the major confounding scenario and calibrate the contour plot according to ρ

```
dml_irm_rf.sensitivity_analysis(cf_y
                                cf_d
                                rho
```

```
print(dml_irm_rf.sensitivity_summary
```

```
================== Sensitivity Analysis ==================

------------------ Scenario          ------------------
Significance Level: level=0.95
Sensitivity parameters: cf_y=0.14598551560922302;
cf_d=0.17533317267482074, rho=0.21120271145715136

------------------ Bounds with CI    ------------------
          CI lower  theta lower        theta   theta upper
CI upper
e401   1206.761225   3127.196673   8121.564764   13115.932855
15041.493382

------------------ Robustness Values ------------------
      H_0      RV (%)     RVa (%)
e401   0.0   24.837451   19.469718
```

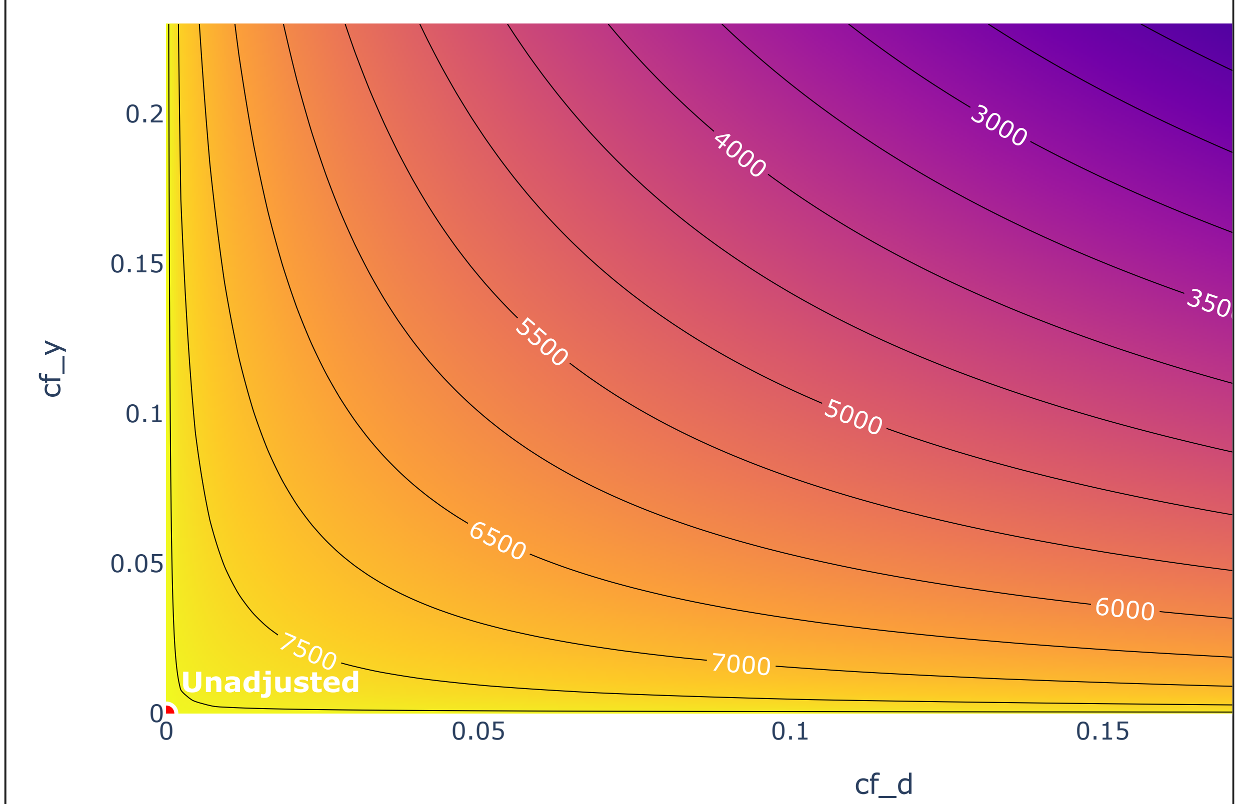
Sensitivity Analysis in DoubleML

Benchmarking

- We can use the benchmarking scenario as the major confounding scenario and calibrate the contour plot according to ρ

```
dml_irm_rf.sensitivity_analysis(cf_y
                                cf_d
                                rho
```

```
dml_irm_rf.sensitivity_plot(grid_bou
```



Sensitivity Analysis in DoubleML

Conclusion: Workflow example

- If we believe that we likely miss a confounder with similarly strong relationships with D and Y as the benchmark variable `inc`, the results do not seem to be robust
- However, if we believe that excluding such a confounder is unlikely, we can be more confident in the results (e.g., compare to other benchmarking variables)

General comment 💡

- Conclusions from sensitivity analyses will generally not be unambiguous - they depend on the context of the study and need to be interpreted based on domain expertise

Appendix

Appendix: Sensitivity Analysis for Causal ML

- Nonparametric partial R^2

$$\frac{\mathrm{Var}(\mathbb{E}[Y|D,X,A]) - \mathrm{Var}(\mathbb{E}[Y|D,X])}{\mathrm{Var}(Y) - \mathrm{Var}(\mathbb{E}[Y|D,X])}$$

Appendix: Sensitivity Analysis for IRM

Riesz-Fréchet representation for IRM (ATE)

$$\begin{aligned}\theta_0 &= \mathbb{E}(m(W,g)) \\ &= \mathbb{E}(g(1,X) - g(0,X)),\end{aligned}$$

with $g(d,X) = \mathbb{E}[Y|D=d,X]$. The Riesz-representation theorem says that we can re-write the θ_0 as

$$\theta_0 = \mathbb{E}[g_0(W)\underbrace{\alpha_0(W)}_{RR}].$$

In the IRM we have

$$\alpha_0(W) = \frac{D}{m(X)} - \frac{1-D}{1-m(X)}.$$

Appendix: Sensitivity Analysis for IRM

Riesz-Frechet representation for IRM (ATE)

The Riesz-Representer (RR) points down a debiased / orthogonal score function, see Chernozhukov, Newey, and Singh (2022).[1]

$$\psi(W,\theta_0,g,\alpha) = m(W,g) - \theta_0 + \alpha(W)\{Y - g(X)\}$$

For the ATE in the IRM (Example 3 in Chernozhukov, Newey, and Singh (2022)), we have

$$\begin{aligned}\psi(\cdot) :=& g(1,X) - g(0,X) - \theta \\ &+ \frac{D(Y-g(1,X))}{m(X)} - \frac{(1-D)(Y-g(0,X))}{1-m(x)}\end{aligned}$$

which is the doubly-robust score.

1. We drop the expectations here and directly consider the score function $\psi(\cdot)$.

Appendix: Sensitivity Analysis for IRM

- Sensitivity parameter `cf_d`$:= \frac{C_D^2}{1+C_D^2}$ with

$$C_D^2 = \frac{\mathbb{E}\Big[\big(P(D=1|X,A)(1-P(D=1|X,A))\big)^{-1}\Big]}{\mathbb{E}\Big[\big(P(D=1|X)(1-P(D=1|X))\big)^{-1}\Big]} - 1$$

References

References

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