Novel Generic Bounds on the Sum Rate of MIMO ZF Receivers

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***Abstract*—This paper introduces some novel upper and lower bounds on the achievable sum rate of multiple-input multiple-output (MIMO) systems with zero-forcing (ZF) receivers. The presented bounds are not only tractable but also generic since they apply for different fading models of interest, such as uncorrelated/correlated Rayleigh fading and Ricean fading. We further formulate a new relationship between the sum rate and the first negative moment of the unordered eigenvalue of the instantaneous correlation matrix. The derived expressions are explicitly compared with some existing results on MIMO systems operating with optimal and minimum mean-squared error (MMSE) receivers. Based on our analytical results, we gain valuable insights into the implications of the model parameters, such as the number of antennas, spatial correlation and Ricean-K factor, on the sum rate of MIMO ZF receivers.**

***Index Terms*—Correlated fading, multiple-input multiple-output (MIMO) systems, sum rate, zero-forcing receivers.**

I. Introduction

The dramatic capacity improvement that multiple-input multiple-output (MIMO) systems can offer, has brought them to the forefront of wireless communications over the past years. This capacity growth is achievable with no extra bandwidth and power requirements but by exploiting the surrounding spatial environment [1], [2]. For this reason, a vast amount of literature has been devoted to the statistical and capacity characterization of MIMO technology (see, e.g., [3]–[11], among others). A common characteristic of the above mentioned papers is that they implicitly assume the presence of optimal nonlinear receivers, which minimize the error probability when all data vectors are equally likely by performing an exhaustive search. The main disadvantage of such schemes, however, is their high complexity and implementation cost, especially for large number of antennas, which makes their employment prohibitive in practical applications.

In light of this fact, several suboptimal techniques have been proposed with the simplest one being the linear zero-forcing (ZF), which is relatively simpler compared to successive interference cancellation (SIC) or minimum mean-square error (MMSE); yet, when independent decoding is used, it suffers from an inherent noise enhancement [12], [13]. In the following, our main focus will be on MIMO ZF receivers whose performance has not been thoroughly addressed in the literature due to the difficulty in statistically characterizing the inverse of the instantaneous MIMO correlation matrix.

In the context of MIMO ZF receivers, we first note the work of [13], which capitalized on the pioneering work of [14] and derived a closed-form expression for the postprocessing signal-to-noise ratio (SNR) of the ZF receiver operating in semi-correlated Rayleigh fading, with correlation only on the side with the minimum number of antennas. The case of correlation on the side with the maximum number of antennas was explored in [15], where the authors upper and lower bounded the SNR using a linear transformation of Hermitian quadratic variables. Later, [16] extended the analysis to the doubly correlated Rayleigh fading channels. However, analytical results were derived only for dual 2×2 MIMO configurations. The case of uncorrelated and semi-correlated Ricean fading, were addressed in [17] and [18], respectively. Both studies, however, rely on a classical result from random matrix theory that approximates a noncentral Wishart distribution with a central Wishart distribution of modified correlation structure [19]. While this approximation drastically simplifies the mathematical formulations, its accuracy is low even at moderate Ricean K-factors.

In [20], an exact, low and high-SNR analysis for multiuser MIMO ZF receivers was performed, although the results were limited to the tractable case of independent and identically distributed (iid) Rayleigh fading channels. Moreover, [21] elaborated on the sum rates of MIMO ZF and MMSE receivers with the aid of the asymptotic diversity-multiplexing tradeoff. It is also worth mentioning the seminal work of [22], in which the authors formulated a generic analytical framework for statistically characterizing the sum rate of MIMO MMSE receivers. On the other hand, to the best of the authors' knowledge, a similar analysis for MIMO ZF receivers seems to be missing; in light of this fact, we have tried to bridge this gap by analytically investigating the sum rate of ZF receivers.

In particular, the contributions of the paper can now be summarized as follows.

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- Motivated by some recent advances in the area of random matrix theory, we first present three novel bounding techniques for the achievable sum rate of MIMO ZF receivers. The derived bounds apply for a finite number of antennas and remain tight across the entire SNR range. Contrary to the majority of reported studies reported in the corresponding literature [13]–[18], our analysis is generic since it applies for several fading models of interest. In particular, we hereafter consider both uncorrelated and correlated Rayleigh fading along with uncorrelated Ricean fading. We note that the cases of doubly correlated Rayleigh and uncorrelated Ricean fading induce some significant difficulties in the statistical description of the postprocessing SNR, and therefore have been rarely examined so far. Despite these inherent difficulties, all the analytical expressions presented herein can be rather easily evaluated and therefore allow for fast and efficient computation.
- In addition, we devise a new relationship between the first upper bound and the first negative moment of the unordered eigenvalue of the instantaneous correlation matrix. Since the negative moment does not always exist, we establish a straightforward condition for its existence, that is only dependent on the number of antennas. It is explicitly shown that apart from the case of semi-correlated Rayleigh fading with correlation on the side with the maximum number of antennas, the moment does exist for all the considered fading models if the fading matrix is rectangular. On the contrary, the second upper bound and the lower bound apply even for square matrices with equal number of transmit and receive antennas. Under these circumstances, some new closed-form expressions are derived that complement and extend previous results.
- The proposed bounds reveal some significant implications of the model parameters on the achievable sum rate. For instance, the effects of the number of antennas, SNR, spatial correlation and Ricean K-factor are assessed in detail. In the high-SNR regime, the effects of transmit/receive correlation and deterministic LoS components are effectively decoupled. We also provide the link between the presented results for ZF receivers and those previously reported for optimal/MMSE receivers.

The rest of the paper is organized as follows. In Section II, the MIMO ZF signal model used throughout is introduced. The novel, generic bounds on the achievable sum rate are presented in Section III which are then particularized in Section IV to different fading models of interest. Finally, Section V concludes the paper.

Notation: We use upper and lower case boldface to denote matrices and vectors, respectively. The $n \times n$ identity matrix reads as $\mathbf{I}_n$ while a $n \times m$ full of zeros matrix as $\mathbf{0}_{n\times m}$. The (i,j)th minor of a matrix is denoted by $\mathbf{A}_{ij}$, while $\mathbf{A}_i$ is $\mathbf{A}$ with the ith column removed. The expectation is given by $\mathcal{E}[\cdot]$ while the matrix determinant and trace by $\det(\cdot)$ and $\mathrm{tr}(\cdot)$. The symbols $(\cdot)^\dagger$ and $(\cdot)^H$ represent the pseudo-inverse and Hermitian transpose of a matrix, respectively, while $\otimes$ denotes the Kronecker product. The symbol $\sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma})$ denotes a complex Gaussian matrix with mean $\mathbf{M}$ and covariance $\mathbf{\Sigma}$. The notation $\Gamma(\cdot)$ stands for the well-known Gamma function [23, eq. (8.310.1)], $\mathtt{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$ is the exponential integral function [23, eq. (8.360.1)], while $\psi(x)$ is the Euler's digamma function [23, (8.360.1)]. Finally, ${}_pF_q(\cdot)$ is the generalized hypergeometric function with p, q nonnegative integers [23, eq. (9.14.1)].

II. MIMO Signal Model and ZF Reception

Let us consider a typical point-to-point MIMO system equipped with N_r receive antennas and N_t transmit antennas with $N_r \geq N_t$. The wireless channel can be effectively characterized by the fading matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$, whose entries represent the complex responses between each antenna pair. In the following, we assume that the receiver has perfect channel state information (CSI) while the transmitter knows neither the statistics nor the instantaneous CSI. Then, in a typical spatial multiplexing system, the transmitter uniformly allocates the available power, P, to all data streams. The discrete-time input–output relationship is given by

$$\mathbf{y} = \sqrt{\frac{P}{N_t}} \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector, $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ is the vector containing the transmitted symbols which are drawn from a unit-power constellation; the complex noise term is Gaussian distributed with zero-mean and covariance $\mathcal{E}\left[\mathbf{n}\mathbf{n}^H\right] = N_0 \mathbf{I}_{N_r}$, where N_0 is the noise power.

We now apply the concept of ZF reception on (1) to obtain the ZF filter matrix according to $\mathbf{G} = \left(\frac{P}{N_t}\right)^{-\frac{1}{2}} \mathbf{H}^\dagger$ [12]. Please note that the noise term is now colored and as such the components of $\hat{\mathbf{s}}$ are in general not independent. Yet, introducing the simple detection scheme of [13], we consider that each component of $\hat{\mathbf{s}}$ is independently decoded. Thus, the received signal after detection, which is used to recover the spatially multiplexed data streams, becomes

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{s} + (P/N_t)^{-1/2} \mathbf{H}^\dagger \mathbf{n} \tag{2}$$

with the instantaneous received SNR at the mth ZF output being equal to [12]–[14]

$$\gamma_m \triangleq \frac{\rho}{N_t \left[\left(\mathbf{H}^H \mathbf{H}\right)^{-1}\right]_{mm}}, \ m = 1, \ldots, N_t \tag{3}$$

where $\rho = \frac{P}{N_0}$ is the average SNR and $[\cdot]_{mm}$ returns the mth diagonal element of a matrix. The achievable sum rate, assuming independent decoding at the receiver, is essentially the sum of throughputs contributed from all subchannels, or

$$R \triangleq \sum_{m=1}^{N_t} \mathcal{E}\left[\log_2(1 + \gamma_m)\right] \tag{4}$$

where the expectation is taken over all channel realizations of $\mathbf{H}$ and the channel is assumed to be ergodic. Clearly, a sum rate analysis of MIMO ZF receivers requires precise knowledge of the statistics of γ_m which however are available only for iid Rayleigh or semi-correlated Rayleigh fading channels [13]. On this basis, in the following section we introduce three novel and generic bounds on the sum rate of MIMO ZF receivers, which apply for different fading models.

III. Generic Bounds on the Sum Rate of MIMO ZF Receivers

This section introduces some new upper and lower bounds on the sum rate of MIMO ZF receivers that apply for a finite number of antennas and for arbitrary SNRs. Note that in the following, the instantaneous MIMO correlation matrix $\mathbf{W} \triangleq \mathbf{H}^H\mathbf{H}$, will be extensively used. We begin with the following theorem which returns a novel upper bound on the sum rate.

Theorem 1: The achievable sum rate of MIMO ZF receivers is upper bounded by $R \leq R_{u,1}$ with

$$R_{u,1} = N_t \log_2\left(\mathcal{E}\left[\lambda^{-1}\right] + \frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}^H\mathbf{H}\right)\right)\right] - \frac{1}{\ln 2}\sum_{m=1}^{N_t}\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}_m^H\mathbf{H}_m\right)\right)\right] \quad (5)$$

where λ is an unordered eigenvalue of $\mathbf{H}^H\mathbf{H}$.

Proof: Starting from (4), we can obtain the alternative expressions

$$\begin{aligned} R &= \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left(1+\frac{\rho}{N_t}\frac{1}{[\mathbf{W}^{-1}]_{mm}}\right)\right] \\ &= \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left([\mathbf{W}^{-1}]_{mm}+\frac{\rho}{N_t}\right) - \log_2\left([\mathbf{W}^{-1}]_{mm}\right)\right] \\ &= \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left([\mathbf{W}^{-1}]_{mm}+\frac{\rho}{N_t}\right)\right] - \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left([\mathbf{W}^{-1}]_{mm}\right)\right]. \end{aligned} \quad (6)$$

As a next step, the first term in (6) can be successively upper bounded by

$$\begin{aligned} 1_{\text{term}} &\leq N_t\mathcal{E}\left[\log_2\left(\frac{1}{N_t}\sum_{m=1}^{N_t}\left([\mathbf{W}^{-1}]_{mm}+\frac{\rho}{N_t}\right)\right)\right] && (7) \\ &= N_t\mathcal{E}\left[\log_2\left(\frac{1}{N_t}\mathrm{tr}\left(\mathbf{W}^{-1}\right)+\frac{\rho}{N_t}\right)\right] && (8) \\ &\leq N_t\log_2\left(\frac{1}{N_t}\mathcal{E}\left[\mathrm{tr}\left(\mathbf{W}^{-1}\right)\right]+\frac{\rho}{N_t}\right) && (9) \\ &= N_t\log_2\left(\mathcal{E}\left[\lambda^{-1}\right]+\frac{\rho}{N_t}\right) && (10) \end{aligned}$$

where from (6) to (7) we have used the inequality of arithmetic and geometric means, while (9) is a result of Jensen's inequality since $\log_2(\cdot)$ is a concave function. Combining (6) with (9) and with the aid of the following key matrix property, which was also used in some previous works on ZF and MMSE receivers (e.g., [13] and [22]),

$$\left[\mathbf{W}^{-1}\right]_{mm} = \frac{\det\left(\mathbf{W}_{mm}\right)}{\det\left(\mathbf{W}\right)} = \frac{\det\left(\mathbf{H}_m^H\mathbf{H}_m\right)}{\det\left(\mathbf{H}^H\mathbf{H}\right)} \quad (11)$$

we conclude the proof. ∎

Clearly, the evaluation of (5) requires the existence of the first negative moment of the unordered eigenvalue. While the positive eigenvalue moments have been thoroughly investigated in the MIMO context (see, e.g., [24]–[26] and references therein), the negative moments are scarcely addressed. In light of this fact, the following lemma will be particularly useful.

Lemma 1 [27]: For a continuous random variable X with probability density function $f_X(x)$, its first negative moment does not exist if it has a positive mass at $X = 0$, i.e., $f_X(0) > 0$.

Theorem 2: The achievable sum rate of MIMO ZF receivers is upper bounded by $R \leq R_{u,2}$ with

$$R_{u,2} = \sum_{m=1}^{N_t}\log_2\left(\mathcal{E}\left[\det(\mathbf{H}_m^H\mathbf{H}_m)\right] + \frac{\rho}{N_t}\mathcal{E}\left[\det(\mathbf{H}^H\mathbf{H})\right]\right) - \frac{1}{\ln 2}\sum_{m=1}^{N_t}\mathcal{E}\left[\ln\left(\det(\mathbf{H}_m^H\mathbf{H}_m)\right)\right]. \quad (12)$$

Proof: The derivation of the second upper bound is based on the subsequent methodology

$$R = \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left(1+\frac{\rho}{N_t}\frac{1}{[\mathbf{W}^{-1}]_{mm}}\right)\right] \quad (13)$$

$$= \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left(\det(\mathbf{W}_{mm}) + \frac{\rho}{N_t}\det(\mathbf{W})\right) - \log_2\left(\det(\mathbf{W}_{mm})\right)\right] \quad (14)$$

where from (13) to (14) we have used (11). Once more, applying Jensen's inequality on (14), we can easily obtain (12). ∎

Theorem 3: The achievable sum rate of MIMO ZF receivers is lower bounded by $R \geq R_L$ with

$$R_L = \sum_{m=1}^{N_t}\log_2\left(1+\frac{\rho}{N_t}\exp\left(\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}^H\mathbf{H}\right)\right) - \ln\left(\det\left(\mathbf{H}_m^H\mathbf{H}_m\right)\right)\right]\right)\right). \quad (15)$$

Proof: The proof relies on the general bounding technique, originally proposed in [6, Theorem 1] and later adopted by [9, eq. (41)], [11, Theorem 6], for lower bounding the ergodic MIMO capacity with optimal receivers. In particular, we can re-express (4), according to

$$R = \sum_{m=1}^{N_t}\mathcal{E}\left[\log_2\left(1+\frac{\rho}{N_t}\exp\left(\ln\left(\frac{1}{[\mathbf{W}^{-1}]_{mm}}\right)\right)\right)\right]. \quad (16)$$

Exploiting the fact that $\log_2(1+\alpha\exp(x))$ is convex in x for $\alpha > 0$, and thereafter applying Jensen's inequality, we can obtain (15) via (11). ∎

IV. Sum Rate Bounds in Fading Channels

In this section, we particularize the generic bounds presented in Section III to the most common fading models in the MIMO context.

A. Uncorrelated Rayleigh Fading

The uncorrelated Rayleigh model represents a rich scattering environment with no LoS path between the transmitter and receiver and large antenna spacings. In this case, the fading matrix can be modeled as

$$\mathbf{H} = \mathbf{H}_w \tag{17}$$

where the entries of $\mathbf{H}_w$ entries are iid complex zero-mean, unit-variance random variables.

Proposition 1: For iid Rayleigh fading, the sum rate of MIMO ZF receivers is upper bounded by

$$R_{u,1} = N_t \log_2\left(\frac{1}{N_r - N_t} + \frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) \tag{18}$$

$$R_{u,2} = N_t \log_2\left(\frac{N_r!}{(N_r - N_t + 1)!} + \frac{\rho}{N_t}\frac{N_r!}{(N_r - N_t)!}\right) - \frac{N_t}{\ln 2}\sum_{k=1}^{N_t-1}\psi\left(N_r + 1 - k\right). \tag{19}$$

Proof: The proof starts by invoking the following results for an $N_r \times N_t$ (with $N_r \geq N_t$) central (zero-mean) Wishart matrix

$$\mathcal{E}\left[\det\left(\mathbf{H}_w^H\mathbf{H}_w\right)\right] = \frac{N_r!}{(N_r - N_t)!} \tag{20}$$

$$\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}_w^H\mathbf{H}_w\right)\right)\right] = \sum_{k=0}^{N_t-1}\psi\left(N_r - k\right) \tag{21}$$

which are obtained from [4, eq. (A.7.1)] and [4, eq. (A.8.1)], respectively. The unordered eigenvalue of $\mathbf{H}_w^H\mathbf{H}_w$ has the following distribution [28] (used also in [1]):

$$f_\lambda(x) = \frac{1}{N_t}\sum_{k=0}^{N_t-1}\frac{k!(L_k^{N_r-N_t}(x))^2}{(N_r - N_t + k)!}e^{-x}x^{N_r-N_t} \tag{22}$$

where $L_k^r(\cdot)$ denotes the generalized Laguerre polynomial of order k [23, eq. (8.970.1)]. We note that the above distribution has a zero mass at $x = 0$ only if $N_r - N_t > 0$, since $L_k^0(0) = 1$; once this condition is fulfilled, the first negative moment of λ is equal to [29, Lemma 6]

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \frac{1}{N_r - N_t}, \text{ for } N_r \geq N_t + 1. \tag{23}$$

Substituting (20), (21) and (23) into (5), (12), we can obtain (18), (19) after noting that

$$\mathbf{H}_m \sim \mathcal{CN}(\mathbf{0}_{N_r \times N_t - 1}, \mathbf{I}_{N_r} \otimes \mathbf{I}_{N_t - 1}) \tag{24}$$

and some simple simplifications. ■

Proposition 2: For iid Rayleigh fading, the sum rate of MIMO ZF receivers is lower bounded by

$$R_L = N_t \log_2\left(1 + \frac{\rho}{N_t}\exp\left(\psi\left(N_r - N_t + 1\right)\right)\right). \tag{25}$$

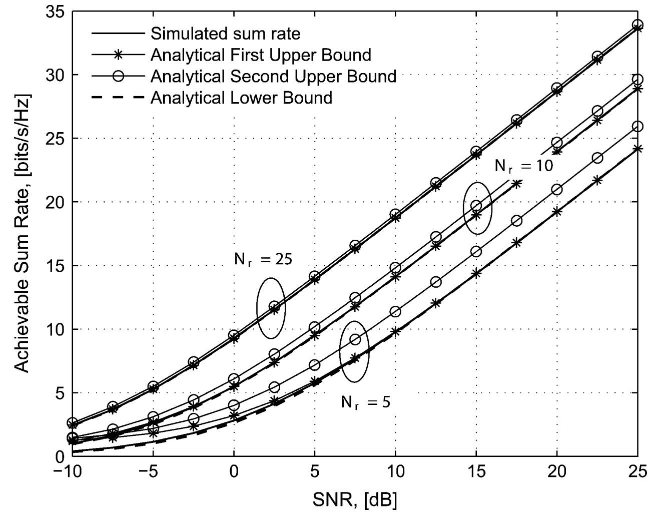

Fig. 1. Simulated sum rate and upper/lower bounds against the SNR ρ ($N_t = 3$).

Proof: Follows trivially by combining (15) with (21) and (24). ■

Corollary 1: For iid Rayleigh fading, $R_{u,1}$ and R_L become exact at high SNRs and equal to

$$R_{u,1}^\infty = R_L^\infty = N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) \tag{26}$$

$$\leq N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{1}{\ln 2}\sum_{k=0}^{N_t-1}\psi\left(N_r - k\right). \tag{27}$$

Proof: The proof follows by taking ρ large in (18) and (25) and simplifying. ■

We note that (27) corresponds to the high-SNR average mutual information of an iid Rayleigh fading MIMO system with optimal receivers under $N_r \geq N_t$ (also identical to the ergodic capacity for this specific scenario) [4, eq. (9)], [6, eq. (12)]. In addition, (26) and (27) validate the well-known feature of MIMO systems, that is the asymptotically (in terms of SNR) linear sum rate (or capacity) scaling with the minimum number of antennas [1], [4]–[6], [9]. Note also that (26) is strongly related with [22, Proposition 2] which was derived in the context of MIMO MMSE receivers. This phenomenon is anticipated, since at high-SNRs the ZF and MMSE receivers behave equivalently in terms of sum rate [12].

In Fig. 1, the simulated achievable sum rate along with the proposed analytical bounds of (18), (19), and (25) are plotted against the average SNR, ρ. We consider different MIMO configurations by keeping $N_t = 3$ and increasing only N_r.

The graph indicates that adding more receive antennas significantly stabilizes the MIMO link by improving the receive diversity and reducing the noise enhancement effect. Clearly, all bounds remain sufficiently tight across the entire SNR range. As anticipated, both $R_{u,1}$ and R_L become exact even at moderate SNRs, which implies that (26) can explicitly predict the exact sum rate for most practical SNR values with a lower computational load. Regarding $R_{u,2}$, we first note that although is, in general, less tight than $R_{u,1}$ in the high-SNR regime, it can outperform the first bound in the low-SNR regime. Further, its tightness improves with an increasing N_r. In fact, in the limit of large number of receive antennas, all bounds become exact due to the law of large numbers which states that $\lim_{N_r \to \infty} \frac{\mathbf{H}_w^H \mathbf{H}_w}{N_r} \to \mathbf{I}_{N_t}$.

In order to obtain some extra insights into the "large-antenna limit," we now present the following perceptive result.

Corollary 2: For iid Rayleigh fading, as the number of receive antennas grows to infinity and N_t is kept fixed, all bounds become exact and converge to the same asymptote

$$R_{u,1} = R_{u,2} = R_L \overset{N_r \to \infty}{\approx} N_t \log_2\left(\frac{\rho}{N_t}\right) + N_t \log_2 N_r. \tag{28}$$

Proof: A detailed proof is relegated in Appendix I. ■

It is easily seen that, asymptotically, the sum rate increases logarithmically with the number of receive antennas. Note that similar results were observed for iid Rayleigh MIMO channels with optimal receivers [4, eq. (8)], [10, eq. (13)]. This implies that as $N_r \to \infty$, the effects of fading can be completely eliminated by MIMO ZF receivers. As an aside, we also examine the case when both N_r, N_t grow infinitely large with a fixed and finite ratio.

Corollary 3: For iid Rayleigh fading, as the number of receive and transmit antennas grows to infinity with a fixed ratio $\beta = \frac{N_r}{N_t} \geq 1$, $R_{u,1}$ and R_L become exact and converge to the same asymptote

$$\frac{R_{u,1}}{N_t} = \frac{R_L}{N_t} \overset{N_r, N_t \to \infty}{\approx} \log_2\left(1 + (\beta - 1)\rho\right). \tag{29}$$

Proof: For $R_{u,1}$, we have from (18)

$$\begin{aligned} R_{u,1} &\overset{(64)}{\approx} N_t \log_2\left(\frac{1}{N_r - N_t} + \frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\ln(N_r - N_t) \\ &= N_t \log_2\left((N_r - N_t)\left(\frac{1}{N_r - N_t} + \frac{\rho}{N_t}\right)\right) \\ &= N_t \log_2\left(1 + (\beta - 1)\rho\right) \end{aligned}$$

while the lower bound R_L admits the following simplification:

$$\begin{aligned} R_L &\overset{(64)}{\approx} N_t \log_2\left(1 + \frac{\rho}{N_t}\exp\left(\ln(N_r - N_t)\right)\right) \\ &= N_t \log_2\left(1 + \frac{\rho}{N_t}(N_r - N_t)\right) \\ &= N_t \log_2\left(1 + (\beta - 1)\rho\right). \end{aligned}$$

Invoking that $\lim_{N_r, N_t \to \infty} \left[\left(\mathbf{H}_w^H \mathbf{H}_w\right)^{-1}\right]_{kk} = \frac{1}{N_t(\beta - 1)}$ [30, eq. (20)], we can explicitly verify that both bounds become asymptotically exact. ■

The above result indicates that for $\beta = 1$, the sum rate saturates since there are no degrees of freedom to cancel out an infinitely high-number of interferers. As β gets larger a monotonic increase in the sum rate is achieved. These results are consistent with those in [4] and [10].

B. Min Semi-Correlated Rayleigh Fading

This is a scenario in which spatial correlation occurs only on the side with the minimum number of antennas (i.e., transmitter). This can occur due to either limited transmit angular spread or small interelement spacings in the transmit antenna array (e.g., mobile handset). Under these circumstances, the channel matrix can be modeled as[1]

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_{\mathrm{T}}^{\frac{1}{2}} \tag{30}$$

where $\mathbf{R}_{\mathrm{T}} \in \mathbb{C}^{N_t \times N_t}$ denotes the Hermitian, positive definite transmit correlation matrix. Our analysis begins with the following new result for the first negative moment of an arbitrary eigenvalue of $\mathbf{W}$.

Theorem 4: For semi-correlated Rayleigh fading with correlation on the side with the minimum number of antennas, the first negative moment of an arbitrary eigenvalue, λ, of $\mathbf{H}^H \mathbf{H}$ is

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \frac{\sum_{i=1}^{N_t} \beta_i^{-1}}{N_t(N_r - N_t)}, \text{ for } N_r \geq N_t + 1 \tag{31}$$

where $\beta_i, i = 1, \ldots, N_t$ are the real, positive eigenvalues of $\mathbf{R}_{\mathrm{T}}$.

Proof: A detailed proof is given in Appendix II. ■

As stated in Appendix II, the first negative moment of λ exists only if $N_r \geq N_t + 1$, which implies that the same condition needs to be satisfied as for the iid Rayleigh case (see Lemma 1). Having this result in hand, we can now evaluate the upper bounds of Theorem 1 and 2 as follows.

Proposition 3: For semi-correlated Rayleigh fading with correlation on the side with the minimum number of antennas, the sum rate of MIMO ZF receivers is upper bounded by

$$\begin{aligned} R_{u,1} = {} & N_t \log_2\left(\frac{\mathrm{tr}\left(\mathbf{R}_{\mathrm{T}}^{-1}\right)}{N_t(N_r - N_t)} + \frac{\rho}{N_t}\right) \\ & + \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) - \sum_{m=1}^{N_t} \log_2(\sigma_m) \end{aligned} \tag{32}$$

$$\begin{aligned} R_{u,2} = {} & \sum_{m=1}^{N_t} \log_2\left(\frac{N_r! \det(\mathbf{R}_{\mathrm{T},mm})}{(N_r - N_t + 1)!} + \frac{\rho}{N_t}\frac{N_r! \det(\mathbf{R}_{\mathrm{T}})}{(N_r - N_t)!}\right) \\ & - \frac{N_t}{\ln 2}\sum_{k=1}^{N_t - 1} \psi\left(N_r + 1 - k\right) \\ & - \sum_{m=1}^{N_t} \log_2\left(\det(\mathbf{R}_{\mathrm{T},mm})\right) \end{aligned} \tag{33}$$

where σ_m is the mth diagonal entry of $\mathbf{R}_{\mathrm{T}}^{-1}$, while $\mathbf{R}_{\mathrm{T},mm}$ is the (m, m)th minor of $\mathbf{R}_{\mathrm{T}}$.

Proof: For both bounds, we first note that

$$\mathbf{H}_m \sim \mathcal{CN}(\mathbf{0}_{N_r \times N_t - 1}, \mathbf{I}_{N_r} \otimes \mathbf{R}_{\mathrm{T},mm}) \tag{34}$$

and we thereafter successively use the property of square matrices $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$ along with (11). Then, the upper bound in (32) stems from (5) upon application of (21) and Theorem 4; likewise, (33) is directly inferred from (12) with the aid of (20)–(21). ■

[1]We recall that this product model, normally referred to as the Kronecker model, has been widely used in the MIMO literature to assess the effects of correlation (see, e.g., [3], [5], [6], [9], [13] and references therein.)

Proposition 4: For semi-correlated Rayleigh fading with correlation on the side with the minimum number of antennas, the sum rate of MIMO ZF receivers is lower bounded by

$$R_L = \sum_{m=1}^{N_t} \log_2 \left(1 + \frac{\rho}{N_t} \exp\left(\psi\left(N_r - N_t + 1\right) - \ln(\sigma_m)\right)\right). \tag{35}$$

Proof: The result is readily obtained by applying the same methodology of (32) on (15) and simplifying. ■

Corollary 4: For semi-correlated Rayleigh fading with correlation on the side with the minimum number of antennas, $R_{u,1}$ and R_L become exact at high SNRs and equal to

$$R_{u,1}^{\infty} = R_L^{\infty} = N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) - \sum_{m=1}^{N_t} \log_2(\sigma_m) \tag{36}$$

$$\leq N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{1}{\ln 2}\sum_{k=0}^{N_t - 1}\psi\left(N_r - k\right) + \log_2\left(\det(\mathbf{R}_{\mathrm{T}})\right). \tag{37}$$

Proof: The proof follows by taking ρ large in (32), (35) and simplifying. ■

The expression (36) is intuitive as it indicates that at high SNRs the effects of Rayleigh fading and spatial correlation are decoupled, which is in line with [6], [9], [10], and [22]. We also validate the diminishing effects of spatial correlation on the sum rate since, due to Hadamard's inequality, $1 \leq \det\left(\mathbf{R}_{\mathrm{T}}^{-1}\right) \leq \prod_{m=1}^{N_t} \sigma_m$, with equality holding for $\mathbf{R}_{\mathrm{T}} = \mathbf{I}_{N_t}$. Note that (37) corresponds to the high-SNR average mutual information of a semi-correlated Rayleigh fading MIMO system with optimal receivers under $N_r \geq N_t$ [6, eq. (18)], [9, eq. (84)]. In Fig. 2, the effects of transmit spatial correlation are addressed. The transmit correlation matrix is constructed via the exponential correlation model, and as such its entries are $\{\mathbf{R}_{\mathrm{T}}\}_{i,j} = \rho_t^{|i-j|}$ with $\rho_t \in [0, 1)$ being the transmit correlation coefficient [31].

We can easily notice the diminishing effects of spatial correlation on the sum rate of MIMO ZF receivers, which validates some well-known results in the MIMO literature [3], [5]–[7], [9]. This is due to the reduced transmit diversity since the spatial signatures of the impinging wavefronts become identical with a higher ρ_t, thereby decreasing the degrees of freedom of the MIMO channel. While $R_{u,1}$ and R_L are tight across the entire for all values of ρ_t, the tightness of $R_{u,2}$ is significantly improved in the high-correlation regime.

As a final result for this fading scenario, we provide the following "large-system" expressions.

Corollary 5: For semi-correlated Rayleigh fading with correlation on the side with the minimum number of antennas, as the number of receive antennas grows to infinity and N_t is kept fixed, all bounds become exact and converge to the same asymptote

$$R_{u,1} = R_{u,2} = R_L \overset{N_r \to \infty}{\approx} N_t \log_2\left(\frac{\rho}{N_t}\right) + \sum_{m=1}^{N_t} \log_2\left(\frac{N_r}{\sigma_m}\right) \tag{38}$$

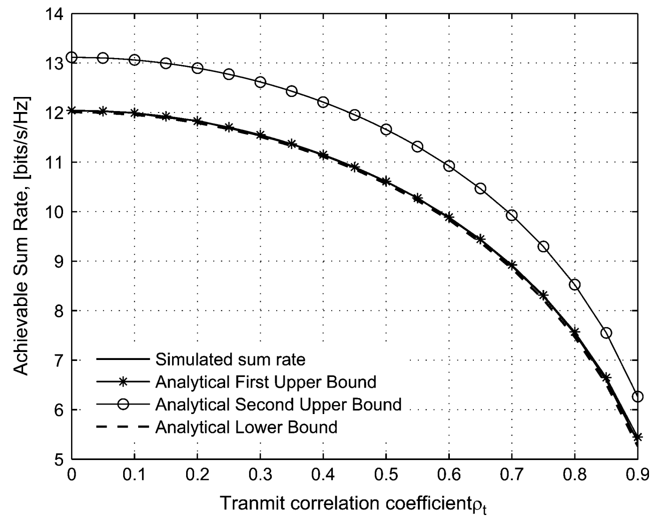

Fig. 2. Simulated sum rate and upper/lower bounds against the transmit correlation coefficient ρ_t ($N_r = 7, N_t = 3, \rho = 10$ dB).

while as the number of receive and transmit antennas grows to infinity with a fixed ratio $\beta = \frac{N_r}{N_t} \geq 1$, R_L converges to

$$R_L \overset{N_r \to \infty}{\approx} \sum_{m=1}^{N_t} \log_2\left(1 + \frac{(\beta - 1)\rho}{\sigma_m}\right). \tag{39}$$

Proof: The result in (38) follows by taking N_r large in (32), (33) and (35) and thereafter use the methodology of Appendix I to simplify. Likewise, we can deduce (39) with the aid of Corollary 3. ■

C. Doubly Correlated Rayleigh Fading

As a generalization to the min semi-correlated Rayleigh fading scenario, we now investigate the so-called doubly correlated Rayleigh fading case. Under these conditions, spatial correlation occurs on both sides of the MIMO link and as such, we get

$$\mathbf{H} = \mathbf{R}_{\mathrm{R}}^{\frac{1}{2}} \mathbf{H}_w \mathbf{R}_{\mathrm{T}}^{\frac{1}{2}} \tag{40}$$

where $\mathbf{R}_{\mathrm{R}} \in \mathbb{C}^{N_r \times N_r}$ denotes the Hermitian, positive definite receive correlation matrix.

In general, this scenario of doubly correlated Rayleigh fading is not amenable to tractable manipulations since the random term $\det\left(\mathbf{H}^H\mathbf{H}\right) = \det(\mathbf{R}_{\mathrm{T}}) \det\left(\mathbf{H}_w^H \mathbf{R}_{\mathrm{R}} \mathbf{H}_w\right)$ can not be further decomposed [32], [33]. Besides, that is the main reason for this scenario not to be thoroughly addressed in the context of MIMO ZF receivers. The only relevant study has been reported in [16], although the presented analysis was tied to 2×2 MIMO configurations.

However, using some recent advances in random matrix theory as a starting point, we now present the following result for the mean value of $\det\left(\mathbf{H}_w^H \mathbf{R}_{\mathrm{R}} \mathbf{H}_w\right)$:

Lemma 2: For semi-correlated Rayleigh fading with correlation on the side with the maximum number of antennas, the first moment of the generalized variance is equal to

$$\mathcal{E}\left[\det\left(\mathbf{H}_w^H \mathbf{R}_{\mathrm{R}} \mathbf{H}_w\right)\right] = \frac{N_t! \det(\mathbf{X})}{\prod_{i<j}^{N_r}(\gamma_j - \gamma_i)} \tag{41}$$

where $\gamma_i,\ i=1,\ldots,N_r$ are the real, positive eigenvalues of $\mathbf{R}_{\mathrm{R}}$, while $\mathbf{X}$ is a $N_r\times N_r$ matrix with entries

$$\{\mathbf{X}\}_{i,j}=\begin{cases}\gamma_i^{j-1}, & j=1,\ldots,N_r-N_t\\ \gamma_i^{j}, & j=N_r-N_t+1,\ldots,N_r.\end{cases}$$

Proof: The proof is based on a similar methodology as that in [25, Appendix I-E]. More specifically, by simple rearrangement of [25, eq. (128)], we can get

$$\mathcal{E}\left[\det(\mathbf{H}_w^H\mathbf{R}_{\mathrm{R}}\mathbf{H}_w)\right]=\frac{\det(\boldsymbol{\Delta})}{\prod_{i=1}^{N_t}\Gamma(N_t-i+1)\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)} \tag{42}$$

where $\boldsymbol{\Delta}$ is an $N_r\times N_r$ matrix defined at the bottom of the page. The second branch of $\boldsymbol{\Delta}$ can be further simplified through (53), to yield

$$\{\boldsymbol{\Delta}\}_{i,j}=\gamma_i^j\Gamma(j-N_r+N_t+1),\quad j=N_r-N_t+1,\ldots,N_r. \tag{43}$$

The proof concludes after factorizing the common terms from $\det(\boldsymbol{\Delta})$ in (42) and simplifying. ■

We point out that the result in (41) is rather tractable and, at the same time, applies for arbitrary-dimensional fading matrices $\mathbf{H}$. More importantly, it can be very easily evaluated and efficiently programmed. In contrast, the results of [5], [9] rely on the complicated theory of zonal polynomials while the final expressions involve hypergeometric functions of matrix arguments.

Proposition 5: For doubly correlated Rayleigh fading, the sum rate of MIMO ZF receivers is upper bounded by

$$R_{u,2}=\sum_{m=1}^{N_t}\log_2\left(\frac{(N_t-1)!\left(\det(\tilde{\mathbf{X}})+\frac{\rho}{\sigma_m}\det(\mathbf{X})\right)}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}\right)-\frac{N_t}{\ln 2}\left(\sum_{k=1}^{N_t-1}\psi(k)+\frac{\sum_{k=N_r-N_t+2}^{N_r}\det(\mathbf{Y}_k)}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}\right) \tag{44}$$

where the matrix $\tilde{\mathbf{X}}$ is directly related to $\mathbf{X}$ with its entries given by

$$\left\{\tilde{\mathbf{X}}\right\}_{i,j}=\begin{cases}\gamma_i^{j-1}, & j=1,\ldots,N_r-N_t+1\\ \gamma_i^{j}, & j=N_r-N_t+2,\ldots,N_r\end{cases}$$

while $\mathbf{Y}_k$ is an $N_r\times N_r$ matrix with entries

$$\{\mathbf{Y}_k\}_{i,j}=\begin{cases}\gamma_i^{j-1}, & j\neq k\\ \gamma_i^{j-1}\ln\gamma_i, & j=k.\end{cases} \tag{45}$$

Proof: A detailed proof is given in Appendix III. ■

Regarding the upper bound $R_{u,1}$, its evaluation requires statistical knowledge of the unordered eigenvalue, which, however, is an extremely challenging problem that remains unsolved.[2] In fact, there are very few analytical results on the eigenstatistics of doubly correlated Rayleigh fading MIMO systems [35]–[37]. For this reason, we now focus on the still practical case of correlation on the side with the maximum number of antennas (i.e., $\mathbf{R}_{\mathrm{T}}=\mathbf{I}_{N_t}$). Then, we have the following result.

Corollary 6: For semi-correlated Rayleigh fading with correlation on the side with the maximum number of antennas, the upper bound $R_{u,1}$ does not exist.

Proof: The proof starts by considering the marginal density of λ, which for the case of correlation on the receive side reads as [25, eq. (14)]

$$f_\lambda(x)=\frac{1}{N_t\prod_{i<j}^{N_t}(\gamma_j-\gamma_i)}\times\sum_{\ell=1}^{N_r}\sum_{k=N_r-N_t+1}^{N_r}\frac{x^{N_t-N_r+k-1}e^{\frac{-x}{\gamma_\ell}}\gamma_\ell^{N_r-N_t-1}}{\Gamma(N_t-N_r+k)}D_{\ell k}$$

which has a zero mass at $x=0$, only if $N_t-N_r+k-1>0$. Since the minimum value of $k=N_r-N_t+1$ this condition is not satisfied; thus, according to Lemma 1, the first negative moment of λ, does not exist. This concludes the proof. ■

Interestingly, the above result demonstrates a fundamental difference between the fading models considered in Sections IV-A–IV-C, which lies in the nonexistence of the first upper bound for the case of semi-correlated Rayleigh fading with correlation on the side with the maximum number of antennas.

Proposition 6: For doubly correlated Rayleigh fading, the sum rate of MIMO ZF receivers is lower bounded by

$$R_L=\sum_{m=1}^{N_t}\log_2\left(1+\frac{\rho}{N_t}\exp\left(\psi(N_t)-\ln(\sigma_m)+\frac{\det(\mathbf{Y}_{N_r-N_t+1})}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}\right)\right). \tag{46}$$

Proof: In this case, both expectations in (15) can be evaluated by combining (73) with (74) and simplifying. ■

It should be emphasized that for both bounds in (44) and (46), the effects of transmit and receive correlation are decoupled while the implications of the latter feature are introduced through the eigenvalues of the corresponding correlation matrix. Yet, the overall impact of receive correlation is not straightforwardly inferred due to the Vandermonde determinants that appear in both (44) and (46). For this reason, we consider the high-SNR regime via the following corollary.

[2]The main difficulty in this case is that the joint eigenvalue density can not be expressed via a product of determinants as for the case of uncorrelated/semi-correlated Rayleigh fading [34].

$$\{\boldsymbol{\Delta}\}_{i,j}=\begin{cases}\gamma_i^{j-1}, & j=1,\ldots,N_r-N_t\\ \gamma_i^{N_r-N_t-1}\int_0^\infty\lambda^{j-N_r+N_t}e^{\frac{-\lambda}{\gamma_i}}d\lambda, & j=N_r-N_t+1,\ldots,N_r.\end{cases}$$

Corollary 7: For doubly correlated Rayleigh fading, R_L becomes exact at high SNRs and equal to

$$\begin{aligned} R_L^{\infty} &= N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi(N_t) \\ &\quad - \sum_{m=1}^{N_t} \log_2(\sigma_m) + \frac{N_t}{\ln 2}\frac{\det(\mathbf{Y}_{N_r-N_t+1})}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)} \quad (47) \\ &\le N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{1}{\ln 2}\sum_{k=1}^{N_t}\psi(k) \\ &\quad + \log_2\det(\mathbf{R}_{\mathrm{T}}) + \frac{\sum_{k=N_r-N_t+1}^{N_r}\det(\mathbf{Y}_k)}{\ln 2\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}. \quad (48) \end{aligned}$$

Proof: The proof for (47) follows by taking ρ large in (46) and simplifying. For the evaluation of (48), we have used the standard expression for the average MIMO mutual information with optimal receivers under $N_r \geq N_t$ [1], [2]

$$I^{\mathrm{opt}} = \mathcal{E}_{\mathbf{H}}\left[\log_2\left(\det\left(\mathbf{I}_{N_t} + \frac{\rho}{N_t}\mathbf{H}^H\mathbf{H}\right)\right)\right]. \quad (49)$$

The proof concludes by taking ρ large in (49), dropping out the identity term, and then using (74) to evaluate the remaining nonlinear logdet function. ■

To the best of our knowledge the expression in (48) is new and does not require the MIMO matrix to be square, as in [9, eq. (88)]. Note that the equality between (47) and (48) holds for $N_t = 1$. This is anticipated since ZF detection becomes optimal for a single transmit antenna because all N_r degrees of freedom are devoted to the recovery of the corresponding multiplexed stream. What is more, the expression in (47) is in agreement with an associated result on the high-SNR sum rate of MIMO MMSE receivers [22, Prop. 6].

In Fig. 3, the proposed bounds in (44) and (46) are evaluated as a function of the SNR. As above, the entries of the receive correlation matrix are modeled as $\{\mathbf{R}_{\mathrm{R}}\}_{i,j} = \rho_r^{|i-j|}$ with $\rho_r \in [0, 1)$ being the receive correlation coefficient. Two different pairs of correlation coefficients are considered while $N_r = 7, N_t = 3$. Interestingly, the sum rate loss is more severe when correlation occurs on the transmit, rather on the receive, side. This is line with the results of [22].

In Fig. 4, the analytical high-SNR approximations for ZF and optimal receivers are compared, based on (47) and (48), respectively.

We can clearly observe that as N_t increases, the offset between the two expressions increases which demonstrates the increasingly poor interference cancellation capabilities of ZF receivers. In both cases, however, the sum rate (or respectively average mutual information) increases linearly with the minimum number of antennas.

D. Uncorrelated Ricean Fading

This model is suitable when there is a direct LoS or specular path between the transmitter and receiver. Then, the channel matrix consists of a deterministic component, and a Rayleigh-distributed random component, which accounts for the scattered signals. We hereafter focus on the case of rank-1 deterministic component, since the mathematical manipulations are tractable while an extension to the arbitrary rank case seems to be tedious. This scenario occurs when the interelement distances are small and the LoS rays' phases become identical [38]–[40]. Then, the fading matrix $\mathbf{H}$ reads

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{\mathrm{L}} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w \quad (50)$$

where K stands for the Ricean K-factor, representing the ratio of the power of the deterministic component to the power of the fading component.

As above, we first analytically derive the first negative moment of an arbitrary eigenvalue, λ.

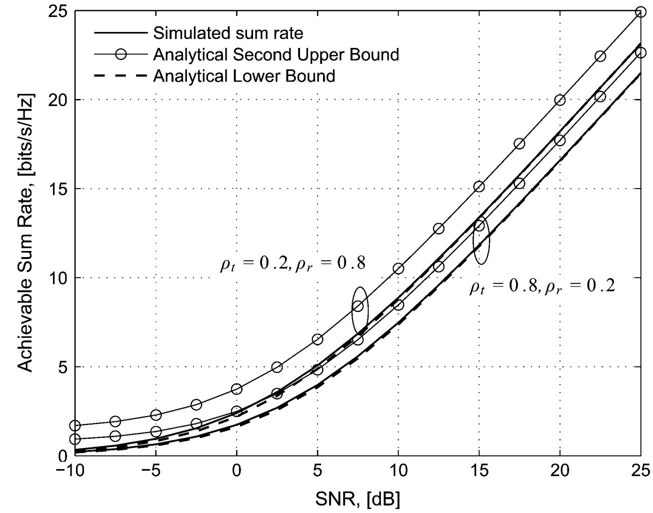

Fig. 3. Simulated sum rate and upper/lower bounds against the SNR ρ $(N_r = 7, N_t = 3)$.

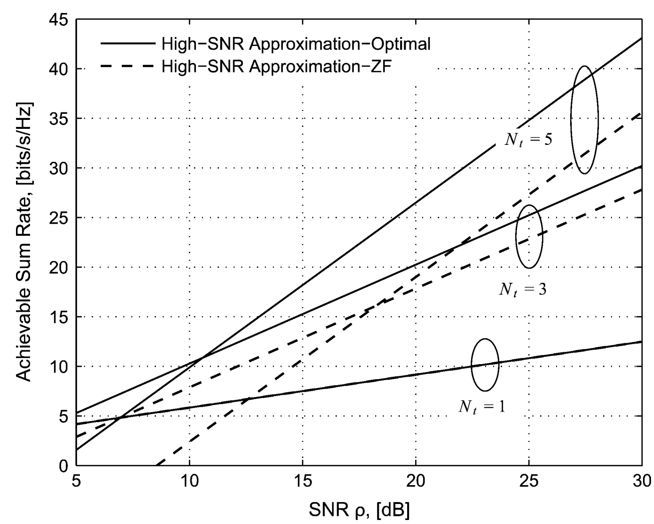

Fig. 4. Analytical high-SNR approximations for ZF and optimal receivers against the SNR ρ $(N_r = 7, \rho_t = 0.3, \rho_r = 0.8)$.

Theorem 5: For uncorrelated Ricean fading with a rank-1 LoS component, the first negative moment of an arbitrary eigenvalue, λ, of $\mathbf{H}^H\mathbf{H}$ is (for $N_r \geq N_t + 1$)

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \frac{e^{-\Delta}(K+1)}{N_t((N_r-N_t)!^{N_t})}\sum_{\ell=1}^{N_t}\frac{1}{\Delta^{N_t-1}\prod_{i=1}^{N_t-1}\Gamma(i)} \times\left(\sum_{k=1}^{N_t-1}\frac{\Gamma(N_r-N_t+\ell+k-2)}{(K+1)^{k-1}(N_r-N_t+1)_{k-1}}\mathcal{A}_{k\ell} + \Gamma(N_r-N_t+\ell-1) \times {}_1F_1(N_r-N_t+\ell-1;N_r-N_t+1;\Delta)\mathcal{A}_{N_t\ell}\right) \quad (51)$$

where $\Delta = KN_tN_r$. The term $\mathcal{A}_{k\ell}$ denotes the (k,ℓ)th cofactor of a $N_t \times N_t$ matrix $\mathcal{A}$ whose (i,j)th entry is

$$\{\mathcal{A}\}_{i,j} = \begin{cases} \frac{(N_r-N_t+i+j-2)!}{(N_r-N_t+1)_{i-1}}, & 1 \leq i \leq N_t - 1 \\ \frac{{}_1F_1(N_r-N_t+j;N_r-N_t+1;\Delta)}{((N_r-N_t+j-1)!)^{-1}}, & i = N_t. \end{cases}$$

In the above equation, $(\alpha)_n = \alpha(\alpha+1)\ldots(\alpha+n-1) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}$ is the Pochhammer symbol [41, eq. (6.1.22)].

Proof: In this case, we invoke [42, Corollary 1] to express the density of the unordered eigenvalue according to

$$f_\lambda(x) = \frac{e^{-(\Delta+x(K+1))}}{xN_t((N_r-N_t)!^{N_t})}\sum_{\ell=1}^{N_t}\frac{(x(K+1))^{N_r-N_t+\ell}}{\Delta^{N_t-1}\prod_{i=1}^{N_t-1}\Gamma(i)} \times\left(\sum_{k=1}^{N_t-1}\frac{x^{k-1}}{(N_r-N_t+1)_{k-1}}\mathcal{A}_{k\ell} + {}_0F_1(N_r-N_t+1;x(K+1)\Delta)\mathcal{A}_{N_t\ell}\right). \quad (52)$$

Note that $f_\lambda(0) = 0$ as long as $N_r - N_t > 0$. The final result is then obtained by evaluating the corresponding integrals in (67) with the aid of [23, eq. (3.381.4)]

$$\int_0^\infty x^{\nu-1}\exp(-\mu x) = \mu^{-\nu}\Gamma(\nu),\ \mathrm{Re}(\mu,\nu) > 0 \quad (53)$$

and [23, eq. (7.522.5)]

$$\int_0^\infty e^{-x}x^{\nu-1}{}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;\alpha x) = \Gamma(\nu){}_{p+1}F_q(\nu,a_1,\ldots,a_p;b_1,\ldots,b_q;\alpha) \quad (54)$$

for $\mathrm{Re}(\nu) > 0$ and $p < q$. ■

Setting $\mathcal{G}(\lambda) = \mathcal{E}\left[\frac{1}{\lambda}\right]$ for the sake of clarity, we can now present two novel upper bounds on the sum rate of MIMO ZF receivers under Ricean fading:

Proposition 7: For uncorrelated Ricean fading with a rank-1 LoS component, the sum rate of MIMO ZF receivers is upper bounded by

$$R_{u,1} = N_t\log_2\left(\mathcal{G}(\lambda) + \frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}(\psi(N_r-N_t+1) + g(N_r,\Delta) - g(N_r,\Delta_1) - \ln(K+1)) \quad (55)$$

$$R_{u,2} = N_t\log_2\left(\frac{N_r!}{(N_r-N_t+1)!}\frac{1+K(N_t-1)}{(K+1)^{N_t-1}} + \frac{\rho}{N_t}\frac{N_r!}{(N_r-N_t)!}\frac{1+KN_t}{(K+1)^{N_t}}\right) - \frac{N_t}{\ln 2}\left(g(N_r,\Delta_1) - (N_t-1)\ln(K+1) + \sum_{k=1}^{N_t-2}\psi(N_r-k)\right) \quad (56)$$

where $\Delta_1 = K(N_t-1)N_r$, while the auxiliary function $g(n,x)$ is defined according to

$$g(n,x) \triangleq \ln(x) - \mathtt{Ei}(-x) + \sum_{k=1}^{n-1}\left(-\frac{1}{x}\right)^k\left(e^{-x}(k-1)! - \frac{(n-1)!}{k(n-k-1)!}\right). \quad (57)$$

Proof: A detailed proof is given in Appendix IV. ■

Proposition 8: For uncorrelated Ricean fading with a rank-1 LoS component, the sum rate of MIMO ZF receivers is lower bounded by

$$R_L = N_t\log_2\left(1 + \frac{\rho}{N_t}\exp\big(\psi(N_r-N_t+1) - \ln(K+1) + g(N_r,\Delta) - g(N_r,\Delta_1)\big)\right). \quad (58)$$

Proof: The proof is trivial and therefore omitted. ■

Once more, we can observe that the second upper bound is more general since it is exists even for the $N_r = N_t$ case, while the first upper bound exists only if $N_r \geq N_t + 1$. Since it is difficult to obtain insights into the implications of the Ricean K-factor from (55), (56) and (58), we now focus on the high-SNR regime:

Corollary 8: For uncorrelated Ricean fading with a rank-1 LoS component, $R_{u,1}$ and R_L become exact at high SNRs and equal to

$$R_{u,1}^\infty = R_L^\infty = N_t\log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi(N_r-N_t+1) + \frac{N_t}{\ln 2}(g(N_r,\Delta) - g(N_r,\Delta_1) - \ln(K+1)) \quad (59)$$

$$\leq N_t\log_2\left(\frac{\rho}{N_t}\right) + \frac{1}{\ln 2}\sum_{k=1}^{N_t-1}\psi(N_r-k) + \frac{1}{\ln 2}g(N_r,\Delta) - N_t\log_2(K+1). \quad (60)$$

Proof: The proof follows by taking ρ large in (55) and (58) and simplifying. ■

From (59), we conjecture the deleterious effects of the rank-1 deterministic component on the sum rate, since the term $g(N_r,\Delta) - g(N_r,\Delta_1) - \ln(K+1)$ was numerically found to be a monotonically decreasing function in K. We also note that (60) corresponds to the high-SNR average mutual information of a rank-1 uncorrelated Ricean fading MIMO system with optimal receivers under $N_r \geq N_t$ [40, eq. (38)]. In Fig. 5, the effects of the Ricean K-factor on the performance of the proposed bounds are investigated.

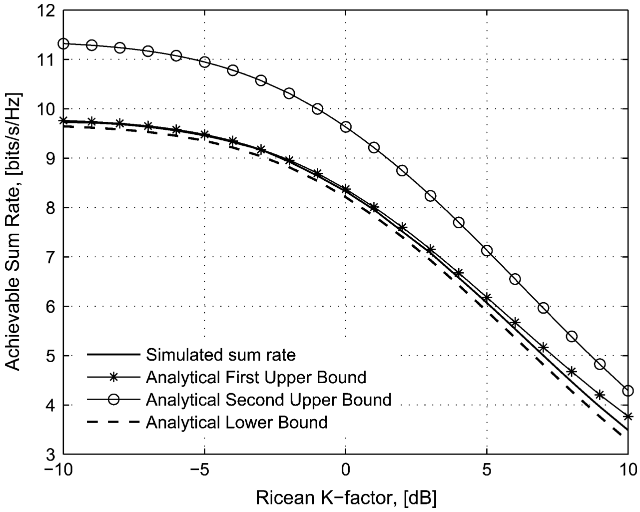

Fig. 5. Simulated sum rate, and upper/lower bounds against the Ricean K-factor ($N_r = 5, N_t = 3, \rho = 10$ dB).

It can be observed that a higher K-factor extensively limits the advantages of MIMO technology for the considered rank-1 configurations. This observation is in line with [38]–[40]. In the limit, $K \longrightarrow \infty$dB, those configurations degenerate into a single-path link. This can be justified by considering the high-SNR/ high K-factor regime, which leads to the following insightful result for the sum rate:

$$R_{u,1}^{\infty} = R_L^{\infty} \overset{K \longrightarrow \infty}{\approx} N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\psi\left(N_r - N_t + 1\right) - N_t \log_2(K). \quad (61)$$

Note that (61) originates from (59) by taking K large and noticing that $g(N_r, \Delta) \approx \ln(\Delta)$ for $\Delta \longrightarrow \infty$[40]. The above expression demonstrates that increasing K tends to logarithmically decrease the achievable sum rate. We finally note that $R_{u,1}$ and R_L remain notably tight across the entire K-factor regime, while $R_{u,2}$ becomes tighter with an increasing Ricean K-factor.

V. Conclusion

In this paper, a detailed sum rate analysis of MIMO ZF receivers was presented. Specifically, three novel upper/lower bounds were devised that can be applied for arbitrary number of antennas and remain sufficiently tight across the entire SNR range. More importantly, the proposed bounds are generic since they encompass most fading models of practical interest and can be very easily evaluated. Through these expressions, we gained valuable insights into the implications of the various system and fading parameters on the performance of ZF receivers. For instance, we established an interesting relationship between the sum rate and the first negative moment of the unordered eigenvalue of the instantaneous correlation matrix. In this context, it was shown that the second upper bound is more general since it is exists even for the $N_r = N_t$ case, while the first upper bound exists only if $N_r \geq N_t + 1$.

Moreover, we examined in detail (both theoretically and via numerical simulations) the impact of spatial correlation, Ricean K-factor, number of antennas and SNR. Finally, the presented closed-form expressions were compared with previous results that have been reported for both MIMO optimal and MMSE receivers.

Appendix I
Proof of Corollary 2

The proof starts by rewriting (18) according to

$$\begin{aligned} R_{u,1} &= N_t \log_2\left(\frac{N_t + \rho(N_r - N_t)}{N_t(N_r - N_t)}\right) \\ &\quad + \frac{N_t}{\ln 2}\psi(N_r - N_t + 1) \\ &\approx N_t \log_2\left(\frac{\rho}{N_t}\right) + \frac{N_t}{\ln 2}\ln(N_r - N_t) \qquad (62) \\ &\approx N_t \log_2\left(\frac{\rho}{N_t}\right) + N_t \log_2 N_r \qquad (63) \end{aligned}$$

where from (62) to (63), we have utilized the fact that [41, eq. (6.3.18)]

$$\psi(x) \approx \ln(x), \text{ if } x \longrightarrow \infty. \quad (64)$$

The second upper bound in (19) can be simplified after invoking the Stirling's approximation $\ln x! \approx x \ln x - x$, if $x \longrightarrow \infty$ [23, eq. (8.327.2)]. Hence, we get

$$\begin{aligned} R_{u,2} &\overset{(64)}{\approx} N_t \log_2\left(\frac{1}{N_r - N_t + 1} + \frac{\rho}{N_t}\right) \\ &\quad + N_t \log_2\left(\frac{N_r!}{(N_r - N_t)!}\right) - \frac{N_t}{\ln 2}(N_t - 1)\ln N_r \\ &\approx N_t \log_2\left(\frac{\rho}{N_t}\right) - N_t(N_t - 1)\log_2 N_r \\ &\quad + \frac{N_t}{\ln 2}\left(N_r \ln N_r - (N_r - N_t)\ln(N_r - N_t) - N_t\right) \\ &\approx N_t \log_2\left(\frac{\rho}{N_t}\right) - N_t(N_t - 1)\log_2 N_r + N_t^2 \log_2 N_r \\ &= N_t \log_2\left(\frac{\rho}{N_t}\right) + N_t \log_2 N_r. \end{aligned}$$

The proof for the lower bound in (19) follows a similar line of reasoning as in (62)–(63) and is therefore omitted.

Appendix II
Proof of Theorem 4

The proof starts from the density of an unordered eigenvalue of a semi-correlated Wishart matrix presented recently in [25, Lemma 1].[3] For the case of correlation on the side with the minimum number of antennas, the expression [25, eq. (14)] particularizes to

$$\begin{aligned} f_\lambda(x) = &\frac{1}{N_t \prod_{i<j}^{N_t}(\beta_j - \beta_i)} \\ &\times \sum_{\ell=1}^{N_t}\sum_{k=1}^{N_t} \frac{x^{N_r - N_t + k - 1} e^{-\frac{x}{\beta_\ell}} \beta_\ell^{N_t - N_r - 1}}{\Gamma(N_r - N_t + k)} D_{\ell k} \quad (65) \end{aligned}$$

[3]Note that the results in [25] represent a unified and analytically friendlier version of those in [43].

where $D_{\ell k}$ is the (ℓ, k)th cofactor of a $N_t \times N_t$ matrix $\mathbf{D}$ whose (i, j)th entry is

$$\{\mathbf{D}\}_{i,j} = \beta_i^{j-1}. \tag{66}$$

We emphasize the fact that $f_\lambda(0) = 0$ holds only if $N_r \geq N_t+1$ since for $N_r = N_t$, we get $f_\lambda(0) > 0$. Once this condition is fulfilled, the first negative moment can be computed through (65), according to

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \int_0^\infty \frac{f_\lambda(x)}{x}dx \tag{67}$$

$$= \frac{1}{N_t \prod_{i<j}^{N_t}(\beta_j - \beta_i)} \sum_{\ell=1}^{N_t}\sum_{k=1}^{N_t} \frac{\beta_\ell^{k-2} D_{\ell k}}{N_r - N_t + k - 1} \tag{68}$$

where we have used (53) to solve the involved integral.

From Laplace's expansion of matrix determinant along the kth column, we can rewrite (68) as

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \frac{1}{N_t \prod_{i<j}^{N_t}(\beta_j - \beta_i)} \sum_{k=1}^{N_t} \frac{\det(\mathbf{D}_k)}{N_r - N_t + k - 1} \tag{69}$$

where $\mathbf{D}_k$ is a $N_t \times N_t$ matrix with entries defined as

$$\{\mathbf{D}_k\}_{i,j} = \begin{cases} \beta_i^{j-1}, & j \neq k \\ \beta_i^{j-2}, & j = k. \end{cases} \tag{70}$$

It is easy to observe that the matrix $\mathbf{D}_k$ contains identical columns when $k > 1$, hence $\det(\mathbf{D}_k) = 0$. Thus, the only nonzero term stems from $k = 1$, which yields

$$\mathcal{E}\left[\frac{1}{\lambda}\right] = \frac{1}{N_t \prod_{i<j}^{N_t}(\beta_j - \beta_i)} \frac{\det(\mathbf{D}_1)}{N_r - N_t}. \tag{71}$$

After some basic algebraic manipulations, which involve the definition of a Vandermonde determinant, we can explicitly compute $\det(\mathbf{D}_1)$

$$\det(\mathbf{D}_1) = \prod_{i<j}^{N_t}(\beta_j - \beta_i)\left(\sum_{i=1}^{N_t}\beta_i^{-1}\right) \tag{72}$$

which when substituted into (71), returns (31).

Appendix III
Proof of Proposition 5

In order to evaluate the first term in (12), we notice that

$$\mathbf{H}_m \sim \mathcal{CN}(\mathbf{0}_{N_r \times N_t - 1}, \mathbf{R}_{\rm R} \otimes \mathbf{R}_{{\rm T},mm}) \tag{73}$$

which implies that we can directly employ the results of Lemma 2, with the only difference pertaining to the replacement of $\mathbf{X}$ with $\tilde{\mathbf{X}}$. For the last term in (12), we invoke a recent result from [25, Lemma 4], for an $N_r \times N_t$ semi-correlated MIMO matrix, that is

$$\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}_w^H \mathbf{R}_{\rm R} \mathbf{H}_w\right)\right)\right] = \sum_{k=1}^{N_t}\psi(k) + \frac{\sum_{k=N_r-N_t+1}^{N_r}\det(\mathbf{Y}_k)}{\prod_{i<j}^{N_r}(\gamma_j - \gamma_i)} \tag{74}$$

where $\mathbf{Y}_k$ is defined in (45). Combining these results together,

$$\begin{aligned} R_{u,2} = &\sum_{m=1}^{N_t}\log_2\left(\frac{(N_t-1)!\det(\mathbf{R}_{{\rm T},mm})\det(\tilde{\mathbf{X}})}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)} + \frac{\rho N_t!\det(\mathbf{R}_{\rm T})\det(\mathbf{X})}{N_t\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}\right) \\ &- \sum_{m=1}^{N_t}\log_2\left(\det\left(\mathbf{R}_{{\rm T},mm}\right)\right) \\ &- \frac{N_t}{\ln 2}\left(\sum_{k=1}^{N_t-1}\psi(k) + \frac{\sum_{k=N_r-N_t+2}^{N_r}\det(\mathbf{Y}_k)}{\prod_{i<j}^{N_r}(\gamma_j-\gamma_i)}\right). \end{aligned} \tag{75}$$

The desired result in (44) is then obtained by applying (11) on $\mathbf{R}_{\rm T}$ and simplifying (75).

Appendix IV
Proof of Proposition 7

The proof starts by invoking the following results for a $N_r \times N_t$ (with $N_r \geq N_t$) noncentral Wishart matrix with a rank-1 LoS component

$$\mathcal{E}\left[\det\left(\mathbf{H}^H\mathbf{H}\right)\right] = \frac{N_r!}{(N_r - N_t)!}\frac{1 + KN_t}{(K+1)^{N_t}} \tag{76}$$

$$\mathcal{E}\left[\ln\left(\det\left(\mathbf{H}^H\mathbf{H}\right)\right)\right] = g(N_r, \Delta) - N_t\ln(K+1) + \sum_{k=1}^{N_t-1}\psi(N_r - k) \tag{77}$$

which are respectively obtained from [40, Theorem 1] and [40, Theorem 2]. We note that $g(N_r, 0) = \psi(N_r)$.[4] Evidently, for Rayleigh fading conditions (i.e., $K = 0$) (76)–(77) respectively coincide with (20)–(21). Then, in order to evaluate the last term in (5), we notice that

$$\mathbf{H}_m \sim \mathcal{CN}\left(\sqrt{\frac{K}{K+1}}\mathbf{H}_{{\rm L},m}, \frac{1}{K+1}\mathbf{I}_{N_r}\otimes\mathbf{I}_{N_t-1}\right) \tag{78}$$

where $\mathbf{H}_{{\rm L},m}$ corresponds to $\mathbf{H}_{\rm L}$ with the mth column removed. Clearly, we can apply the results of Theorem 5 and (77) on (5) to obtain (55). Likewise, the upper bound in (56) is again obtained through (12), (78) with the aid of (76)–(77).

[4]Note that the expression in [40, Theorem 2] is a compact and efficient combination of results originally presented in [6], [44, Appendix X].

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