# GraphicalModels version 1.2 -- What's new in version 1.2

We updated gaussianRing and are in the process of making GraphicalModels compatible with the new Graphs package.
The Gaussian ring of a graph acts just like before.

```
i1 : gG=graph {{1,3},{2,4}}

o1 = Graph{1 => {3}}
           2 => {4}
           3 => {1}
           4 => {2}

o1 : Graph
```

```
i2 : RG=gaussianRing(gG);
```

```
i3 : print toString gens RG
{k_(1,1), k_(2,2), k_(3,3), k_(4,4), k_(1,3), k_(2,4), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i4 : undirectedEdgesMatrix(RG)

o4 = | k_(1,1) 0       k_(1,3) 0       |
     | 0       k_(2,2) 0       k_(2,4) |
     | k_(1,3) 0       k_(3,3) 0       |
     | 0       k_(2,4) 0       k_(4,4) |

             4        4
o4 : Matrix RG  <--- RG
```

Note: "directedEdgesMatrix(RG)" and "bidirectedEdgesMatrix(RG)" and "gaussianParametrization(RG)" will give errors.
Here is an example of a mixed graph that has only undirected edges.

```
i5 : g=graph {{1,3},{2,4}};
```

```
i6 : H=new HashTable from {Graph=>g, Digraph=>digraph {}, Bigraph => bigraph {} };
```

```
i7 : gG = new MixedGraph from { {graph,H}, {cache,new CacheTable from {}} }

o7 = MixedGraph{Bigraph => Bigraph{}    }
                Digraph => Digraph{}
                Graph => Graph{1 => {3}}
                               2 => {4}
                               3 => {1}
                               4 => {2}

o7 : MixedGraph
```

```
i8 : RG=gaussianRing(gG);
```

```
i9 : print toString gens RG
{k_(1,1), k_(2,2), k_(3,3), k_(4,4), k_(1,3), k_(2,4), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i10 : undirectedEdgesMatrix(RG)

o10 = | k_(1,1) 0       k_(1,3) 0       |
      | 0       k_(2,2) 0       k_(2,4) |
      | k_(1,3) 0       k_(3,3) 0       |
      | 0       k_(2,4) 0       k_(4,4) |

              4        4
o10 : Matrix RG  <--- RG
```

```
i11 : directedEdgesMatrix(RG)

o11 = 0

              4        4
o11 : Matrix RG  <--- RG
```

```
i12 : bidirectedEdgesMatrix(RG)

o12 = 0

o12 : Matrix 0 <--- 0
```

```
i13 : gaussianParametrization(RG)

o13 = | k_(1,1) 0       k_(1,3) 0       |
      | 0       k_(2,2) 0       k_(2,4) |
      | k_(1,3) 0       k_(3,3) 0       |
      | 0       k_(2,4) 0       k_(4,4) |

                     4               4
o13 : Matrix (frac RG)  <--- (frac RG)
```

Here is an example of a digraph.

```
i14 : gD = digraph {{1,{2}},{2,{3,4}}};
```

```
i15 : RD=gaussianRing(gD);
```

```
i16 : print toString gens RD
{s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

"undirectedEdgesMatrix(RD)", "directedEdgesMatrix(RD)", "bidirectedEdgesMatrix(RD)", and "gaussianParametrization(RD)" will give errors.
Here is an example of a digraph embedded in a mixed graph.

```
i17 : gD = mixedGraph(digraph {{1,{2}},{2,{3,4}}});
```

```
i18 : RD=gaussianRing(gD);
```

```
i19 : print toString gens RD
```

```
{l_(1,2), l_(2,3), l_(2,4), p_(1,1), p_(2,2), p_(3,3), p_(4,4), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i20 : undirectedEdgesMatrix(RD)

o20 = 0

o20 : Matrix 0 <--- 0
```

```
i21 : directedEdgesMatrix(RD)

o21 = | 0 l_(1,2) 0       0       |
      | 0 0       l_(2,3) l_(2,4) |
      | 0 0       0       0       |
      | 0 0       0       0       |

               4        4
o21 : Matrix RD  <--- RD
```

```
i22 : bidirectedEdgesMatrix(RD)

o22 = | p_(1,1) 0       0       0       |
      | 0       p_(2,2) 0       0       |
      | 0       0       p_(3,3) 0       |
      | 0       0       0       p_(4,4) |

               4        4
o22 : Matrix RD  <--- RD
```

```
i23 : gaussianParametrization(RD)

o23 = | p_(1,1)                l_(1,2)p_(1,1)
      | l_(1,2)p_(1,1)         l_(1,2)^2p_(1,1)+p_(2,2)
      | l_(1,2)l_(2,3)p_(1,1) l_(1,2)^2l_(2,3)p_(1,1)+l_(2,3)p_(2,2)
      | l_(1,2)l_(2,4)p_(1,1) l_(1,2)^2l_(2,4)p_(1,1)+l_(2,4)p_(2,2)
      ------------------------------------------------------------------------
      l_(1,2)l_(2,3)p_(1,1)
      l_(1,2)^2l_(2,3)p_(1,1)+l_(2,3)p_(2,2)
      l_(1,2)^2l_(2,3)^2p_(1,1)+l_(2,3)^2p_(2,2)+p_(3,3)
      l_(1,2)^2l_(2,3)l_(2,4)p_(1,1)+l_(2,3)l_(2,4)p_(2,2)
      ------------------------------------------------------------------------
      l_(1,2)l_(2,4)p_(1,1)                                 |
      l_(1,2)^2l_(2,4)p_(1,1)+l_(2,4)p_(2,2)                |
      l_(1,2)^2l_(2,3)l_(2,4)p_(1,1)+l_(2,3)l_(2,4)p_(2,2) |
      l_(1,2)^2l_(2,4)^2p_(1,1)+l_(2,4)^2p_(2,2)+p_(4,4)    |

                      4                4
o23 : Matrix (frac RD)  <--- (frac RD)
```

Here is an example of a chain graph (a graph with no bidirected edges).

```
i24 : gGD = mixedGraph(graph {{1,3},{2,4}} , digraph {{1,{2}},{2,{3,4}}});
```

```
i25 : RGD = gaussianRing(gGD);
```

```
i26 : print toString gens RGD
{l_(1,2), l_(2,3), l_(2,4), k_(1,1), k_(2,2), k_(3,3), k_(4,4), k_(1,3), k_(2,4), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i27 : undirectedEdgesMatrix(RGD)

o27 = | k_(1,1) 0       k_(1,3) 0       |
      | 0       k_(2,2) 0       k_(2,4) |
      | k_(1,3) 0       k_(3,3) 0       |
      | 0       k_(2,4) 0       k_(4,4) |

                4         4
o27 : Matrix RGD  <--- RGD
```

```
i28 : directedEdgesMatrix(RGD)

o28 = | 0 l_(1,2) 0       0       |
      | 0 0       l_(2,3) l_(2,4) |
      | 0 0       0       0       |
      | 0 0       0       0       |

                4         4
o28 : Matrix RGD  <--- RGD
```

```
i29 : bidirectedEdgesMatrix(RGD)

o29 = 0

o29 : Matrix 0 <--- 0
```

```
i30 : gaussianParametrization(RGD)

o30 = | k_(1,1)
      | l_(1,2)k_(1,1)
      | l_(1,2)l_(2,3)k_(1,1)+k_(1,3)
      | l_(1,2)l_(2,4)k_(1,1)
      ------------------------------------------------------------------------
      l_(1,2)k_(1,1)
      l_(1,2)^2k_(1,1)+k_(2,2)
      l_(1,2)^2l_(2,3)k_(1,1)+l_(2,3)k_(2,2)+l_(1,2)k_(1,3)
      l_(1,2)^2l_(2,4)k_(1,1)+l_(2,4)k_(2,2)+k_(2,4)
      ------------------------------------------------------------------------
      l_(1,2)l_(2,3)k_(1,1)+k_(1,3)
      l_(1,2)^2l_(2,3)k_(1,1)+l_(2,3)k_(2,2)+l_(1,2)k_(1,3)
      l_(1,2)^2l_(2,3)^2k_(1,1)+l_(2,3)^2k_(2,2)+2l_(1,2)l_(2,3)k_(1,3)+k_(3,
      l_(1,2)^2l_(2,3)l_(2,4)k_(1,1)+l_(2,3)l_(2,4)k_(2,2)+l_(1,2)l_(2,4)k_(1
      ------------------------------------------------------------------------
                         l_(1,2)l_(2,4)k_(1,1)
                         l_(1,2)^2l_(2,4)k_(1,1)+l_(2,4)k_(2,2)+k_(2,4)
      3)                 l_(1,2)^2l_(2,3)l_(2,4)k_(1,1)+l_(2,3)l_(2,4)k_(2,2
      ,3)+l_(2,3)k_(2,4) l_(1,2)^2l_(2,4)^2k_(1,1)+l_(2,4)^2k_(2,2)+2l_(2,4)
      ------------------------------------------------------------------------
                                             |
                                             |
      )+l_(1,2)l_(2,4)k_(1,3)+l_(2,3)k_(2,4) |
      k_(2,4)+k_(4,4)                        |

                       4                 4
o30 : Matrix (frac RGD)  <--- (frac RGD)
```

Here is an example of a mixed graph with no undirected edges.

```
i31 : gDB = mixedGraph(digraph {{1,{2}},{2,{3,4}}}, bigraph {{1,3},{2,4}})

o31 = MixedGraph{Bigraph => Bigraph{1 => {3}}   }
                                    2 => {4}
                                    3 => {1}
                                    4 => {2}
                 Digraph => Digraph{1 => {2}   }
                                    2 => {3, 4}
                                    3 => {}
                                    4 => {}
                 Graph => Graph{}

o31 : MixedGraph
```

```
i32 : RDB=gaussianRing(gDB);
```

```
i33 : print toString gens RDB
{l_(1,2), l_(2,3), l_(2,4), p_(1,1), p_(2,2), p_(3,3), p_(4,4), p_(1,3), p_(2,4), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i34 : undirectedEdgesMatrix(RDB)

o34 = 0

o34 : Matrix 0 <--- 0
```

```
i35 : directedEdgesMatrix(RDB)

o35 = | 0 l_(1,2) 0       0       |
      | 0 0       l_(2,3) l_(2,4) |
      | 0 0       0       0       |
      | 0 0       0       0       |

                4         4
o35 : Matrix RDB  <--- RDB
```

```
i36 : bidirectedEdgesMatrix(RDB)

o36 = | p_(1,1) 0       p_(1,3) 0       |
      | 0       p_(2,2) 0       p_(2,4) |
      | p_(1,3) 0       p_(3,3) 0       |
      | 0       p_(2,4) 0       p_(4,4) |

                4         4
o36 : Matrix RDB  <--- RDB
```

```
i37 : gaussianParametrization(RDB)

o37 = | p_(1,1)
      | l_(1,2)p_(1,1)
      | l_(1,2)l_(2,3)p_(1,1)+p_(1,3)
      | l_(1,2)l_(2,4)p_(1,1)
      ------------------------------------------------------------------------
      l_(1,2)p_(1,1)
      l_(1,2)^2p_(1,1)+p_(2,2)
      l_(1,2)^2l_(2,3)p_(1,1)+l_(2,3)p_(2,2)+l_(1,2)p_(1,3)
      l_(1,2)^2l_(2,4)p_(1,1)+l_(2,4)p_(2,2)+p_(2,4)
      ------------------------------------------------------------------------
      l_(1,2)l_(2,3)p_(1,1)+p_(1,3)
      l_(1,2)^2l_(2,3)p_(1,1)+l_(2,3)p_(2,2)+l_(1,2)p_(1,3)
      l_(1,2)^2l_(2,3)^2p_(1,1)+l_(2,3)^2p_(2,2)+2l_(1,2)l_(2,3)p_(1,3)+p_(3,
      l_(1,2)^2l_(2,3)l_(2,4)p_(1,1)+l_(2,3)l_(2,4)p_(2,2)+l_(1,2)l_(2,4)p_(1
      ------------------------------------------------------------------------
                         l_(1,2)l_(2,4)p_(1,1)
                         l_(1,2)^2l_(2,4)p_(1,1)+l_(2,4)p_(2,2)+p_(2,4)
      3)                 l_(1,2)^2l_(2,3)l_(2,4)p_(1,1)+l_(2,3)l_(2,4)p_(2,2
      ,3)+l_(2,3)p_(2,4) l_(1,2)^2l_(2,4)^2p_(1,1)+l_(2,4)^2p_(2,2)+2l_(2,4)
      ------------------------------------------------------------------------
                                             |
                                             |
      )+l_(1,2)l_(2,4)p_(1,3)+l_(2,3)p_(2,4) |
      p_(2,4)+p_(4,4)                        |

                       4                 4
o37 : Matrix (frac RDB)  <--- (frac RDB)
```

Here is a mixed graph with all three kinds of edges.

```
i38 : gGDB = mixedGraph(graph {{1,2}},digraph {{1,{2}},{2,{3,4}}},bigraph {{3,4}});
```

```
i39 : RGDB= gaussianRing(gGDB);
```

```
i40 : print toString gens RGDB
{l_(1,2), l_(2,3), l_(2,4), p_(3,3), p_(4,4), p_(3,4), k_(1,1), k_(2,2), k_(1,2), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(2,2), s_(2,3), s_(2,4), s_(3,3), s_(3,4), s_(4,4)}
```

```
i41 : undirectedEdgesMatrix(RGDB)

o41 = | k_(1,1) k_(1,2) |
      | k_(1,2) k_(2,2) |

                 2          2
o41 : Matrix RGDB  <--- RGDB
```

```
i42 : directedEdgesMatrix(RGDB)

o42 = | 0 l_(1,2) 0       0       |
      | 0 0       l_(2,3) l_(2,4) |
      | 0 0       0       0       |
      | 0 0       0       0       |

                 4          4
o42 : Matrix RGDB  <--- RGDB
```

```
i43 : bidirectedEdgesMatrix(RGDB)

o43 = | p_(3,3) p_(3,4) |
      | p_(3,4) p_(4,4) |

                 2          2
o43 : Matrix RGDB  <--- RGDB
```

```
i44 : gaussianParametrization(RGDB)

o44 = | k_(2,2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
      (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2k_(2,2)-2l_(1,2)k_(1,2)+k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+l_(2,3)k_(1,1))/(k_(1,1
      (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,4)k_(1,1))/(k_(1,1
      ------------------------------------------------------------------------
                         (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k
                         (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)^2k_(2,2)-2l_(1,2)l_(2,3)^2k_(1
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l
      ------------------------------------------------------------------------
      _(2,2)-k_(1,2)^2)
      l_(2,3)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ,2)+l_(2,3)^2k_(1,1)+p_(3,3)k_(1,1)k_(2,2)-p_(3,3)k_(1,2)^2)/(k_(1,1)k
      _(2,4)k_(1,2)+l_(2,3)l_(2,4)k_(1,1)+p_(3,4)k_(1,1)k_(2,2)-p_(3,4)k_(1,
      ------------------------------------------------------------------------


      _(2,2)-k_(1,2)^2)
      2)^2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
      (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,4)k_(1,1))/(k_(1,
      (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l_(2,4)k_(1,2)+l_(2,3)l
      (l_(1,2)^2l_(2,4)^2k_(2,2)-2l_(1,2)l_(2,4)^2k_(1,2)+l_(2,4)^2k_(1,1)+p
      ------------------------------------------------------------------------

      1)k_(2,2)-k_(1,2)^2)
      _(2,4)k_(1,1)+p_(3,4)k_(1,1)k_(2,2)-p_(3,4)k_(1,2)^2)/(k_(1,1)k_(2,2)-k
      _(4,4)k_(1,1)k_(2,2)-p_(4,4)k_(1,2)^2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
                |
                |
      _(1,2)^2) |
                |

                        4                 4
o44 : Matrix (frac RGDB)  <--- (frac RGDB)
```

The next two examples are examples of a graph with more than one possible UW decomposition in the sense of Sullivant, Talaska, and Draisma. Here is the Gaussian ring that arises from the package’s default UW decomposition.

```
i45 : gGDB5 = mixedGraph(graph {{1,2}},digraph {{1,{2}},{2,{3,4,5}}},bigraph {{4,5}});
```

```
i46 : RGDB5= gaussianRing(gGDB5);
```

```
i47 : print toString gens RGDB5
{l_(1,2), l_(2,3), l_(2,4), l_(2,5), p_(4,4), p_(5,5), p_(4,5), k_(1,1), k_(2,2), k_(3,3), k_(1,2), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(1,5), s_(2,2), s_(2,3), s_(2,4), s_(2,5), s_
```

```
i48 : undirectedEdgesMatrix(RGDB5)

o48 = | k_(1,1) k_(1,2) 0       |
      | k_(1,2) k_(2,2) 0       |
      | 0       0       k_(3,3) |

               3           3
o48 : Matrix RGDB5  <--- RGDB5
```

```
i49 : directedEdgesMatrix(RGDB5)

o49 = | 0 l_(1,2) 0       0       0       |
      | 0 0       l_(2,3) l_(2,4) l_(2,5) |
      | 0 0       0       0       0       |
      | 0 0       0       0       0       |
      | 0 0       0       0       0       |

               5           5
o49 : Matrix RGDB5  <--- RGDB5
```

```
i50 : bidirectedEdgesMatrix(RGDB5)

o50 = | p_(4,4) p_(4,5) |
      | p_(4,5) p_(5,5) |

               2           2
o50 : Matrix RGDB5  <--- RGDB5
```

```
i51 : gaussianParametrization(RGDB5)

o51 = | k_(2,2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,5)k_(2,2)-l_(2,5)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
      (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2k_(2,2)-2l_(1,2)k_(1,2)+k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+l_(2,3)k_(1,1))/(k_(1,1
      (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,4)k_(1,1))/(k_(1,1
      (l_(1,2)^2l_(2,5)k_(2,2)-2l_(1,2)l_(2,5)k_(1,2)+l_(2,5)k_(1,1))/(k_(1,1
      ------------------------------------------------------------------------
                         (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k
                         (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)^2k_(2,2)k_(3,3)-2l_(1,2)l_(2,3
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,5)k_(2,2)-2l_(1,2)l_(2,3)l
      ------------------------------------------------------------------------
      _(2,2)-k_(1,2)^2)
      l_(2,3)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      )^2k_(3,3)k_(1,2)+l_(2,3)^2k_(1,1)k_(3,3)+k_(1,1)k_(2,2)-k_(1,2)^2)/(k
      _(2,4)k_(1,2)+l_(2,3)l_(2,4)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      _(2,5)k_(1,2)+l_(2,3)l_(2,5)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
```

```
      _(1,1)k_(2,2)k_(3,3)-k_(3,3)k_(1,2)^2)


      ------------------------------------------------------------------------
      (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,4)k_(1,1))/(k_(1,
      (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l_(2,4)k_(1,2)+l_(2,3)l
      (l_(1,2)^2l_(2,4)^2k_(2,2)-2l_(1,2)l_(2,4)^2k_(1,2)+l_(2,4)^2k_(1,1)+p
      (l_(1,2)^2l_(2,4)l_(2,5)k_(2,2)-2l_(1,2)l_(2,4)l_(2,5)k_(1,2)+l_(2,4)l
      ------------------------------------------------------------------------

      1)k_(2,2)-k_(1,2)^2)
      _(2,4)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      _(4,4)k_(1,1)k_(2,2)-p_(4,4)k_(1,2)^2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      _(2,5)k_(1,1)+p_(4,5)k_(1,1)k_(2,2)-p_(4,5)k_(1,2)^2)/(k_(1,1)k_(2,2)-k
      ------------------------------------------------------------------------
                (l_(1,2)l_(2,5)k_(2,2)-l_(2,5)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,
                (l_(1,2)^2l_(2,5)k_(2,2)-2l_(1,2)l_(2,5)k_(1,2)+l_(2,5)k_(1,
                (l_(1,2)^2l_(2,3)l_(2,5)k_(2,2)-2l_(1,2)l_(2,3)l_(2,5)k_(1,2
                (l_(1,2)^2l_(2,4)l_(2,5)k_(2,2)-2l_(1,2)l_(2,4)l_(2,5)k_(1,2
      _(1,2)^2) (l_(1,2)^2l_(2,5)^2k_(2,2)-2l_(1,2)l_(2,5)^2k_(1,2)+l_(2,5)^
      ------------------------------------------------------------------------
      2)^2)
      1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      )+l_(2,3)l_(2,5)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      )+l_(2,4)l_(2,5)k_(1,1)+p_(4,5)k_(1,1)k_(2,2)-p_(4,5)k_(1,2)^2)/(k_(1,1
      2k_(1,1)+p_(5,5)k_(1,1)k_(2,2)-p_(5,5)k_(1,2)^2)/(k_(1,1)k_(2,2)-k_(1,2
      ------------------------------------------------------------------------
                          |
                          |
                          |
      )k_(2,2)-k_(1,2)^2) |
      )^2)                |

                        5                   5
o51 : Matrix (frac RGDB5)  <--- (frac RGDB5)
```

The user may specify the UW decomposition with an optional input.

```
i52 : gGDBo = mixedGraph(graph {{1,2}},digraph {{1,{2}},{2,{3,4,5}}},bigraph {{4,5}});
```

```
i53 : RGDBo= gaussianRing(gGDBo,verticesInU=>{1,2});
```

```
i54 : print toString gens RGDBo
{l_(1,2), l_(2,3), l_(2,4), l_(2,5), p_(3,3), p_(4,4), p_(5,5), p_(4,5), k_(1,1), k_(2,2), k_(1,2), s_(1,1), s_(1,2), s_(1,3), s_(1,4), s_(1,5), s_(2,2), s_(2,3), s_(2,4), s_(2,5), s_
```

```
i55 : undirectedEdgesMatrix(RGDBo)

o55 = | k_(1,1) k_(1,2) |
      | k_(1,2) k_(2,2) |

               2           2
o55 : Matrix RGDBo  <--- RGDBo
```

```
i56 : directedEdgesMatrix(RGDBo)

o56 = | 0 l_(1,2) 0       0       0       |
      | 0 0       l_(2,3) l_(2,4) l_(2,5) |
      | 0 0       0       0       0       |
      | 0 0       0       0       0       |
      | 0 0       0       0       0       |

               5           5
o56 : Matrix RGDBo  <--- RGDBo
```

```
i57 : bidirectedEdgesMatrix(RGDBo)

o57 = | p_(3,3) 0       0       |
      | 0       p_(4,4) p_(4,5) |
      | 0       p_(4,5) p_(5,5) |

               3           3
o57 : Matrix RGDBo  <--- RGDBo
```

```
i58 : gaussianParametrization(RGDBo)

o58 = | k_(2,2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      | (l_(1,2)l_(2,5)k_(2,2)-l_(2,5)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
      (l_(1,2)k_(2,2)-k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2k_(2,2)-2l_(1,2)k_(1,2)+k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+l_(2,3)k_(1,1))/(k_(1,1
      (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,4)k_(1,1))/(k_(1,1
      (l_(1,2)^2l_(2,5)k_(2,2)-2l_(1,2)l_(2,5)k_(1,2)+l_(2,5)k_(1,1))/(k_(1,1
      ------------------------------------------------------------------------
                          (l_(1,2)l_(2,3)k_(2,2)-l_(2,3)k_(1,2))/(k_(1,1)k
                          (l_(1,2)^2l_(2,3)k_(2,2)-2l_(1,2)l_(2,3)k_(1,2)+
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)^2k_(2,2)-2l_(1,2)l_(2,3)^2k_(1
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l
      )k_(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,5)k_(2,2)-2l_(1,2)l_(2,3)l
      ------------------------------------------------------------------------
      _(2,2)-k_(1,2)^2)
      l_(2,3)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ,2)+l_(2,3)^2k_(1,1)+p_(3,3)k_(1,1)k_(2,2)-p_(3,3)k_(1,2)^2)/(k_(1,1)k
      _(2,4)k_(1,2)+l_(2,3)l_(2,4)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      _(2,5)k_(1,2)+l_(2,3)l_(2,5)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      ------------------------------------------------------------------------
                        (l_(1,2)l_(2,4)k_(2,2)-l_(2,4)k_(1,2))/(k_(1,1)k_(2,2
                        (l_(1,2)^2l_(2,4)k_(2,2)-2l_(1,2)l_(2,4)k_(1,2)+l_(2,
      _(2,2)-k_(1,2)^2) (l_(1,2)^2l_(2,3)l_(2,4)k_(2,2)-2l_(1,2)l_(2,3)l_(2,4
                        (l_(1,2)^2l_(2,4)^2k_(2,2)-2l_(1,2)l_(2,4)^2k_(1,2)+l
                        (l_(1,2)^2l_(2,4)l_(2,5)k_(2,2)-2l_(1,2)l_(2,4)l_(2,5
      ------------------------------------------------------------------------
      )-k_(1,2)^2)
      4)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      )k_(1,2)+l_(2,3)l_(2,4)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
      _(2,4)^2k_(1,1)+p_(4,4)k_(1,1)k_(2,2)-p_(4,4)k_(1,2)^2)/(k_(1,1)k_(2,
      )k_(1,2)+l_(2,4)l_(2,5)k_(1,1)+p_(4,5)k_(1,1)k_(2,2)-p_(4,5)k_(1,2)^2
```

```
     ----------------------------------------------------------------------



     2)-k_(1,2)^2)
     )/(k_(1,1)k_(2,2)-k_(1,2)^2)
     ----------------------------------------------------------------------
     (l_(1,2)l_(2,5)k_(2,2)-l_(2,5)k_(1,2))/(k_(1,1)k_(2,2)-k_(1,2)^2)
     (l_(1,2)^2l_(2,5)k_(2,2)-2l_(1,2)l_(2,5)k_(1,2)+l_(2,5)k_(1,1))/(k_(1,
     (l_(1,2)^2l_(2,3)l_(2,5)k_(2,2)-2l_(1,2)l_(2,3)l_(2,5)k_(1,2)+l_(2,3)l
     (l_(1,2)^2l_(2,4)l_(2,5)k_(2,2)-2l_(1,2)l_(2,4)l_(2,5)k_(1,2)+l_(2,4)l
     (l_(1,2)^2l_(2,5)^2k_(2,2)-2l_(1,2)l_(2,5)^2k_(1,2)+l_(2,5)^2k_(1,1)+p
     ----------------------------------------------------------------------

     1)k_(2,2)-k_(1,2)^2)
     _(2,5)k_(1,1))/(k_(1,1)k_(2,2)-k_(1,2)^2)
     _(2,5)k_(1,1)+p_(4,5)k_(1,1)k_(2,2)-p_(4,5)k_(1,2)^2)/(k_(1,1)k_(2,2)-k
     _(5,5)k_(1,1)k_(2,2)-p_(5,5)k_(1,2)^2)/(k_(1,1)k_(2,2)-k_(1,2)^2)
     ----------------------------------------------------------------------
                 |
                 |
                 |
     _(1,2)^2) |
                 |

                              5                  5
o58 : Matrix (frac RGDBo)  <--- (frac RGDBo)
```