Karl Dorogy(20168380)
Douglas Chafee (20144996)
Tom Lin,
Matthew Kurpac

Group 48 Draft #1

Project Summary

- Connect four is a game where color black or red must connect four units in a row by dropping units down different columns; eventually forming a line of length 4 in the finite playing area, whether it be horizontally, vertically, or diagonally.
- The goal of this project is to assess the mechanics of playing connect four and how they change the state of a game.
- The model will represent the current state of a connect four game and how the game's state will change after 1 move, determining whether no one side has won or whether black or red had won in that one move from the current state.
- If the model does not contain any black units or red units, then the current state of the game is at the start with black placing/moving first.

Propositions

$B_{x\gamma}$: This value is true when position (x, y) is a black unit on the board

$R_{x\gamma}$: This value is true when position (x, y) is a red unit on the board

P: This value is true when there are either three black or red units in increments of one with an unoccupied unit on either side

W: This value is true when the game has a winner

$U_{x\gamma}$: This is true when the tile at position (x, y) is occupied by either a red or black unit

L: This is true when its black's turn in the current state of the game

J: This is true when its red's turn in the current state of the game

H_x: This value is true when row x has either four black or red units in increments of one in a row (winning row).

C_γ: This value is true when column y has either four black or red units in increments of one in a column (winning column).

D_i : This value is true and can only be true if x values are 4 or higher and when there are either four black or red units in increments of one in a diagonal where the y increases proportionally or inversely to x.

Z: This value is true when x is 0 (The floor).

Constraints

The game's turn state cannot be both red and blacks turn:

$$\neg (L \wedge J)$$

The game's tile cannot be occupied by both a red and black unit at the same time:

$$\neg (B_{x\gamma} \wedge R_{x\gamma})$$

In order to place a block down there must be a block or the floor underneath that position in that column:

$$(B_{x-1\gamma} \vee R_{x-1\gamma} \vee Z)$$

A game's tile must consist of either a black unit, red unit, or neither:

$$(B_{x\gamma} \vee R_{x\gamma} \vee (\neg B_{x\gamma} \wedge \neg R_{x\gamma}))$$

A row proposition holds only when there are either four black or red units in increments of one in a row:

$$H_i \leftrightarrow ((B_{x\gamma} \wedge B_{x\gamma+1} \wedge B_{x\gamma+2} \wedge B_{x\gamma+3}) \vee (R_{x\gamma} \wedge R_{x\gamma+1} \wedge R_{x\gamma+2} \wedge R_{x\gamma+3}))$$

A column proposition holds only when there are either four black or red units in increments of one in a column:

$$Ci \leftrightarrow ((B_{x\gamma} \wedge B_{x+1\gamma} \wedge B_{x+2\gamma} \wedge B_{x+3\gamma}) \vee (R_{x\gamma} \wedge R_{x+1\gamma} \wedge R_{x+2\gamma} \wedge R_{x+3\gamma}))$$

A diagonal proposition holds only when there are either four black or red units in increments of one in a diagonal where the y increases proportionally or inversely to x.

$$D_i \leftrightarrow (((B_{x\gamma} \wedge B_{(x+1)(\gamma+1)} \wedge B_{(x+2)(\gamma+2)} \wedge B_{(x+3))(\gamma+3)}) \vee (B_{x\gamma} \wedge B_{(x+1)(\gamma-1)} \wedge B_{(x+2)(\gamma-2)} \wedge B_{(x+3))(\gamma-3)})) \vee ((R_{x\gamma} \wedge R_{(x+1)(\gamma+1)} \wedge R_{(x+2)(\gamma+2)} \wedge R_{(x+3))(\gamma+3)}) \vee (R_{x\gamma} \wedge R_{(x+1)(\gamma-1)} \wedge R_{(x+2)(\gamma-2)} \wedge R_{(x+3))(\gamma-3)})))$$

- A winning proposition holds true if either black or red has the option to win in one turn, where there is either three black or red units in increments of one with an unoccupied unit on either side

$$P \leftrightarrow (((B \wedge B \wedge B \wedge \neg U) \vee (\neg U \wedge B \wedge B \wedge B)) \vee ((R \wedge R \wedge R \wedge \neg U) \vee (\neg U \wedge R \wedge R \wedge R)))$$

- A won proposition holds true if either a row proposition, column proposition, or a diagonal proposition holds true.

$$W \leftrightarrow (H_x \vee C_\gamma \vee D_{x\gamma}$$

Model Exploration

Propositional Jape Proofs:

Rows: $H_i \leftrightarrow ((B_{x\gamma} \wedge B_{x\gamma+1} \wedge B_{x\gamma+2} \wedge B_{x\gamma+3}) \vee (R_{x\gamma} \wedge R_{x\gamma+1} \wedge R_{x\gamma+2} \wedge R_{x\gamma+3}))$

$H_x \rightarrow (((B_{x,1} \wedge B_{x,2} \wedge B_{x,3} \wedge B_{x,4}) \vee (B_{x,2} \wedge B_{x,3} \wedge B_{x,4} \wedge B_{x,5}) \vee (B_{x,3} \wedge B_{x,4} \wedge B_{x,5} \wedge B_{x,6}) \vee (B_{x,4} \wedge B_{x,5} \wedge B_{x,6} \wedge B_{x,7})) \vee ((R_{x,1} \wedge R_{x,2} \wedge R_{x,3} \wedge R_{x,4}) \vee (R_{x,2} \wedge R_{x,3} \wedge R_{x,4} \wedge R_{x,5}) \vee (R_{x,3} \wedge R_{x,4} \wedge R_{x,5} \wedge R_{x,6}) \vee (R_{x,4} \wedge R_{x,5} \wedge R_{x,6} \wedge R_{x,7})))$

$(((B_{x,1} \wedge B_{x,2} \wedge B_{x,3} \wedge B_{x,4}) \vee (B_{x,2} \wedge B_{x,3} \wedge B_{x,4} \wedge B_{x,5}) \vee (B_{x,3} \wedge B_{x,4} \wedge B_{x,5} \wedge B_{x,6}) \vee (B_{x,4} \wedge B_{x,5} \wedge B_{x,6} \wedge B_{x,7})) \vee ((R_{x,1} \wedge R_{x,2} \wedge R_{x,3} \wedge R_{x,4}) \vee (R_{x,2} \wedge R_{x,3} \wedge R_{x,4} \wedge R_{x,5}) \vee (R_{x,3} \wedge R_{x,4} \wedge R_{x,5} \wedge R_{x,6}) \vee (R_{x,4} \wedge R_{x,5} \wedge R_{x,6} \wedge R_{x,7}))) \rightarrow H_x$

Columns: $C_i \leftrightarrow ((B_{x\gamma} \wedge B_{x+1\gamma} \wedge B_{x+2\gamma} \wedge B_{x+3\gamma}) \vee (R_{x\gamma} \wedge R_{x+1\gamma} \wedge R_{x+2\gamma} \wedge R_{x+3\gamma}))$

$C_\gamma \rightarrow (((B_{1,\gamma} \wedge B_{2,\gamma} \wedge B_{3,\gamma} \wedge B_{4,\gamma}) \vee (B_{2,\gamma} \wedge B_{3,\gamma} \wedge B_{4,\gamma} \wedge B_{5,\gamma}) \vee (B_{3,\gamma} \wedge B_{4,\gamma} \wedge B_{5,\gamma} \wedge B_{6,\gamma})) \vee ((R_{1,\gamma} \wedge R_{2,\gamma} \wedge R_{3,\gamma} \wedge R_{4,\gamma}) \vee (R_{2,\gamma} \wedge R_{3,\gamma} \wedge R_{4,\gamma} \wedge R_{5,\gamma}) \vee (R_{3,\gamma} \wedge R_{4,\gamma} \wedge R_{5,\gamma} \wedge R_{6,\gamma})))$

$(((B_{1,\gamma} \wedge B_{2,\gamma} \wedge B_{3,\gamma} \wedge B_{4,\gamma}) \vee (B_{2,\gamma} \wedge B_{3,\gamma} \wedge B_{4,\gamma} \wedge B_{5,\gamma}) \vee (B_{3,\gamma} \wedge B_{4,\gamma} \wedge B_{5,\gamma} \wedge B_{6,\gamma})) \vee ((R_{1,\gamma} \wedge R_{2,\gamma} \wedge R_{3,\gamma} \wedge R_{4,\gamma}) \vee (R_{2,\gamma} \wedge R_{3,\gamma} \wedge R_{4,\gamma} \wedge R_{5,\gamma}) \vee (R_{3,\gamma} \wedge R_{4,\gamma} \wedge R_{5,\gamma} \wedge R_{6,\gamma}))) \rightarrow C_\gamma$

Diaganols: $D_i \leftrightarrow (((B_{x\gamma} \wedge B_{(x+1)(\gamma+1)} \wedge B_{(x+2)(\gamma+2)} \wedge B_{(x+3))(\gamma+3)}) \vee (B_{x\gamma} \wedge B_{(x+1)(\gamma-1)} \wedge B_{(x+2)(\gamma-2)} \wedge B_{(x+3))(\gamma-3)})) \vee ((R_{x\gamma} \wedge R_{(x+1)(\gamma+1)} \wedge R_{(x+2)(\gamma+2)} \wedge R_{(x+3))(\gamma+3)}) \vee (R_{x\gamma} \wedge R_{(x+1)(\gamma-1)} \wedge R_{(x+2)(\gamma-2)} \wedge R_{(x+3))(\gamma-3)})))$

$D_4 \rightarrow (((B_{1,1} \wedge B_{2,2} \wedge B_{3,3} \wedge B_{4,4}) \vee (B_{2,2} \wedge B_{3,3} \wedge B_{4,4} \wedge B_{5,5}) \vee (B_{3,3} \wedge B_{4,4} \wedge B_{5,5} \wedge B_{6,6})) \vee ((B_{1,7} \wedge B_{2,6} \wedge B_{3,5} \wedge B_{4,4}) \vee (B_{2,6} \wedge B_{3,5} \wedge B_{4,4} \wedge B_{5,3}) \vee (B_{3,5} \wedge B_{4,4} \wedge B_{5,3} \wedge B_{6,2})) \vee ((R_{1,1} \wedge R_{2,2} \wedge R_{3,3} \wedge R_{4,4}) \vee (R_{2,2} \wedge R_{3,3} \wedge R_{4,4} \wedge R_{5,5}) \vee (R_{3,3} \wedge R_{4,4} \wedge R_{5,5} \wedge R_{6,6})) \vee ((R_{1,7} \wedge R_{2,6} \wedge R_{3,5} \wedge R_{4,4}) \vee (R_{2,6} \wedge R_{3,5} \wedge R_{4,4} \wedge R_{5,3}) \vee (R_{3,5} \wedge R_{4,4} \wedge R_{5,3} \wedge R_{6,2}))$

$(((B_{1,1} \wedge B_{2,2} \wedge B_{3,3} \wedge B_{4,4}) \vee (B_{2,2} \wedge B_{3,3} \wedge B_{4,4} \wedge B_{5,5}) \vee (B_{3,3} \wedge B_{4,4} \wedge B_{5,5} \wedge B_{6,6})) \vee ((B_{1,7} \wedge B_{2,6} \wedge B_{3,5} \wedge B_{4,4}) \vee (B_{2,6} \wedge B_{3,5} \wedge B_{4,4} \wedge B_{5,3}) \vee (B_{3,5} \wedge B_{4,4} \wedge B_{5,3} \wedge B_{6,2})) \vee ((R_{1,1} \wedge R_{2,2} \wedge R_{3,3} \wedge R_{4,4}) \vee (R_{2,2} \wedge R_{3,3} \wedge R_{4,4} \wedge R_{5,5}) \vee (R_{3,3} \wedge R_{4,4} \wedge R_{5,5} \wedge R_{6,6})) \vee ((R_{1,7} \wedge R_{2,6} \wedge R_{3,5} \wedge R_{4,4}) \vee (R_{2,6} \wedge R_{3,5} \wedge R_{4,4} \wedge R_{5,3}) \vee (R_{3,5} \wedge R_{4,4} \wedge R_{5,3} \wedge R_{6,2})) \rightarrow D_4$

First-Order Extension

Describe how you might extend your model to a predicate logic setting, including how both the propositions and constraints would be updated. There is no need to implement this extension!

Row:

$H_i, \forall k.(H_k \rightarrow (B_{k,\gamma} \wedge B_{k,(\gamma+1)} \wedge B_{k,(\gamma+2)} \wedge B_{k,(\gamma+3)})) \vdash (B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$

Solution:

1. H_i **(premise)**
2. $\forall k.(H_k \rightarrow (B_{k,\gamma} \wedge B_{k,(\gamma+1)} \wedge B_{k,(\gamma+2)} \wedge B_{k,(\gamma+3)}))$ **(premise)**
3. actual(i) **(copy 1)**
4. $H_i \rightarrow (B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$ **($\forall$elimination 2, 3)**
5. $(B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$ **($\rightarrow$ elimination 4, 1)**

Column:

$C_i, \forall k.(C_k \rightarrow (B_x,k \wedge B_{x+1},k \wedge B_{x+2},k \wedge B_{x+3},k)) \vdash (B_i,k \wedge B_{i+1},k \wedge B_{i+2},k \wedge B_{i+3},k)$

Solution:

1. C_i **(premise)**
2. $\forall k.(C_k \rightarrow (B_x,k \wedge B_{x+1},k \wedge B_{x+2},k \wedge B_{x+3},k))$ **(premise)**
3. actual(i) **(copy 1)**
4. $C_i \rightarrow (B_i,k \wedge B_{i+1},k \wedge B_{i+2},k \wedge B_{i+3},k$ **($\forall$elimination 2, 3)**
5. $(B_i,k \wedge B_{i+1},k \wedge B_{i+2},k \wedge B_{i+3},k)$ **($\rightarrow$ elimination 4, 1)**