$$
\begin{array}{rcl}
\lfloor X \rceil & := & \{x \mid X(x)\} \\
\lceil X \rceil & := & \lfloor X \rceil \cup k_X \\
\lfloor X \rfloor & := & \lfloor X \rceil \setminus k_X \\
\overline{x} & := & \{e \mid e \notin x\}
\end{array}
$$

$$
\begin{array}{rcl}
\lfloor \neg X \rceil & = & \{x \mid \neg X(x)\} \\
 & = & \overline{\{x \mid X(x)\}} \\
 & = & \overline{\lfloor X \rceil}
\end{array}
$$

$$
\begin{array}{rcl}
\lceil \neg X \rceil & = & \lfloor \neg X \rceil \cup k_X \\
 & = & \{e \mid \neg X(e) \vee e \in k_X\} \\
 & = & \overline{\overline{\{e \mid \neg X(e) \vee e \in k_X\}}} \\
 & = & \overline{\{e \mid \neg(\neg X(e) \vee e \in k_X)\}} \\
 & = & \overline{\{e \mid X(e) \wedge e \notin k_X\}} \\
 & = & \overline{\lfloor X \rceil \setminus k_X} \\
 & = & \overline{\lfloor X \rfloor}
\end{array}
$$

$$
\lfloor \neg X \rfloor = \overline{\lceil X \rceil}
$$

$$
\begin{array}{rcl}
\lceil A \wedge B \rceil & := & \lceil A \rceil \cap \lceil B \rceil \\
\lfloor A \wedge B \rceil & \subseteq & \lceil A \wedge B \rceil \\
 & = & \lceil A \rceil \cap \lceil B \rceil \\
 & = & (\lfloor A \rceil \cup k_A) \cap (\lfloor B \rceil \cup k_B) \\
 & = & (\lfloor A \rceil \cap \lfloor B \rceil) \cup ((\lfloor A \rceil \cap k_B) \cup (k_A \cap \lfloor B \rceil) \cup (k_A \cap k_B)) \\
 & = & \{e \mid e \in \lfloor A \rceil \wedge e \in \lfloor B \rceil\} \cup ((\lfloor A \rceil \cap k_B) \cup (k_A \cap \lfloor B \rceil) \cup (k_A \cap k_B)) \\
 & = & \{e \mid A(e) \wedge B(e)\} \cup ((\lfloor A \rceil \cap k_B) \cup (k_A \cap \lfloor B \rceil) \cup (k_A \cap k_B)) \\
 & = & \lfloor A \wedge B \rceil \cup ((\lfloor A \rceil \cap k_B) \cup (k_A \cap \lfloor B \rceil) \cup (k_A \cap k_B))
\end{array}
$$

$$
\begin{array}{rcl}
\lfloor A \wedge B \rfloor & := & \lfloor A \rfloor \cap \lfloor B \rfloor \\
\lfloor A \wedge B \rceil & \supseteq & \lfloor A \wedge B \rfloor \\
 & = & \lfloor A \rfloor \cap \lfloor B \rfloor \\
 & = & (\lfloor A \rceil \setminus k_A) \cap (\lfloor B \rceil \setminus k_B) \\
 & = & (\lfloor A \rceil \cap \overline{k_A}) \cap (\lfloor B \rceil \cap \overline{k_B}) \\
 & = & (\lfloor A \rceil \cap \lfloor B \rceil) \cap (\overline{k_A} \cap \overline{k_B}) \\
 & = & \{e \mid e \in \lfloor A \rceil \wedge e \in \lfloor B \rceil\} \cap (\overline{k_A} \cap \overline{k_B}) \\
 & = & \{e \mid A(e) \wedge B(e)\} \cap (\overline{k_A} \cap \overline{k_B}) \\
 & = & \lfloor A \wedge B \rceil \cap (\overline{k_A} \cap \overline{k_B})
\end{array}
$$