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Group 48 Draft #1

Project Summary

- Connect four is a game where color black or red must connect four units in a row by dropping units down different columns; eventually forming a line of length 4 in the finite playing area, whether it be horizontally, vertically, or diagonally.
- The goal of this project is to assess the mechanics of playing connect four and how they change the state of a game.
- The model will represent the current state of a connect four game and how the game's state will change after 1 move, determining whether no one side has won or whether black or red had won in that one move from the current state.
- If the model does not contain any black units or red units, then the current state of the game is at the start with black placing/moving first.

Propositions

B_{xy}: This value is true when position (x, y) is a black unit on the board

R_{xy}: This value is true when position (x, y) is a red unit on the board

P: This value is true when there are either three black or red units in increments of one with an unoccupied unit on either side

W: This value is true when the game has a winner

U_{xy}: This is true when the tile at position (x, y) is occupied by either a red or black unit

L: This is true when its black's turn in the current state of the game

J: This is true when its red's turn in the current state of the game

H_x: This value is true when row x has either four black or red units in increments of one in a row (winning row).

C_y: This value is true when column y has either four black or red units in increments of one in a column (winning column).

D_i : This value is true and can only be true if x values are 4 or higher and when there are either four black or red units in increments of one in a diagonal where the y increases proportionally or inversely to x.

Z: This value is true when x is 0 (The floor).

Constraints

The game's turn state cannot be both red and blacks turn:

$$\neg (L \wedge J)$$

The game's tile cannot be occupied by both a red and black unit at the same time:

$$\neg (B_{x\gamma} \wedge R_{x\gamma})$$

In order to place a block down there must be a block or the floor underneath that position in that column:

$$(B_{x-1\gamma} \vee R_{x-1\gamma} \vee Z)$$

A game's tile must consist of either a black unit, red unit, or neither:

$$(B_{x\gamma} \vee R_{x\gamma} \vee (\neg B_{x\gamma} \wedge \neg R_{x\gamma}))$$

A row proposition holds only when there are either four black or red units in increments of one in a row:

$$H_i \leftrightarrow ((B_{x\gamma} \wedge B_{x\gamma+1} \wedge B_{x\gamma+2} \wedge B_{x\gamma+3}) \vee (R_{x\gamma} \wedge R_{x\gamma+1} \wedge R_{x\gamma+2} \wedge R_{x\gamma+3}))$$

A column proposition holds only when there are either four black or red units in increments of one in a column:

$$Ci \leftrightarrow ((B_{x\gamma} \wedge B_{x+1\gamma} \wedge B_{x+2\gamma} \wedge B_{x+3\gamma}) \vee (R_{x\gamma} \wedge R_{x+1\gamma} \wedge R_{x+2\gamma} \wedge R_{x+3\gamma}))$$

A diagonal proposition holds only when there are either four black or red units in increments of one in a diagonal where the y increases proportionally or inversely to x.

$$D_i \leftrightarrow (((B_{x\gamma} \wedge B_{(x+1)(\gamma+1)} \wedge B_{(x+2)(\gamma+2)} \wedge B_{(x+3))(\gamma+3)}) \vee (B_{x\gamma} \wedge B_{(x+1)(\gamma-1)} \wedge B_{(x+2)(\gamma-2)} \wedge B_{(x+3))(\gamma-3)})) \vee ((R_{x\gamma} \wedge R_{(x+1)(\gamma+1)} \wedge R_{(x+2)(\gamma+2)} \wedge R_{(x+3))(\gamma+3)}) \vee (R_{x\gamma} \wedge R_{(x+1)(\gamma-1)} \wedge R_{(x+2)(\gamma-2)} \wedge R_{(x+3))(\gamma-3)})))$$

- A winning proposition holds true if either black or red has the option to win in one turn, where there is either three black or red units in increments of one with an unoccupied unit on either side

$$Pi \leftrightarrow (((Bi \wedge Bi \wedge Bi \wedge \neg Ui) \vee (\neg Ui \wedge Bi \wedge Bi \wedge Bi)) \vee ((Ri \wedge Ri \wedge Ri \wedge \neg Ui) \vee (\neg Ui \wedge Ri \wedge Ri \wedge Ri)))$$

- A won proposition holds true if either a row proposition, column proposition, or a diagonal proposition holds true.

$$W \leftrightarrow (H_x \vee C_\gamma \vee D_{x\gamma}$$

Model Exploration

Propositional Jape Proofs:

Rows: $H_i \leftrightarrow ((B_{x\gamma} \land B_{x\gamma+1} \land B_{x\gamma+2} \land B_{x\gamma+3}) \lor (R_{x\gamma} \land R_{x\gamma+1} \land R_{x\gamma+2} \land R_{x\gamma+3}))$

$H_x \rightarrow (((B_{x,1} \land B_{x,2} \land B_{x,3} \land B_{x,4}) \lor (B_{x,2} \land B_{x,3} \land B_{x,4} \land B_{x,5}) \lor (B_{x,3} \land B_{x,4} \land B_{x,5} \land B_{x,6}) \lor (B_{x,4} \land B_{x,5} \land B_{x,6} \land B_{x,7})) \lor ((R_{x,1} \land R_{x,2} \land R_{x,3} \land R_{x,4}) \lor (R_{x,2} \land R_{x,3} \land R_{x,4} \land R_{x,5}) \lor (R_{x,3} \land R_{x,4} \land R_{x,5} \land R_{x,6}) \lor (R_{x,4} \land R_{x,5} \land R_{x,6} \land R_{x,7})))$

$(((B_{x,1} \land B_{x,2} \land B_{x,3} \land B_{x,4}) \lor (B_{x,2} \land B_{x,3} \land B_{x,4} \land B_{x,5}) \lor (B_{x,3} \land B_{x,4} \land B_{x,5} \land B_{x,6}) \lor (B_{x,4} \land B_{x,5} \land B_{x,6} \land B_{x,7})) \lor ((R_{x,1} \land R_{x,2} \land R_{x,3} \land R_{x,4}) \lor (R_{x,2} \land R_{x,3} \land R_{x,4} \land R_{x,5}) \lor (R_{x,3} \land R_{x,4} \land R_{x,5} \land R_{x,6}) \lor (R_{x,4} \land R_{x,5} \land R_{x,6} \land R_{x,7}))) \rightarrow H_x$

Columns: $C_i \leftrightarrow ((B_{x\gamma} \land B_{x+1\gamma} \land B_{x+2\gamma} \land B_{x+3\gamma}) \lor (R_{x\gamma} \land R_{x+1\gamma} \land R_{x+2\gamma} \land R_{x+3\gamma}))$

$C_\gamma \rightarrow (((B_{1,\gamma} \land B_{2,\gamma} \land B_{3,\gamma} \land B_{4,\gamma}) \lor (B_{2,\gamma} \land B_{3,\gamma} \land B_{4,\gamma} \land B_{5,\gamma}) \lor (B_{3,\gamma} \land B_{4,\gamma} \land B_{5,\gamma} \land B_{6,\gamma})) \lor ((R_{1,\gamma} \land R_{2,\gamma} \land R_{3,\gamma} \land R_{4,\gamma}) \lor (R_{2,\gamma} \land R_{3,\gamma} \land R_{4,\gamma} \land R_{5,\gamma}) \lor (R_{3,\gamma} \land R_{4,\gamma} \land R_{5,\gamma} \land R_{6,\gamma})))$

$(((B_{1,\gamma} \land B_{2,\gamma} \land B_{3,\gamma} \land B_{4,\gamma}) \lor (B_{2,\gamma} \land B_{3,\gamma} \land B_{4,\gamma} \land B_{5,\gamma}) \lor (B_{3,\gamma} \land B_{4,\gamma} \land B_{5,\gamma} \land B_{6,\gamma})) \lor ((R_{1,\gamma} \land R_{2,\gamma} \land R_{3,\gamma} \land R_{4,\gamma}) \lor (R_{2,\gamma} \land R_{3,\gamma} \land R_{4,\gamma} \land R_{5,\gamma}) \lor (R_{3,\gamma} \land R_{4,\gamma} \land R_{5,\gamma} \land R_{6,\gamma}))) \rightarrow C_\gamma$

Diagonals: $D_i \leftrightarrow (((B_{x\gamma} \land B_{(x+1)(\gamma+1)} \land B_{(x+2)(\gamma+2)} \land B_{(x+3))(\gamma+3)}) \lor (B_{x\gamma} \land B_{(x+1)(\gamma-1)} \land B_{(x+2)(\gamma-2)} \land B_{(x+3))(\gamma-3)})) \lor ((R_{x\gamma} \land R_{(x+1)(\gamma+1)} \land R_{(x+2)(\gamma+2)} \land R_{(x+3))(\gamma+3)}) \lor (R_{x\gamma} \land R_{(x+1)(\gamma-1)} \land R_{(x+2)(\gamma-2)} \land R_{(x+3))(\gamma-3)})))$

$D_4 \rightarrow (((B_{1,1} \land B_{2,2} \land B_{3,3} \land B_{4,4}) \lor (B_{2,2} \land B_{3,3} \land B_{4,4} \land B_{5,5}) \lor (B_{3,3} \land B_{4,4} \land B_{5,5} \land B_{6,6})) \lor ((B_{1,7} \land B_{2,6} \land B_{3,5} \land B_{4,4}) \lor (B_{2,6} \land B_{3,5} \land B_{4,4} \land B_{5,3}) \lor (B_{3,5} \land B_{4,4} \land B_{5,3} \land B_{6,2})) \lor ((R_{1,1} \land R_{2,2} \land R_{3,3} \land R_{4,4}) \lor (R_{2,2} \land R_{3,3} \land R_{4,4} \land R_{5,5}) \lor (R_{3,3} \land R_{4,4} \land R_{5,5} \land R_{6,6})) \lor ((R_{1,7} \land R_{2,6} \land R_{3,5} \land R_{4,4}) \lor (R_{2,6} \land R_{3,5} \land R_{4,4} \land R_{5,3}) \lor (R_{3,5} \land R_{4,4} \land R_{5,3} \land R_{6,2}))$

$(((B_{1,1} \land B_{2,2} \land B_{3,3} \land B_{4,4}) \lor (B_{2,2} \land B_{3,3} \land B_{4,4} \land B_{5,5}) \lor (B_{3,3} \land B_{4,4} \land B_{5,5} \land B_{6,6})) \lor ((B_{1,7} \land B_{2,6} \land B_{3,5} \land B_{4,4}) \lor (B_{2,6} \land B_{3,5} \land B_{4,4} \land B_{5,3}) \lor (B_{3,5} \land B_{4,4} \land B_{5,3} \land B_{6,2})) \lor ((R_{1,1} \land R_{2,2} \land R_{3,3} \land R_{4,4}) \lor (R_{2,2} \land R_{3,3} \land R_{4,4} \land R_{5,5}) \lor (R_{3,3} \land R_{4,4} \land R_{5,5} \land R_{6,6})) \lor ((R_{1,7} \land R_{2,6} \land R_{3,5} \land R_{4,4}) \lor (R_{2,6} \land R_{3,5} \land R_{4,4} \land R_{5,3}) \lor (R_{3,5} \land R_{4,4} \land R_{5,3} \land R_{6,2})) \rightarrow D_4$

First-Order Extension

We could explore our model using predicate logic by checking every possible winning situation for a row, column, or diagonal instead of checking every possible row, column, or diagonal individually (our current constraints). Continuing our predicate logic we can then assume there exists a winning row, column, or diagonal, and we would then check for that winning row, column, or diagonal within the scope of all rows, columns, or diagonals. We could also check if there exists a situation in which it's possible for either team to be a single move away from making the current model a winning configuration by taking the same logic used in the constraints for being one away and then check for said logic within the scope of all rows, columns, and diagonals.

Row:

$H_i, \forall k.(H_k \rightarrow (B_{k,\gamma} \wedge B_{k,(\gamma+1)} \wedge B_{k,(\gamma+2)} \wedge B_{k,(\gamma+3)})) \vdash (B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$

Solution:

1. H_i **(premise)**
2. $\forall k.(H_k \rightarrow (B_{k,\gamma} \wedge B_{k,(\gamma+1)} \wedge B_{k,(\gamma+2)} \wedge B_{k,(\gamma+3)}))$ **(premise)**
3. actual(i) **(copy 1)**
4. $H_i \rightarrow (B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$ **($\forall$ elimination 2, 3)**
5. $(B_{k,i} \wedge B_{k,(i+1)} \wedge B_{k,(i+2)} \wedge B_{k,(i+3)})$ **($\rightarrow$ elimination 4, 1)**

Column:

$C_i, \forall k.(C_k \rightarrow (B_{x,k} \wedge B_{x+1,k} \wedge B_{x+2,k} \wedge B_{x+3,k})) \vdash (B_{i,k} \wedge B_{i+1,k} \wedge B_{i+2,k} \wedge B_{i+3,k})$

Solution:

1. C_i **(premise)**
2. $\forall k.(C_k \rightarrow (B_{x,k} \wedge B_{x+1,k} \wedge B_{x+2,k} \wedge B_{x+3,k}))$ **(premise)**
3. actual(i) **(copy 1)**
4. $C_i \rightarrow (B_{i,k} \wedge B_{i+1,k} \wedge B_{i+2,k} \wedge B_{i+3,k}$ **($\forall$ elimination 2, 3)**
5. $(B_{i,k} \wedge B_{i+1,k} \wedge B_{i+2,k} \wedge B_{i+3,k})$ **($\rightarrow$ elimination 4, 1)**

Requested Feedback:

1. **How can the project go about introducing a connect four board and its x and y values to differentiate the different black and red units on the board? So far, the proofs, when written in Python, have no differentiation between the different black/red pieces on the board, so we end up with formulas like B&B&B&B with no positional differences.**
2. **Has our project improved in terms of complexity to the level that allows us to develop sufficient constraints, propositions, and models? So far, the group has moved from determining a winning state of a board to modelling the mechanics of a single game/turn in addition to determining a winning state, but we are unsure how to or if we even should make it more complex beyond what we have currently.**
3. **How should the constraints and propositions look like in Python code in order for the model to be solved by the Python library? So far, we have functions for every possible outcome of our connect four models, but each model only currently holds their corresponding constraints of the program.**