February 2, 2015

Handout

Ondrej Škopek <oskopek@matfyz.cz>

100 prisoners problem (The Locker Problem) [2]

The director of a prison offers 100 prisoners on death row, which are numbered from 1 to 100, a last chance. In a room there is a cupboard with 100 drawers. The director puts in each drawer the number of exactly one prisoner in random order and closes the drawers afterwards. The prisoners enter the room one after another. Each prisoner may open and look into 50 drawers in any order and the drawers are closed again afterwards. If during this search every prisoner finds his number in one of the drawers, all prisoners are pardoned. If just one prisoner does not find his number, all prisoners have to die. Before the first prisoner enters the room, the prisoners may discuss their strategy, afterwards no communication of any means is possible. What is the best strategy for the prisoners?

Three Prisoners problem (The Monty Hall Problem) [8], [4]

Monty Hall problem: Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, „Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Three Prisoners problem: Three prisoners, A, B and C, are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned, but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the others who is going to be executed. „If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, flip a coin to decide whether to name B or C."

The warden tells A that B is to be executed. Prisoner A is pleased because he believes that his probability of surviving has gone up from 1/3 to 1/2, as it is now between him and C. Prisoner A secretly tells C the news, who is also pleased, because he reasons that A still has a chance of 1/3 to be the pardoned one, but his chance has gone up to 2/3. What is the correct answer?

Prisoner's dilemma [5]

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a Faustian bargain. Each prisoner is given the opportunity either to betray the other, by testifying that the other committed the crime, or to cooperate with the other by remaining silent. Here's how it goes:

- If A and B each betray the other, each of them serves 5 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 20 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

It is implied that the prisoners will have no opportunity to reward or punish their partner other than the prison sentences they get, and that their decision will not affect their reputation in the future.

Bonus problems

Two statisticians in the woods [1]

If two statisticians were to lose each other in an infinite forest, the first thing they would do is get drunk. That way, they would walk more or less randomly, which would give them the best chance of finding each other. However, the statisticians should stay sober if they want to pick mushrooms. Stumbling around drunk and

without purpose would reduce the area of exploration, and make it more likely that the seekers would return to the same spot, where the mushrooms are already gone.

Seven bridges of Kronburg [6]

The problem was to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time; one could not walk halfway onto the bridge and then turn around and later cross the other half from the other side (the walk need NOT start and end at the same spot).

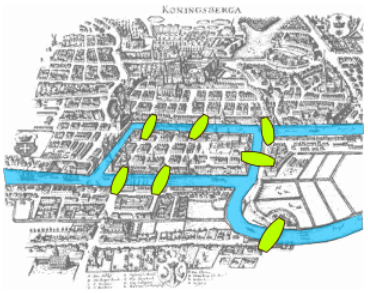

Figure 1: Source: [3]

The King's Wise Men [7]

The King called the three wisest men in the country to his court to decide who would become his new advisor. He placed a hat on each of their heads, such that each wise man could see all of the other hats, but none of them could see their own. Each hat was either white or blue. The king gave his word to the wise men that at least one of them was wearing a blue hat - in other words, there could be one, two, or three blue hats, but not zero. The king also announced that the contest would be fair to all three men. The wise men were also forbidden to speak to each other. The king declared that whichever man stood up first and announced the color of his own hat would become his new advisor. The wise men sat for a very long time before one stood up and correctly announced the answer. What did he say, and how did he work it out?

Presentation: `http://oskopek.com/2015_popmath_gymy/eng_presentation.pdf`
Handout: `http://oskopek.com/2015_popmath_gymy/eng_handouts.pdf`
Solutions: `http://oskopek.com/2015_popmath_gymy/eng_solutions.pdf`

References

[1] Rhett Allain. *How Should Two Lost People Find Each Other?* 2013. URL: `http://www.wired.com/2013/09/two-statisticians-lost-in-the-woods/`.

[2] Wikipedia. *100 prisoners problem*. 2015. URL: `http://en.wikipedia.org/wiki/100_prisoners_problem`.

[3] Wikipedia. *Konigsberg bridges*. 2015. URL: `http://en.wikipedia.org/wiki/File:Konigsberg_bridges.png`.

[4] Wikipedia. *Monty Hall problem*. 2015. URL: `http://en.wikipedia.org/wiki/Monty_Hall_problem`.

[5] Wikipedia. *Prisoner's dilemma*. 2015. URL: `http://en.wikipedia.org/wiki/Prisoner%27s_dilemma`.

[6] Wikipedia. *Seven Bridges of Königsberg*. 2015. URL: `http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg`.

[7] Wikipedia. *The King's Wise Men*. 2015. URL: `http://en.wikipedia.org/wiki/Induction_puzzles#Examples`.

[8] Wikipedia. *Three Prisoners problem*. 2015. URL: `http://en.wikipedia.org/wiki/Three_Prisoners_problem`.