

Faculty of Mathematics and Physics, Charles University

Mathematical problems of prisoners

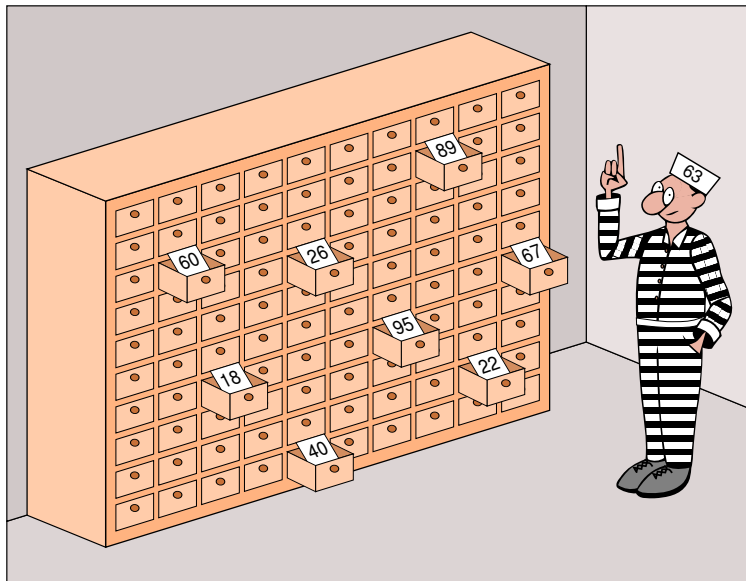
February 8, 2015

Ondrej Škopek

100 prisoners problem [2]

- 100 prisoners, numbered 1-100
- 100 drawers, numbered 1-100
- Every prisoner opens ≤ 50 drawers
- No information exchange allowed during play
- **Everyone** is pardoned, if everyone finds their number
- **No one** is pardoned, if at least one prisoner fails

100 prisoners problem [6]



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Strategy?

- [illegible]

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- Mathematician prisoner: “We each open the drawer with our number and continue to open that drawer, which has the number we found in the previous drawer.”

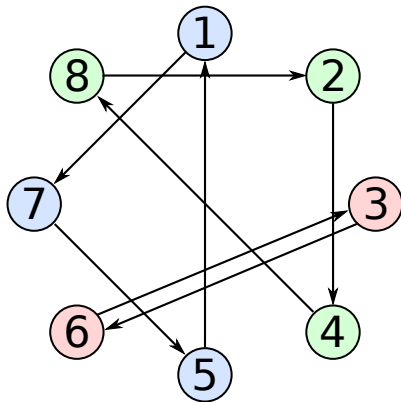
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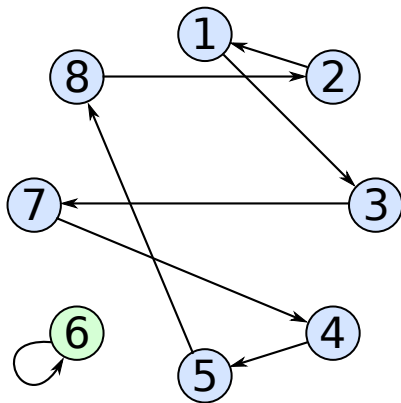
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- Survival probability: > 30% !!!
- But... why?

Example 1 [5]



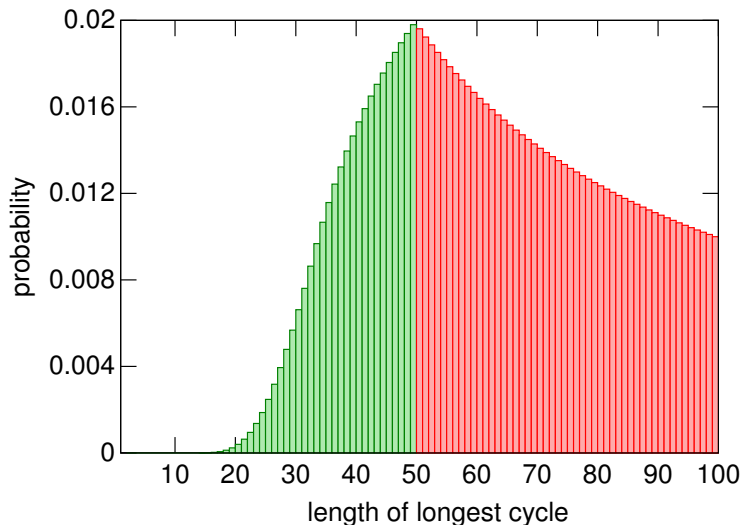
number of drawer	1	2	3	4	5	6	7	8
number of prisoner	7	4	6	8	1	3	5	2

Example 2 [4]



number of drawer	1	2	3	4	5	6	7	8
number of prisoner	3	1	7	5	8	6	4	2

Probability distribution of the length of the longest cycle of a random permutation [9]



Three Prisoners problem [10]

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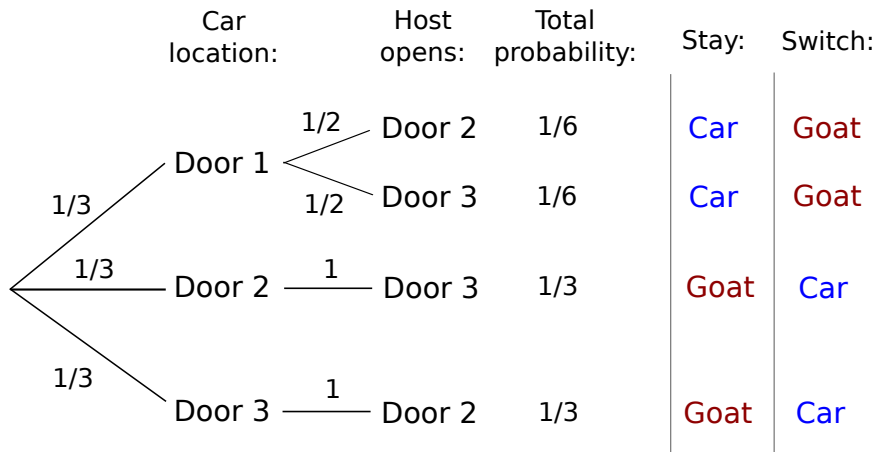
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Monty Hall problem [7]

- Three doors
- Monty knows, behind which door the car is, but can't tell
- Monty tells the player what is behind one of the remaining (not picked) doors
- Which doors are hiding the car with a higher probability?




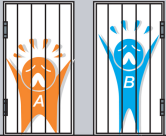


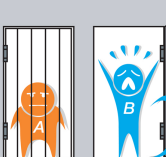
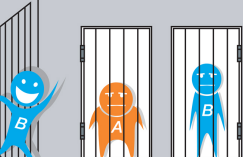
Decision tree [3]



Prisoner's dilemma [8]

- Prisoners can't communicate
- If A and B each betray the other, each of them serves 5 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 20 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)
- How will they (probably) act?

Illustration [1]

Prisoners' dilemma		prisoner B	
		confess 	remain silent 
prisoner A	confess 	 5 years 5 years	 0 year 20 years
	remain silent 	 20 years 0 year	 1 year 1 year

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- Classic “games” (chess, checkers, poker, ...)

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- Programming for Everybody (Python):
<https://www.coursera.org/course/pythonlearn>

Thank you for your attention

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Sources I



Acting Man. *The Prisoner's Dilemma*. 2015. URL: http://www.acting-man.com/blog/media/2014/11/prisoners_dilemma.jpg.



Wikipedia. *100 prisoners problem*. 2015. URL: http://en.wikipedia.org/wiki/100_prisoners_problem.



Wikipedia. *Decision tree for the Monty Hall problem*. 2015. URL: http://en.wikipedia.org/wiki/File:Monty_tree_door1.svg.



Wikipedia. *Graph representation of the permutation (1 3 7 4 5 8 2)(6)*. 2015. URL: http://en.wikipedia.org/wiki/File:Permutation_cycles_qtl2.svg.

Sources II



Wikipedia. *Graph representation of the permutation $(1\ 7\ 5)(2\ 4\ 8)(3\ 6)$* . 2015. URL:
[http://en.wikipedia.org/wiki/File:
Permutation_cycles_qtl1.svg](http://en.wikipedia.org/wiki/File:Permutation_cycles_qtl1.svg).



Wikipedia. *Illustration for the 100 prisoners problem*. 2015. URL: [http://en.wikipedia.org/wiki/File:
100_prisoners_problem_qtl1.svg](http://en.wikipedia.org/wiki/File:100_prisoners_problem_qtl1.svg).



Wikipedia. *Monty Hall problem*. 2015. URL: http://en.wikipedia.org/wiki/Monty_Hall_problem.



Wikipedia. *Prisoner's dilemma*. 2015. URL: http://en.wikipedia.org/wiki/Prisoner%27s_dilemma.

Sources III



Wikipedia. *Probability mass function of the length of the longest cycle of a random permutation of length 100. Cycle lengths ≤ 50 are green, cycle lengths < 50 are red.* 2015. URL: http://en.wikipedia.org/wiki/File:Permutation_longest_cycle_length_pmf_qtl2.svg.



Wikipedia. *Three Prisoners problem.* 2015. URL: http://en.wikipedia.org/wiki/Three_Prisoners_problem.