# Chapter 2
The Physics of the Sun

Abstract The chapter starts by describing briefly the basic features of sun and then proceeds to deriving the relevant equations that allow the calculation of several parameters pertaining to the sun namely the eccentric anomaly, hour angle, and the position of the sun (azimuth and altitude) amongst others. The relevant formulae are valid at any spacial and temporal location on the earth. The rule which allows us to compute the sun's position at any time is cross-checked with the US National Renewable Energy Laboratory's Solar Position Algorithm and is shown to be in good agreement ($<1\,\%$) with the exact sun's position at our present location ($20°17'$ S and $57°33'$ E).

The sun is a huge ball of hot gas subject to the action of gravitational forces that tend to make it shrink in size. This force is balanced by the pressure exerted by the gas, so that an equilibrium size prevails. The core of the sun which extends from the centre to about 20 % of the solar radius is at an extremely high temperature of around 15.7×10^6 K and pressure of 340 billion times earth's air pressure at sea level. Under these extremes, a nuclear fusion reaction takes place that merge four hydrogen nuclei or protons into an α-particle (helium nucleus), resulting in the production of energy from the net change in mass due to the fact that the alpha particle is about 0.7 % less massive than the four protons. This energy is carried to the surface of the sun in about a million years, through a process known as convection, where it is released as light and heat (Hufbauer 1991). Figure 2.1 shows the internal structure of the sun: the radiative surface of the sun, or photosphere, is the surface that emits solar radiation to space and has an average temperature of about 5,777 K. Localized cool areas called sunspots occur in the photosphere. The chromosphere (around 10,000 K) is the region where solar flares composed of gas, electrons, and radiation erupts. The corona forms the outer atmosphere of the sun from which solar wind flows (Mullan 2009).

Fig. 2.1 Structure of the
sun (*Source* http://www.
solarviews.com/eng/sun.htm)
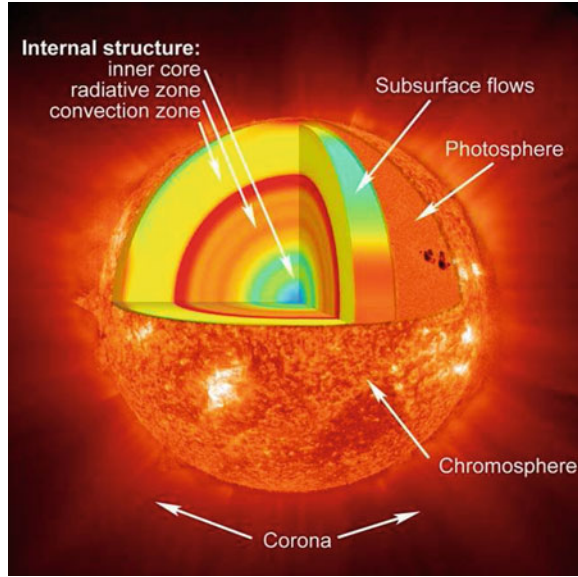
2.1 Irradiance and the Electromagnetic Spectrum

In first approximation, the sun can be considered to be a black-body emitter.
Figure 2.2 shows a comparison between the solar spectral irradiance incident at the
top of the Earth's atmosphere and the spectral irradiance of a black-body source at a
temperature of 5,777 K.

By and large, the spectra are similar. One noteworthy point is that the solar spec-
trum is interspersed with atomic absorption lines from the tenuous layers above the
photosphere.

Valuable estimates of the solar irradiance are obtained from Plank's Law:

$$\rho_T(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda k\theta}} - 1} \tag{2.1}$$

where

ρ_T = energy per unit time per unit emitter surface area per unit solid angle per
unit wavelength (J/m^3−s),

h = Plank's constant (Js),

c = velocity of light in vacuum (m/s),

k = Boltzmann's constant (J/K),

θ = temperature of the object radiating energy (K), and

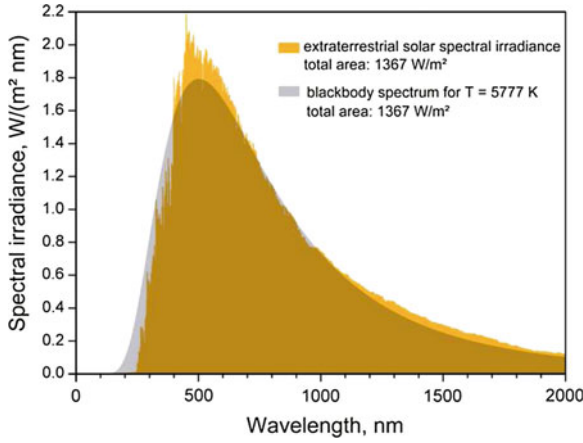λ = wavelength of the radiation (m).

Fig. 2.2 Spectral irradiance of the sun (*Source* http://commons.wikimedia.org/wiki/File: EffectiveTemperature_300dpi_e.png)

The wavelength of radiation λ_{max} at which peak emission occurs from a black-body is dependent upon the temperature of the object and can be calculated using the Wien's-Displacement Law derived from Plank's energy spectrum,

$$\lambda_{max}\theta = 0.2014\frac{hc}{k} = 2.898 \times 10^{-3} \text{ mK}. \tag{2.2}$$

Taking the average surface temperature of the sun to be 5,777 K, we compute $\lambda_{max} = 501$ nm, which is in the yellow-green region of the visible spectrum.

The irradiance E (W/m^2) or total amount of energy per second emitted per unit area of the black-body at temperature θ can be evaluated from Stefan-Boltzmann's Law:

$$E_o = \sigma\theta^4 \tag{2.3}$$

where

E_o = surface irradiance of the object (W/m^2),
$\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4 is the Stefan-Boltzmann constant.

For the sun, we compute $E_\odot = 6.33 \times 10^7$ W/m^2. Over the whole surface area of the sun, $A_\odot$, the amount of radiated power per second called the solar luminosity $L_\odot$ is:

$$L_\odot = E_\odot \times A_\odot = E_\odot \times 4\pi R_\odot^2 = 3.85 \times 10^{26} \text{ W} \tag{2.4}$$

where

$R_\odot$ = radius of the sun (m).

2.2 Theoretical Estimation of the Solar Constant

Solar radiation travelling through a near vacuum and reaching the top of earth's atmosphere is not subject to appreciable scattering or absorption on its path. We define the solar constant $I_\odot$, as the amount of power carried by incoming solar radiation (measured at the outer surface of the earth's atmosphere) per unit surface area perpendicular to the sun rays (Stickler 2003).

$$I_\odot = E_\odot \frac{A_\odot}{A_{sphere}} = E_\odot \frac{4\pi R_\odot^2}{4\pi d_{E-S}^2} \qquad (2.5)$$

$$I_\odot = 1367 \text{ W/m}^2$$

where

    d_{E-S} = earth-sun's distance—1 AU (m),[1]
    $I_\odot$ = irradiance at a distance of 1 AU (W/m^2),
    A_{sphere} = surface area of a sphere of radius 1 AU.

The radiant power available at the surface of the Earth's crust is lower than the solar constant due to a variety of factors, the main ones being (Batey 1998):

- reflection from clouds—cloud cover is one of the main factors blocking the rays of the sun and the amount of radiation reaching the earth on a cloudy day is diminished drastically as compared to a sunny day.
- atmospheric absorption—atmospheric aerosols (ozone, dust layer, air molecules, water vapour etc.) absorb selectively parts of the solar spectrum. A beneficial aspect of this effect is that it prevents destructive ultra-violet rays from damaging our health.
- cosine effect—at high latitudes, sunlight is incident on a level ground at large angles of incidence after travelling through a thick layer of atmosphere. where it undergoes considerable scattering and absorption. This accounts for the reduction in the available solar power, particularly in winter even if the receiving surface is perpendicular to sun rays.

On a very clear day, atmospheric absorption and scattering of incident solar energy cause a reduction of the solar input by about 22 % to a maximum of 60 % (US Department of Energy 1978).

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[1] An Astronomical Unit (AU) is approximately the mean distance between the Earth and the Sun and is equal to 149,597,870,000 ± 6 m.

2.3 Astronomy

2.3.1 The Horizontal (alt–az) System

The horizontal coordinate system as a celestial coordinate system is most immediately related to the observer's impression of being on a flat plane (local horizon) and at the centre of a vast hemisphere across which heavenly bodies move (cf. Fig. 2.3). An observer in the southern hemisphere can define the point directly opposite to the direction in which a plumb-line will hang as the zenith (Bhatnagar and Livingston 2005). There are two coordinates that specify the position of an object in this system:

1. Altitude angle (Alt) or elevation is the angle between an imaginary line from the observer to the sun and the local horizontal plane (shown in 'green' in Fig. 2.3).

   - when the sun is "above the horizon": $0° \leq \text{Alt} \leq 90°$
   - when the sun is "below the horizon": $-90° \leq \text{Alt} \leq 0°$

2. Azimuth angle (Az) is the angle measured clockwise between the northern direction and the projection on horizontal ground of the line of sight to the sun (shown in 'red' in Fig. 2.3). The azimuth ranges from 0 to 360°.

The advantage of the alt–az system is its simplicity to take measurements as only one reference point (North) is needed. However, the main disadvantage is that this system is purely local and observers at different locations on the earth will measure different altitudes and azimuths for the same celestial object even though the measurements are made at the same time.
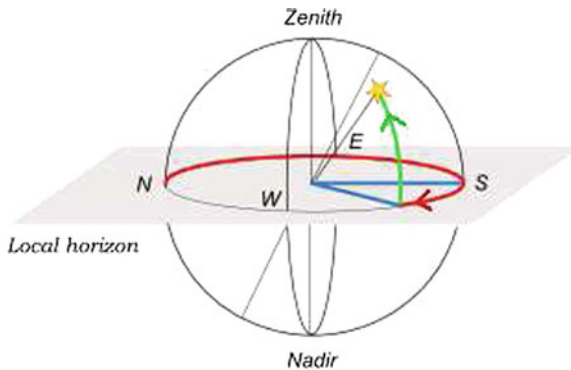

Fig. 2.3 Horizontal coordinate system (*Source* http://en.wikipedia.org/wiki/File:Horizontal_coordinate_system_2.png)

2.3.2 The Equatorial System

The equatorial coordinate system is used to illustrate the motion of heavenly stars on the celestial sphere—an imaginary sphere of radius equal to the distance of stars so that they appear to be lying on its surface. The projection of the earth's equator onto the celestial sphere is called the celestial equator. Similarly, projecting the geographic poles onto the celestial sphere defines the North (NCP in Fig. 2.4) and South (SCP in Fig. 2.4) celestial poles which maintain a relatively fixed direction (in the lifetime of a person) with respect to the distant stars.

Over a much longer term, the earth's rotation axis precesses about the ecliptic North Pole with a period of 25800 years. As a consequence, the North celestial Pole presently pointing towards Polaris will point towards Vega after half the precession period and back towards Polaris after one precession period. Owing to the daily rotation of the earth, the Greenwich Meridian sweeps across the celestial sphere and thus cannot be used as reference for locating stars. Instead, the meridian (longitude) of the vernal equinox is used as the zero celestial meridian. Equinoxes occur twice yearly and correspond to positions of the sun lying in the plane of the celestial equator. The first of the equinoxes, the vernal equinox occurs around March 21 with the sun oriented towards Pisces constellation. Due to the slow precession of the equinoxes, there is a small westward deviation in the direction of the vernal equinox by 50 arc seconds yearly.

The equatorial coordinates of a star are (Bhatnagar and Livingston 2005):

1. Declination (δ) is the angular distance of the sun north or south of the earth's equator. It is analogous to the latitude on planet earth, extrapolated to the celestial sphere. The earth's equator is tilted $23°27'$ with respect to the plane of the earth's orbit around the sun, so at various times during the year, as the earth orbits the sun, declination varies from $+23°27'$ (north) to $-23°27'$ (south). This change in the

Fig. 2.4 Equatorial coordinate system (*Source* http://www.vikdhillon.staff.shef.ac.uk/teaching/phy105/celsphere/equatorial.gif)
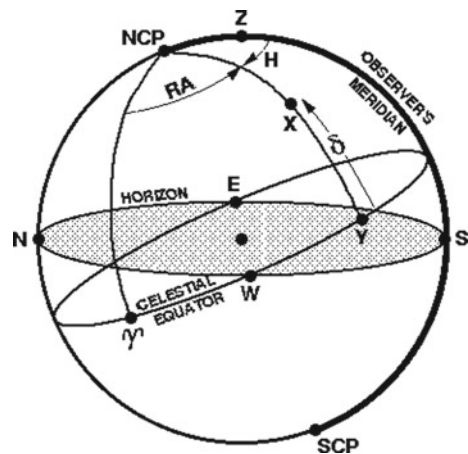
value of declination is responsible for seasonal changes. For more information, please refer to pp.73 in *Astronomy: Principles and Practice* by Roy and Clarke (2003).

2. Right Ascension (*RA*) is measured in hours, minutes and seconds east from the meridian of the vernal equinox (zero meridian) to the star's meridian or hour circle. The hour circle is the great circle that passes through the poles and the stars, that is, right ascension is the time interval between the most recent overhead passage of the meridian of the vernal equinox and the overhead passage of the hour circle. As declination is analogous to latitude on the earth, so is right ascension to longitudes. Alternatively, hour angle can be used in place of *RA*. The hour angle (*H*) indicates the time elapsed since the star transited across the local meridian. Although it is calculated from measurements of time, it may be expressed in angular units (Roy and Clarke 2003).

Unlike the horizontal system, equatorial coordinates do not depend on the observer's location. As a matter of fact, only one pair of coordinates is required for an object at all times.

2.4 The Sun's Position

The change in coordinates of the sun is brought about by a plethora of factors. The first and most obvious motion of the sun is the daily rotation about its north–south axis. The second is a seasonal north–south motion of $\pm 23°27'$ away from the equator. The third motion is a subtle change in the sun's noontime position, brought on mostly by the earth's axial tilt, but with a small additional component produced by the earth's non-circular (elliptical) orbit around the sun.

Since we are to harvest the energy from the sun, it is imperative to know the sun's position at any time. Our formulae should comprise of all the three types of behaviour of the sun. To do so, we will first compute the Julian day number from J2000.0 epoch,

$$JD = 367 \times \text{INT} \left[\frac{7 \times \text{INT} \left[\frac{(M+9)}{12} \right]}{4} \right] + \text{INT} \left[275 \times \frac{M}{9} \right] + D - 730530 + \frac{UT}{24}$$

(2.6)

where,
D is the calendar date,
M is the month,
Y is the year and
UT is the Universal Time in hours only, i.e. the local time less 4 h if daylight saving time (DST) is not applicable and less 5 h if DST is on. INT[] is a function that discards the fractional part and returns the integer part of another function. The following parameters may be calculated directly from the Julian date.

$$\text{Obliquity of eliptic, } OE = 23.4393 - 3.563 \times 10^{-7} \times JD \qquad (2.7)$$

$$\text{Argument of perihelion, } w = 282.9404 + 4.70935 \times 10^{-5} \times JD \qquad (2.8)$$

$$\text{Mean anomaly, } g = 356.0470 + 0.9856002585 \times JD \qquad (2.9)$$

$$\text{Eccentricity, } e = 0.016709 - 1.151 \times 10^{-9} \times JD \qquad (2.10)$$

We can henceforth formulate the eccentric anomaly in degrees as:

$$E = g + e \times \left(\frac{180}{\pi} \right) \times \sin(g) \times (1 + e \times \cos(g)). \qquad (2.11)$$

Next, we will give the expression for the true anomaly, v and the earth-sun's distance, d.

$$d \times \sin(v) = \sqrt{1 - e^2} \times \sin(E) \text{ and} \qquad (2.12)$$

$$d \times \cos(v) = \cos(E) - e. \qquad (2.13)$$

The sun's true longitude, L can now be computed:

$$L = v + w. \qquad (2.14)$$

Since the sun is always at the ecliptic (or extremely close to it), we can use simplified formulae to convert L (the sun's ecliptic longitude) to equatorial coordinate systems:

$$\sin(\delta) = \sin(OE) \times \sin(L) \qquad (2.15)$$

$$\tan(RA) = \frac{\sin(L) \times \cos(OE)}{\cos(L)}. \qquad (2.16)$$

In earth's frame, the alt–az system is easier to implement. We make use of the following relations to compute alt and az from RA and δ. Since on earth, it is easier to work with azimuth and elevation of an object, we need to compute the azimuth and elevation but first we must compute the Local Sidereal Time (LST) of the place and time in question in degrees,

$$LST = 98.9818 + 0.985647352 \times JD + UT \times 15 + lon. \qquad (2.17)$$

where lon is the longitude of the observer.
We compute the Hour Angle (H),

$$H = LST - RA. \qquad (2.18)$$

Now with all the data available, we can compute Alt and Az by applying the cosine formula for spherical geometry:

$$Alt = \sin^{-1}\left[\sin\left(lat\right) \times \sin\left(\delta\right) + \cos\left(lat\right) \times \cos\left(H\right)\right] \tag{2.19}$$

$$Az = \tan^{-1}\left[\frac{\sin\left(H\right)}{\cos\left(H\right) \times \sin\left(lat\right) \times \tan\left(\delta\right) \times \cos\left(lat\right)}\right] + 180 \tag{2.20}$$

where lat is the latitude at the observer's location. To compute when a star rises, we must first calculate when it passes the meridian and the hour angle of rise. Apart from the problem of calculating the sun's position in space relative to earth, one must also calculate the relative motion of the sun at each point on the earth's surface.

To find the meridian time, we compute the Local Sidereal Time at 0 h local time as outlined above and we name that quantity LST0. The Meridian Time, MT, will now be:

$$MT = RA - LST0 \quad 0° \leq MT \leq 360°. \tag{2.21}$$

Now, we compute H for rise, and we name that quantity $H0$:

$$\cos\left(H0\right) = \frac{\sin\left(L0\right)\sin\left(lat\right)\sin\left(\delta\right)}{\cos\left(lat\right)\cos\left(\delta\right)} \tag{2.22}$$

where $L0$ is the selected altitude selected to represent sunrise. The sun would normally appear to be exactly on the horizon when its altitude is zero, except that the atmosphere refracts sunlight when it's low in the sky, and the observer's elevation relative to surrounding terrain also impacts the apparent time of sunrise and sunset. The difference between the time of apparent sunrise or sunset and the time when the sun's altitude is zero is usually on the order of several minutes, so it's necessary to correct for these factors in order to obtain an accurate result. For a purely mathematical horizon, $L0 = 0$ and for a physical solution, accounting for refraction on the atmosphere, $L0 = -35/60°$. The effects of the atmosphere vary with atmospheric pressure, humidity, temperature etc. Errors in sunrise and sunset times can be expected to increase the further away from the equator, because the sun rises and sets at a very shallow angle. And if we want to compute the rise times for the sun's upper limb to appear grazing the horizon, we set $L0 = -50/60°$.

The rise time of the sun is given by:

$$Sunrise = MT - H0. \tag{2.23}$$

The answer from the above equation will be in degrees and is should be converted to hours.

However, in everyday experience, the sunset time prediction is not 'usually' borne out as the atmosphere is disturbed, because one is almost never standing on a flat plain with barriers on the horizon (Duffet-Smith 1988; Meees 1999).

Table 2.1 Table showing the exact values and the theoretical estimates of the solar position

Time	Theoretical azimuth	Theoretical altitude	Exact azimuth	Exact altitude
08 00	105.4°	32.13°	105.4°	32.01°
09 00	103.0°	45.76°	102.9°	45.67°
10 00	101.7°	59.51°	101.6°	59.41°
11 00	103.6°	73.26°	103.4°	73.16°
12 00	140.2°	86.04°	139.7°	86.01°
13 00	252.9°	78.24°	253.0°	78.36°
14 00	258.1°	64.80°	258.2°	64.68°
15 00	257.6°	50.80°	257.7°	50.91°
16 00	255.5°	37.12°	255.6°	37.23°
17 00	252.6°	23.58°	252.7°	23.70°
08 00	105.4°	32.01°	105.3°	32.01°
09 00	102.9°	45.65°	102.9°	45.55°
10 00	101.6°	59.40°	101.5°	59.30°
11 00	103.4°	73.15°	103.1°	73.06°
12 00	138.6°	86.01°	136.9°	85.97°
13 00	253.0°	78.36°	253.1°	78.48°
14 00	258.2°	64.91°	258.4°	64.79°
15 00	257.7°	50.91°	257.8°	51.02°
16 00	255.6°	37.22°	255.7°	37.33°
17 00	252.7°	23.68°	252.7°	23.80°

2.4.1 Validity of Sun's Algorithms

The final equations for the location of the sun were checked on 27 and 28 December 2008 by contrasting the calculated values from the derived equations in Sect. 2.4 to the exact values obtained from National Renewable Energy Laboratory's Solar Position Algorithm available at http://www.nrel.gov/midc/solpos/spa.html in Table 2.1. The latitude and longitude of observation were locked at $20°17'$ S and $57°33'$ E respectively. The complete script with all the relevant comments are shown in Appendix A.

2.4.1.1 Errors in Experiment

From the measurements, we can infer that the calculated solar position reflected the actual solar position most of the time (maximum error of 0.5 %). The algorithm could be enhanced to cater for errors at instants when the sun is in its maximum phase but increasing accuracy is both computer expensive and time-consuming, so a compromise between speed, resource and complexity yields our solar formulae.

2.5 Chapter Summary

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