Advanced Econometrics: Semi-parametric and Simulations

Replication and extension of the paper

Indirect inference for dynamic model, Christian GOURIEROUX et al., 2010.

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Contents

Introduction

It is well known since at least Nickell (1981) that standard estimation methods in the likes of maximum likelihood estimations (MLE) of dynamic panel models with fixed effects yield inconsistent results. Since then, much of the literature has been devoted to the search for consistent estimators; however, as suggested by Gouriéroux et al. (2006), consistency often comes at the cost of a substantial increase in the variance. Hence, the authors suggest using indirect inference methods to ensure both consistency and reduced variance in intractable structural models. This is the main innovation provided by the authors.

In this paper, we reproduce the results of Gouriéroux et al. (2006) and propose a modification to their data-generating process. First, we provide a brief overview of the past literature which motivated the paper we are studying. We then present the theoretical results laid out by the authors. Finally, we reproduce their monte carlo simulations and modify the data-generating process by modifying the variance of the initial condition and we also consider values for the true parameter which are closer to unity.

1 Literature review

The pioneering work of Nickell (1981) provided an expression for the asymptotic bias of the fixed effects maximum likelihood estimator in a dynamic panel model under large N and fixed T asymptotics. The reason for the bias is the endogeneity of the explanatory variable in the de-meaned regression. Applying a first difference transformation yields moment conditions which suggest the use of general methods of moments and instrumental variables (GMM/IV). This approach was developed by Anderson and Hsiao (1981), Anderson and Hsiao (1982), Holtz-Eakin et al. (1988), and Arellano and Bond (1991) which show that GMM/IV estimators yield consistent results under large N and fixed T. However, there are some unwanted properties when dealing with finite samples, especially when the autoregressive coefficient is close to unity which results in substantial bias and large variance. In order to resolve these issues, Han and Phillips (2010b) replace the weak moment conditions that are yielded from the first difference transformation with new moment conditions. The authors show that the resulting GMM/IV estimator is consistent even when the autoregressive parameter is close to unity. Furthermore, Hahn and Kuersteiner (2002a) shows that under large T and large N asymptotics, the resulting MLE is consistent. However, the asymptotic distribution is not centered around the origin and there is a bias in the limiting distribution. Thus, the authors proposed a bias-corrected MLE. The paper shows that such an estimator is consistent and has lower asymptotic variance than the GMM estimator. Thus, the bias-corrected MLE is asymptotically more efficient. However, such an estimator requires an explicit analytical form of the bias function which can be particularly hard to derive. Thus, Gouriéroux et al. (2006) proposes the use of indirect inference to compute the bias function via simulation. In the next section, we present in detail the theoretical results derived by the authors in their use of indirect inference.

2 Theoretical results

The Indirect Inference method was first introduced by Gouriéroux et al. (1993) who developed a two-step framework applicable both to simple cases such as AR(1) processes as well as to dynamic panel models.

2.1 Indirect Inference method in the case of AR(1) process

To get a first intuition of the mechanism at stake, let us take a first example from Gouriéroux et al. (2006) who build on Mariano and Schuermann (2000) by reviewing a simple AR(1) model:

$$y_t = \Phi y_{t-1} + \epsilon_t \tag{1}$$

Where the true value of Φ is Φ_0 which lies in a compact set and where $|\Phi_0| < 1$. In practice, the likelihood of such a model might be computationally intractable; therefore, Gouriéroux et al. (1993) suggest a first step which consists in maximizing a criterion related to an auxiliary model, Suppose that Q_T is the objective function of a certain estimation method related to an auxiliary model indexed by the parameter θ; then, θ might be recovered from observed data by the following program:

$$\hat{\theta}_T = \arg\max_{\theta} \ Q_T(y) \tag{2}$$

Where $y = y_0, y_1, ..., y_T$ is the vector of observations, which, according to the Data Generating Process (DGP) also depends on the parameter Φ to be estimated. This is where the second step of the reasoning occurs: it basically consists in simulating H paths for the vector of potential outcomes y and maximize the following program for each path h:

$$\tilde{\theta_T^h}(\Phi) = \arg\max_{\theta} \ Q_T(\tilde{y^h}(\Phi)) \tag{3}$$

The Indirect Inference estimator $\hat{\Phi_T^{II}}$ is then defined by:

$$\hat{\Phi_T^{II}} = \arg\min_{\Phi} ||\hat{\theta}_t - E(\tilde{\theta_T^h}(\Phi))|| \tag{4}$$

In words, the value of Φ_T^{II} that minimizes the distance between the estimate of θ_T recovered through the auxiliary model from the observed data and the average value of $\tilde{\theta_T^h}(\Phi)$ computed given the H simulated paths of the true model is a consistent estimator of Φ.

2.2 Estimating panel models via indirect inference

The previously described procedure can be applied, with some modifications, to the context of dynamic panel models, where the model to be studied is as follows:

$$y_{it} = \alpha_i + \Phi y_{it-1} + \epsilon_{it} \tag{5}$$

Where $\epsilon_{it} \sim iidN(0, \sigma^2), i = 1, ..., N, t = 1, ..., T$, the true value of ϕ is $\phi_0 \in \Phi$, with Φ a compact set in the stable region, and $|\phi_0| < 1$. Additionally, the initial condition is set to be:

$$y_{i0} = \frac{\alpha_i}{1-\phi} + \frac{\epsilon_{i0}}{\sqrt{1-\phi^2}} \tag{6}$$

In that context, Gouriéroux et al. (2006) suggest defining the so-called binding function as:

$$b_{NT}(\phi) = E(\phi_{NT}^{\tilde{h,ML}}) \tag{7}$$

Then, the Indirect Inference estimator can be defined by:

$$\Phi_{NT}^{\hat{II}} = \arg\min_{\Phi} ||\phi_{NT}^{\hat{ML}} - b_{NT}(\phi)|| \tag{8}$$

In practice, b_{NT} is replaced by the average value of the estimator recovered through the simulations, namely, $\frac{1}{H}\sum_{h=1}^{H}\phi_{NT}^{\tilde{h,ML}}$ where H captures the number of simulated paths. Assuming that the binding function $b_{NT}(.)$ is uniformly continuous and one to one, authors posit the following first theorem:

Theorem 1 *Under the previous assumption, the indirect inference estimator is "b_{NT}-mean-unbiased"; that is:* $b_{NT}^{-1}(E(b_{NT}(\phi_{NT}^{\hat{ML}}))) = \phi_0$

Interestingly, this property of "$b_{NT}-$unbiasedness" does not impose any restrictions on N and T, suggesting that large T, or large N are not necessarily required to provide a consistent estimate of the true parameter. It is also noteworthy that this property does not imply mean-unbiasedness: only in the case where $b_{NT}(\phi)$ is a linear function can we expect $\phi_{NT}^{\hat{II}}$ to be close to mean-unbiasedness. However, further assumptions are required to study the asymptotic behavior of $\phi_{NT}^{\hat{II}}$.

Assuming that (i) $|\phi_0| < 1$,(ii) $N \to \infty$, (iii) $T \xrightarrow{\infty}$, (iv) $0 < lim(N/T) \equiv c < \infty$, (v) $\frac{1}{N}\sum_{i=1}^{N}|\alpha_i|^2 = O(1)$, they posit the second theorem:

Theorem 2 *Under the two above mentioned assumptions, it can be proved that:* $\sqrt{NT}(\phi_{NT}^{\hat{II}} - \phi_0) \Rightarrow N(0, (1-\phi_0^2))$

This implies that the asymptotic distribution of the Indirect Inference estimator is identical to that of Hahn and Kuersteiner (2002a); this is of significant interest as the asymptotic variance of this estimator then achieves a lower bound for regulator estimators: the Indirect Inference estimator is then asymptotically efficient. Additionally, Gouriéroux et al. (2006) observe that the Indirect Inference estimator should inherit the "efficiency" properties of the initial estimator; indeed, the lack of efficiency of any unbiased estimator $\hat{\phi}$ of ϕ might be measured by $\frac{Var(\hat{\phi}}{I^{\phi\phi}}$ where $I^{\phi\phi}$ denotes the element of the inverse of the information matrix corresponding to the parameter ϕ. The lack of efficiency of the indirect inference estimator $\Phi_{NT}^{\hat{II}}$ then writes:

$$Var(\Phi_{NT}^{\hat{II}}) \sim \left(\frac{\partial b_T(\phi_0)}{\partial \phi}\right)^{-2} \frac{Var(\Phi_{NT}^{\hat{ML}})}{I^{\phi\phi}} \tag{9}$$

As the ML estimator has good properties in terms of dispersion, it can be expected that the Indirect Inference estimator will also enjoy a small variance and hence achieve a lower bound among the class of regular estimators.

2.3 Expected advantages of the Indirect Inference estimator

In a nutshell, Gouriéroux et al. (2006) claim this procedure has three main advantages. First of all, it does not impose a structural form on the bias function unlike many other specifications: indeed, the bias is calibrated via simulation and hence, does not require an explicit form. Given that the sample size is two-dimensional in dynamic panel models, such explicit forms can prove very complex, and avoiding to posit them is crucial. Second, this approach might be used with many different estimation methods, and, in turn, benefits from some of the nice properties of the initial estimators as discussed in the previous subsection. Finally, in spite of the fact that Indirect Inference requires simulation, it is computationally inexpensive as long as the initial estimator is a ML. Indeed, since the former usually exhibits a small variance, a small number of paths should be sufficient to ensure an adequate calibration of the bias function.

3 Replication and Extension

3.1 Monte Carlo Simulation

Herein, we provide the design of the different data generator processes used for the Monte Carlo simulation and comment the results obtain by comparing the results of the indirect inference method proposed by the authors to that of the MLE and that of the GMM.

3.1.1 Data Generator Processes

In this simulation part, we have defined, following the authors, three types of DGPs. The three processes are generated according to a linear dynamic panel model with some particularities for each of them.

DGP1 (a): simple linear dynamic panel model

Following Hahn and Kuersteiner (2002b), we generate data from the following linear dynamic panel model:

$$y_{it} = \alpha_i + \Phi_0 y_{it-1} + \epsilon_{it} \tag{10}$$

Where $\epsilon_{it} \sim iidN(0,1), \alpha_i \sim iidN(0,1), \Phi_0 = 0, 0.3, 0.6, 0.9, 0.99$ and α_i and ϵ_{it} are assumed to be independently distributed. Also, the initial condition writes:

$$y_{i0|\alpha_i} \sim N\left(\frac{\alpha_i}{1-\Phi_0}, \frac{1}{1-\Phi_0^2}\right) \tag{11}$$

For robustness check, we change the variance by considering a DGP with variance = $\sqrt{1-\Phi_0^2}$. This DGP is refered as DGP1(b) and the results are given in Table 1 and Table 2.

DGP2: linear dynamic panel model with trend

The above data generator process is slightly modified by adding a trend into the linear dynamic panel model, holding the same assumptions on the initial generated values.

$$y_{it} = \alpha_i + \beta_i t + \Phi_0 y_{it-1} + \epsilon_{it} \tag{12}$$

, where $\beta_i \sim iid\mathcal{N}(0,1)$.

DGP3: linear dynamic panel model with covariates

We considered including a covariate with an assumed normal distribution. However, due to slow compilation, we do not give results related to this DGP, but the code is attached to this project. The main objective was to verify how the methods perform when the DGP is slightly modified and to verify the robustness of the methods to the process that generates the data.

3.1.2 Methodological considerations

For each data generation process, we simulated data with N observations, T periods and such for each value of Φ_0. Overall, for each combination of $N = 100, 200$ and $T = 5, 10, 20$, we employ three methods to estimate Φ_0: ML, GMM, and the Indirect inference, for $H = 10$ and then for $H = 50, 250$, as proposed by the authors. To compare the performances of the methods, and especially the performance of the Indirect Inference to the MLE and the GMM methods, we focus, as done by the authors, on the bias and the Root Squared Mean Error which are calculated after 1000 simulations. The following section presents the results and examines the performance of the Indirect Inference as T, N, or Φ increase.

3.1.3 Results of simulations

General results

In line with the Monte Carlo simulations ran by the authors, our estimates point to four main observations: first of all, both the MLE and the GMM exhibit substantial biases as well as relatively large RMSEs, though lower for the RMSE. Second, our results confirm that these issues are peculiarly relevant for the higher values of ϕ namely when it is set close to unity: this suggests that the GMM and the MLE estimators are not robust to unit root issues. Third, we obtain that the Indirect Inference estimator overcomes most of these flaws, as it exhibits substantially lower biases and RMSEs, whatever the values of T, ϕ and N. Finally, we recover an interesting property of the II estimator, namely that its RMSEs are not systematically increasing in ϕ unlike the other estimators.

Regarding the first aspect mentioned, the results of the first DGP summarized in Table 1 are striking. Indeed, both biases and RMSEs can prove very large, especially for the GMM estimator for which the bias is as large as four times the value of the true parameter when N=100, T=5 and the value of the true parameter is set to 0.99. Increasing T proves inadequate to reduce this bias as well as the RMSE of the GMM estimator. However, the ML estimates are significantly improving when T increases both in terms of bias and RMSE reduction. Indeed, the bias of the ML estimator when ϕ is set to 0.99 and N to 100 drops by close to 78% when T = 20 compared with T = 5. This is consistent with the 74% decrease found by Gouriéroux et al. (2006).

Additionally, it is noteworthy that the bias and the RMSEs are substantially higher when ϕ gets closer to 1.Indeed, the bias in the GMM estimator is more than seven times as large when ϕ is set to 0.99 as when ϕ equals 0 for the case where T = 5. Correspondingly, the RMSE of the GMM is more than three times as large in the case where ϕ is set to 0.99 as in the case where ϕ is worth 0. This statement would be true whatever the values retained for T and N. Such an observation is consistent with the prediction by Gouriéroux et al. (2006) who also identify a unit root issue with the GMM estimation.

In turn, it appears that the Indirect Inference (II) estimator corrects most of the flaws observed for the GMM and the ML estimators. In particular, the II estimator seems robust to the unit root issue, as the values reported prove peculiarly convincing compared with the two other estimators, even when allowing for a modified variance as a robustness check (*see Table 1 and Table 2*). Compared with a -3.966 bias for the GMM and a -2.206 bias for the ML, the II estimator indeed shows a significantly lower 0.72 bias when the true parameter is set close to unity, at 0.99 with N=100 and T = 5. Apart from this unit root issue, the II estimator works well in most of the considered cases: only in the robust framework, when T is set to 20, can the ML estimator outperform the II method. This, however, occurs at the expense of a higher variance for the ML.

Finally, our simulations point to an interesting property of the II estimator, already mentioned by Gouriéroux et al. (2006), namely that its RMSEs are not necessarily increasing in ϕ unlike what is to be seen with the GMM and ML estimators. In particular, the RMSE of the II estimator in the robust case when T=10 and N=100 is lower if the true parameter is set to 0.99 than if the true parameter is set to 0.9 or 0. This is reminiscent of the fact that Gouriéroux et al. (2006) find that the RMSE of the indirect inference estimates is respectively 85.5% and 82.9% smaller than that of the two other estimates.

Table 1: Results with DGP1(b) with N=100

Case		**Bias**			**RMSE**		
T	Φ	II	GMM	ML	II	GMM	ML
5	0	-0.002	-0.530	-0.249	0.105	0.533	0.254
5	0.3	0.281	-1.206	-0.593	0.110	0.862	0.431
5	0.6	0.553	-2.020	-1.042	0.100	1.187	0.625
5	0.9	0.734	-2.971	-1.606	0.143	1.522	0.844
5	0.99	0.720	-3.966	-2.206	0.142	1.819	1.037
10	0	-0.010	-0.526	-0.110	0.163	0.526	0.115
10	0.3	0.358	-1.203	-0.260	0.094	0.859	0.192
10	0.6	0.515	-2.020	-0.460	0.033	1.186	0.279
10	0.9	0.648	-2.967	-0.731	0.146	1.518	0.391
10	0.99	0.831	-3.957	-1.029	0.104	1.813	0.492
20	0	0.051	-0.506	-0.052	0.101	0.507	0.057
20	0.3	0.384	-1.164	-0.124	0.126	0.831	0.094
20	0.6	0.603	-1.969	-0.215	0.088	1.157	0.133
20	0.9	0.727	-2.900	-0.342	0.124	1.486	0.185
20	0.99	0.707	-3.872	-0.491	0.190	1.776	0.238

Note: This table reports the results of our simulations for the three methods when the data are generated from the DGP1b. The number of simulation is set to 500, N to 100 and T varying from in $\{5, 10, 20\}$, $\Phi \in \{0, 0.3, 0.6, 0.9, 0.99\}$

Table 2: Results with DGP1(b) with N=200

Case		**Bias**			**RMSE**		
T	Φ	II	GMM	ML	II	GMM	ML
5	0	-0.060	-0.532	-0.247	0.085	0.533	0.250
5	0.3	0.298	-1.209	-0.592	0.096	0.862	0.427
5	0.6	0.554	-2.023	-1.040	0.069	1.187	0.620
5	0.9	0.736	-2.975	-1.599	0.149	1.522	0.836
5	0.99	0.752	-3.971	-2.198	0.138	1.819	1.029
10	0	-0.049	-0.531	-0.111	0.132	0.531	0.114
10	0.3	0.274	-1.215	-0.261	0.056	0.867	0.190
10	0.6	0.573	-2.038	-0.460	0.109	1.195	0.276
10	0.9	0.535	-2.987	-0.731	0.063	1.526	0.387
10	0.99	0.788	-3.979	-1.029	0.129	1.821	0.489
20	0	0.031	-0.516	-0.053	0.098	0.516	0.056
20	0.3	0.297	-1.185	-0.123	0.074	0.845	0.091
20	0.6	0.574	-2.004	-0.214	0.092	1.177	0.129
20	0.9	0.728	-2.950	-0.341	0.159	1.510	0.181
20	0.99	0.764	-3.939	-0.488	0.129	1.805	0.234

Note: This table reports the results of our simulations for the three methods when the data are generated from the DGP1b. The number of simulation is set to 500, N to 200 and T varying from in $\{5, 10, 20\}$, $\Phi \in \{0, 0.3, 0.6, 0.9, 0.99\}$

Extension and discussions

As mentioned earlier in this paper, the novelty of our approach was first to allow for a ”robust” Data Generating Process (DGP) through which we modify the variance of the initial condition and second to study the behavior of all estimators for a new value of the true parameter which is set even closer to 1 as those studies by Gouriéroux et al. (2006). We discuss briefly in this section the influence of these novelties and the sensibilities of the estimators to those new features.

First, it clearly stands out that the DGP is not excessively sensitive to the alteration of the variance in the initial condition. As Gouriéroux et al. (2006), we find that the II estimator always outperforms the ML estimator both in terms of bias reduction and in terms of RMSEs. While the MLE exhibits biases and RMSEs close to that of the DGP1(b) process , the II estimator proves even more reliable as suggested by the systematic decrease evidenced in Table 3, regardless the value of ϕ or T.

Of interest is also the fact that the II remains robust to the unit root issue in our modified set up, while the ML estimator still exhibits high biases when ϕ gets close to one. Typically, the bias of the ML reaches -1.860 in the robust framework for T=5 and N=200 while it only attains -0.155 for the II estimator, indicating that the latter performs substantially better in terms of bias reduction. In a similar spirit, the RMSE of the ML is definitely not robust to the unit root issue, even when allowing for a modified initial condition, as evidenced by a significant 0.850 RMSE in comparison with the 0.133 value for the II estimator, when, again, T=5 and N=200. This outstanding performance of the II in terms of dispersion properties might be explained by the fact that, as Gouriéroux et al. (2006) put it, the II inherits some of the nice properties of its baseline estimator, including the relatively good dispersion characteristics of the ML estimator.

Table 3: Results of simulation with the DGPa for the II and its used baseline estimator MLE

Case		N=100				N=200			
		Bias		**RMSE**		**Bias**		**RMSE**	
T	Φ	II	ML	II	ML	II	ML	II	ML
5	0	0.070	-0.248	0.069	0.253	-0.034	-0.250	0.127	0.252
5	0.3	0.045	-0.591	0.114	0.429	-0.035	-0.592	0.062	0.426
5	0.6	0.067	-1.021	0.086	0.610	-0.001	-1.021	0.058	0.606
5	0.9	-0.040	-1.456	0.084	0.751	-0.016	-1.457	0.043	0.748
5	0.99	-0.127	-1.860	0.071	0.855	-0.155	-1.860	0.133	0.850
10	0	0.186	-0.110	0.067	0.115	0.235	-0.110	0.034	0.112
10	0.3	0.283	-0.259	0.050	0.191	0.252	-0.260	0.024	0.189
10	0.6	0.204	-0.455	0.060	0.275	0.160	-0.454	0.135	0.272
10	0.9	-0.149	-0.641	0.163	0.333	0.014	-0.640	0.058	0.330
10	0.99	-0.142	-0.788	0.074	0.365	-0.142	-0.787	0.069	0.362
20	0	0.268	-0.052	0.042	0.056	0.248	-0.053	0.032	0.056
20	0.3	0.340	-0.121	0.124	0.092	0.312	-0.123	0.085	0.091
20	0.6	0.240	-0.211	0.057	0.131	0.289	-0.212	0.031	0.128
20	0.9	-0.067	-0.296	0.173	0.156	0.022	-0.295	0.040	0.153
20	0.99	-0.182	-0.345	0.180	0.164	-0.125	-0.344	0.096	0.161

Note: This table reports the results of our simulations for the three methods when the data are generated from the DGP1a. The number of simulation is set to 500, N to 200 and T varying from in {5, 10, 20}, $\Phi \in \{0, 0.3, 0.6, 0.9, 0.99\}$. The table reports the results of the estimations focusing on the Indirect Inference (II) estimator and its baseline estimator used in the maximization step (here the MLE). This is to compare the performance of the II and the MLE performances'.

Conclusion

Biases and efficiency losses in the estimation of the parameters of dynamic panel models can prove significantly large when relying on traditional methods such as the GMM or the ML estimators. In particular, the estimates can suffer severe issues when the true value of the parameter is set close to 1. This, in turn, calls for the development of new estimation methods such as that of Gouriéroux et al. (2006). Following the framework they developed in their 2006 paper, we replicate their results and extend them by modifying the initial condition and by adding one parameter value close to unity. We then check the sensitivity of their main results to the introduction of these assumptions.

Overall, our results confirm the substantial biases encountered by traditional methods, especially when the true parameter nears unity. Also, we establish that the II estimator, most of the time, corrects these flaws. Namely, we bring support to the idea that such an estimator proves peculiarly convenient when the true value of the parameter is set arbitrarily close to 1. However, it is noteworthy that the II is quite sensitive to the definition of the initial condition: building on that introduced by Hahn and Kuersteiner (2002a) we namely prove that the II estimator might sometimes be beaten by the ML estimator, in particular when it comes to RMSE reduction. This, nevertheless, is not that surprising considering the good dispersion properties of the ML which the II will, in turn inherit, as suggested by Gouriéroux et al. (2006)

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